SHOW ALL WORK AND WRITE ALL ANSWERS IN THE SPACES PROVIDED.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Consider the arithmetic sequence -7, 8, 23, ...

(a) Find an expression for the n^{th} term. **(1)**

$$u_n = u_1 + (n-1)d = \boxed{-7 + (n-1)(15)}$$

or $\boxed{u_n = 15n - 22}$

(b) Write down the sum of the first n terms using sigma notation. **(1)**

$$\sum_{i=1}^{n} (15i - 22)$$

(c) Calculate the sum of the first 19 terms. **(2)**

(Total 4 marks)

<u>option 1</u>

$$S_{19} = \frac{19}{2}\left[2(-7) + 18(15)\right]$$
$$= 2432$$

<u>option 2</u> Simply use Sigma command on calc.

$$S_{19} = 2432$$

2. In the arithmetic series with n^{th} term u_n, it is given that $u_4 = 7$ and $u_9 = -23$. Find the minimum value of n so that $u_1 + u_2 + u_3 + \ldots + u_n < -10\,000$.

(Total 5 marks)

$u_4 = 7$ so $u_1 + 3d = 7$

$u_9 = -23$ so $u_1 + 8d = -23$

$\therefore$ $5d = -30$

$d = -6$

then $u_1 + 3(-6) = 7$

$u_1 = 25$

so finally

$S_n = \frac{n}{2}(2u_1 + (n-1)d)$

$-10{,}000 > \frac{n}{2}(50 + (n-1)(-6))$

$-10{,}000 > \frac{n}{2}(-6n + 56)$

$-20{,}000 > -6n^2 + 56n$

$0 > -6n^2 + 56n + 20{,}000$

so look for zeroes of this parabola

$n = 62.6$

so

$n = 63$

3. A geometric sequence has consecutive terms of 4, k, and $k^2 - 1$.

(Total 5 marks)

(a) Determine the value(s) for k.

(3)

$r = \frac{K}{4}$ but $r = \frac{k^2-1}{K}$

So $\frac{K}{4} = \frac{K^2-1}{K}$

So $K^2 = 4K^2 - 4$

$0 = 3k^2 - 4$

$4 = 3k^2$

$\frac{\pm 2}{\sqrt{3}} = K$

(b) These three terms are part of geometric series. Would the series be a converging series or a diverging series? Explain your reasoning.

(2)

if $r = \frac{K}{4}$, then $r = \frac{\frac{\pm 2}{\sqrt{3}}}{4}$

So $r = \frac{\pm 1}{\sqrt{3}}$, meaning that

$|r| < 1$. So our series would

converge

4. A geometric sequence $u_1, u_2, u_3, \ldots$ has $u_1 = 27$ and a sum to infinity of $\frac{81}{2}$.

(a) Find the common ratio of the geometric sequence. **(2)**

$$S_\infty = \frac{81}{2} = \frac{27}{1-r}$$

$$\frac{81(1-r)}{81} = \frac{54}{81}$$

$1 - r = \frac{2}{3}$

so $r = \frac{1}{3}$

An arithmetic sequence $v_1, v_2, v_3, \ldots$ is such that $v_2 = u_2$ and $v_4 = u_4$.

(b) Find the greatest value of N such that $\sum_{n=1}^{N} v_n > 0$.

(5)
(Total 7 marks)

so in the geometric series, find u_2 and u_4

$u_2 = u_1 \cdot r = 27\left(\frac{1}{3}\right) = 9$

$u_4 = u_1 r^3 = 27\left(\frac{1}{3}\right)^3 = 1$

$\therefore$ in arithmetic series, $v_2 = 9$ and $v_4 = 1$

thus $2d = 1 - 9 \Rightarrow$ so $\underline{d = -4}$ and $\underline{v_1 = 13}$

So $\sum_{n=1}^{N}$ $13 + 9 + 5 + 1 + {}^-3 + {}^-7 + {}^-11 + {}^-15$ etc.

if $N = 6 \Rightarrow S_6 = 18$

if $N = 7 \Rightarrow S_7 = 7$

use guess/check

if $N = 8 \Rightarrow S_8 = -8$

formula $\Rightarrow$ $0 < \frac{n}{2}\left[2 \cdot 13 + (n-1)(-4)\right]$

$0 < n(15 - 2n) \Rightarrow$ so $n = 7\frac{1}{2} \Rightarrow n = 7$

DO ALL WORK ON A SEPARATE SHEET AND ALL GRAPHS ON IB GRAPH PAPER.

*Note: **A correct answer with no** indication of the method used will normally receive **no** marks. You are therefore advised to show your working.*

5. (a) The sum of the first six terms of an arithmetic series is 81. The sum of its first eleven terms is 231. Find the first term and the common difference.

(6)

$S_6 = 81 = \frac{6}{2}[2u_1 + 5d] \Rightarrow 81 = 6u_1 + 15d$ ①

$S_{11} = 231 = \frac{11}{2}[2u_1 + 10d] \Rightarrow \underbrace{231 = 11u_1 + 55d}_{\text{simplify}}$ ②

$21 = u_1 + 5d$

$\therefore u_1 = 21 - 5d$

$81 = 6(21 - 5d) + 15d$

$81 = 126 - 30d + 15d$

$-45 = -15d$

$\boxed{3 = d}$ $\boxed{u_1 = 21 - 5(3) = 6}$

(b) The sum of the first two terms of a geometric series is 1 and the sum of its first four terms is 5. If all of its terms are positive, find the first term and the common ratio.

(5)

$S_2 = 1 = \frac{u_1(1-r^2)}{1-r} \Rightarrow 1 - r = u_1(1-r^2)$

$\cancel{1-r} = u_1\cancel{(1-r)}(1+r)$

$\boxed{\frac{1}{1+r} = u_1}$

$S_4 = 5 = \frac{u_1(1-r^4)}{1-r}$

$5 = \frac{1}{(1+r)(1-r)}(1-r^2)(1+r^2)$

$5 = \frac{\cancel{(1-r)}\cancel{(1+r)}(1+r^2)}{\cancel{(1+r)}\cancel{(1-r)}}$

$5 = 1 + r^2$

$\therefore r = \pm 2 \Rightarrow$ but $r > 0$ $\boxed{\text{So } r = 2}$

$\frac{1}{1+2} = u_1$

$\boxed{\frac{1}{3} = u_1}$

(c) The r^{th} term of a new series is defined as the product of the r^{th} term of the arithmetic series and the r^{th} term of the geometric series above. Show that the r^{th} term of this new series is $(r + 1)2^{r-1}$.

(3)

(Total 14 marks)

for the AS $\Rightarrow$ A/S $= 6, 9, 12, \cdots (6 + (n-1)3)$

$\boxed{u_n = 3n + 3}$

for the G/S $\Rightarrow \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3} \cdots \frac{1}{3}(2)^{n-1}$

$u_n = \frac{1}{3}(2)^{n-1}$

Product then is $(3n+3)\left(\frac{1}{3}\right)2^{n-1}$

Product $= \cancel{3}(n+1)\left(\frac{1}{\cancel{3}}\right)2^{n-1}$

Product $= (n+1)2^{(n-1)}$