

- 3 The temperature T of a liquid which has been placed in a refrigerator is given by $T = 4 + 96 \times e^{-0.03t}$ °C, where t is the time in minutes. Find the time required for the temperature to reach: **a** 25°C **b** 5°C.
- 4 The weight W_t of radioactive substance remaining after t years is given by $W_t = 1000 \times 2^{-0.04t}$ grams. Find the time taken for the weight to:
a halve **b** reach 20 grams **c** reach 1% of its original value.
- 5 The weight $W(t)$ of radioactive uranium remaining after t years is given by the formula $W(t) = W_0 \times 2^{-0.0002t}$ grams, $t \geq 0$. Find the time taken for the original weight to fall to: **a** 25% of its original value **b** 0.1% of its original value.
- 6 The current I flowing in a transistor radio t seconds after it is switched off is given by $I = I_0 \times 2^{-0.02t}$ amps. Find the time taken for the current to drop to 10% of its original value.
- 7 A parachutist jumps from the basket of a stationary hot air balloon. His speed of descent is given by $V = 50(1 - 2^{-0.2t})$ ms^{-1} where t is the time in seconds. Find the time taken for his speed to reach 40 ms^{-1} .

REVIEW SET 4A

- 1 Find the following *without* using a calculator. Show all working.
a $\log_4 64$ **b** $\log_2 256$ **c** $\log_2(0.25)$ **d** $\log_{25} 5$ **e** $\log_8 1$
f $\log_6 6$ **g** $\log_{81} 3$ **h** $\log_9(0.\bar{1})$ **i** $\log_{27} 3$ **j** $\log_k \sqrt{k}$
- 2 Without using a calculator, find: **a** $\log \sqrt{10}$ **b** $\log \frac{1}{\sqrt[3]{10}}$ **c** $\log(10^a \times 10^{b+1})$
- 3 Find x if: **a** $\log_2 x = -3$ **b** $\log_5 x \approx 2.743$ **c** $\log_3 x \approx -3.145$
- 4 Write as logarithmic equations: **a** $P = 3 \times b^x$ **b** $m = \frac{n^3}{p^2}$
- 5 Write the following equations without logarithms:
a $\log_2 k \approx 1.699 + x$ **b** $\log_a Q = 3 \log_a P + \log_a R$
c $\log A \approx 5 \log B - 2.602$
- 6 Solve for x , giving your answer correct to 4 significant figures:
a $5^x = 7$ **b** $20 \times 2^{2x+1} = 500$
- 7 The weight of radioactive substance after t years is $W_t = 2500 \times 3^{-\frac{t}{3000}}$ grams.
a Find the initial weight.
b Find the time taken for the weight to reduce to 30% of its original value.
c Find the percentage weight loss after 1500 years.
d Sketch the graph of W_t against t .
- 8 Solve for x : $16^x - 5 \times 8^x = 0$
- 9 Solve the equation $\log_3(10x^2 - x - 2) = 2 + 2 \log_3 x$
- 10 Find the *exact* value of a which satisfies the equation $5^{3a} \times 4^{2a+1} = 10^{3a+2}$
Give your answer in the form $\frac{\ln x}{\ln y}$ where $x, y \in \mathbb{Z}$.

REVIEW SET 4B

- 1 Without using a calculator, find the base 10 logarithms of: a $\sqrt{1000}$ b $\frac{10}{\sqrt[3]{10}}$ c $\frac{10^a}{10^{-b}}$
- 2 Solve for x :
 a $\log x = 3$ b $\log_3(x+2) = 1.732$ c $\log_2\left(\frac{x}{10}\right) = -0.671$
- 3 Write as a single logarithm:
 a $\log 16 + 2\log 3$ b $\log_2 16 - 2\log_2 3$ c $2 + \log_4 5$
- 4 Write the following equations without logarithms:
 a $\log T = 2\log x - \log y$ b $\log_2 K = \log_2 n + \frac{1}{2}\log_2 t$
- 5 Solve for x : a $3^x = 300$ b $30 \times 5^{1-x} = 0.15$ c $3^{x+2} = 2^{1-x}$
- 6 If $A = \log_2 2$ and $B = \log_2 3$, write the following in terms of A and B :
 a $\log_2 36$ b $\log_2 54$ c $\log_2(8\sqrt{3})$ d $\log_2(20.25)$ e $\log_2(0.8)$
- 7 For the function $g: x \mapsto \log_3(x+2) - 2$:
 a Find the domain and range.
 b Find any asymptotes and axes intercepts for the graph of the function.
 c Sketch the graph of $y = g(x)$.
 d Find g^{-1} . Explain how to verify your answer for g^{-1} .
 e Sketch the graphs of g , g^{-1} and $y = x$ on the same axes.
- 8 Solve exactly for a in the equation $\log_4 a^5 + \log_2 a^{\frac{3}{2}} = \log_8 625$.
- 9 A straight line has equation $y = mx + c$. Its gradient is -2 and it passes through the point $(1, \log_5 \frac{3}{25})$.
 a Find the equation of the line.
 b If $y = \log_5 M$, find an expression for M in terms of x .
 c Hence, find the value of x when $M = 25$.
- 10 Solve simultaneously for x and y : $4^x \times 2^y = 16$ and $8^x = 2^{\frac{y}{2}}$.
- 11 Solve $\log_8 \sqrt[4]{x^2 + 7} = \frac{1}{3}$

REVIEW SET 4C

- 1 a On the same set of axes sketch and clearly label graphs of:
 $f: x \mapsto e^x$, $g: x \mapsto e^{-x}$ and $h: x \mapsto -e^{-x}$.
 b What is the geometric connection between: i f and g ii g and h ?
- 2 Sketch on the same set of axes the graphs of $y = e^x$ and $y = 3e^x$.
- 3 A particle moves in a straight line such that its displacement from the origin O is given by $s(t) = 120t - 40e^{-\frac{t}{5}}$ metres, where t is the time in seconds, $t \geq 0$.
 a Find the position of the particle at i $t = 0$ ii $t = 5$ iii $t = 20$.
 b Hence sketch the graph of $s(t) = 120t - 40e^{-\frac{t}{5}}$ for $t \geq 0$.

- 4 Without using a calculator, find: **a** $\ln(e^5)$ **b** $\ln(\sqrt{e})$ **c** $\ln\left(\frac{1}{e}\right)$
- 5 Simplify: **a** $\ln(e^{2x})$ **b** $\ln(e^2 e^x)$ **c** $\ln\left(\frac{e}{e^x}\right)$
- 6 Write as a single logarithm:
a $\ln 6 + \ln 4$ **b** $\ln 60 - \ln 20$ **c** $\ln 4 + \ln 1$ **d** $\ln 200 - \ln 8 + \ln 5$
- 7 Write in the form $a \ln k$ where a and k are positive whole numbers and k is prime:
a $\ln 32$ **b** $\ln 125$ **c** $\ln 729$
- 8 Solve for x , giving answers correct to 3 significant figures:
a $e^x = 400$ **b** $e^{2x+1} = 11$ **c** $25e^{\frac{x}{2}} = 750$ **d** $e^{2x} = 7e^x - 12$
- 9 Solve $12(2^x) = 7 + \frac{10}{2^x}$ giving your answer in the form $m + \log_2 n$, $m, n \in \mathbb{Z}$.

REVIEW SET 4D

- 1 On the same set of axes, sketch and clearly label the graphs of:
 $f: x \mapsto e^x$, $g: x \mapsto e^{-x}$, $h: x \mapsto e^{-x} - 4$.
State the domain and range of each function.
- 2 Sketch on the same set of axes, the graphs of $y = e^x$ and $y = e^{3x}$.
- 3 Without using a calculator, find:
a $\ln(e\sqrt{e})$ **b** $\ln\left(\frac{1}{e^3}\right)$ **c** $\ln\left(\frac{e}{\sqrt{e^5}}\right)$
- 4 Write in the form e^x : **a** 20 **b** 3000 **c** 0.075
- 5 Simplify:
a $4 \ln 2 + 2 \ln 3$ **b** $\frac{1}{2} \ln 9 - \ln 2$ **c** $2 \ln 5 - 1$ **d** $\frac{1}{4} \ln 81$
- 6 Write the following equations without logarithms:
a $\ln P = 1.5 \ln Q + \ln T$ **b** $\ln M = 1.2 - 0.5 \ln N$
- 7 Consider $g: x \mapsto 2e^x - 5$.
a Find the defining equation of g^{-1} .
b Sketch the graphs of g and g^{-1} on the same set of axes.
c State the domain and range of g and g^{-1} .
- 8 The weight W_t grams of radioactive substance remaining after t weeks is given by $W_t = 8000 \times e^{-\frac{t}{20}}$ grams. Find the time for the weight to:
a halve **b** reach 1000 g **c** reach 0.1% of its original value.
- 9 The function f is defined for $x > 4$ by $f(x) = \ln(x^2 - 16) - \ln x - \ln(x - 4)$
a Express $f(x)$ in the form $\ln\left(\frac{x+a}{x}\right)$, stating the value of $a \in \mathbb{Z}$.
b Find an expression for $f^{-1}(x)$.