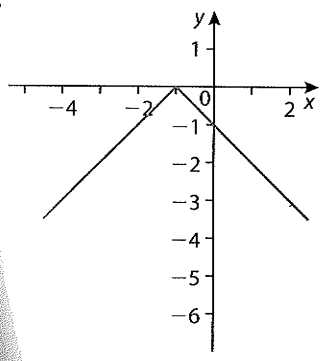
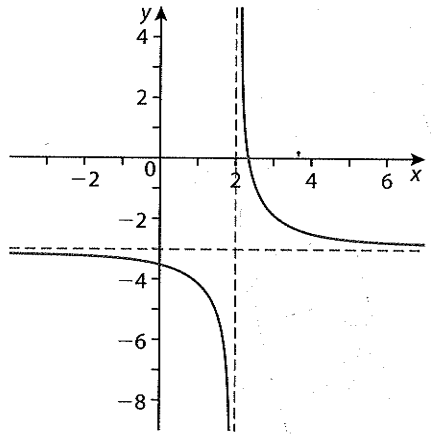
17

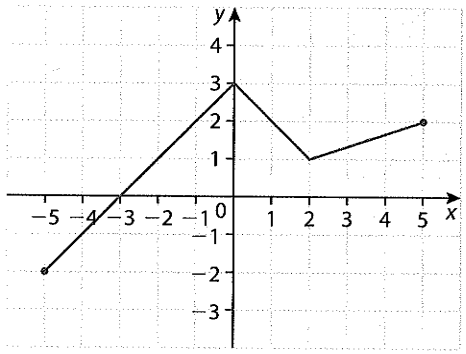


18 Vertical and horizontal asymptotes shown:



19 The graph of f is given. Sketch the graphs of the following functions.

a) $y = f(x) - 3$
b) $y = f(x - 3)$
c) $y = 2f(x)$
d) $y = f(2x)$
e) $y = -f(x)$
f) $y = f(-x)$
g) $y = 2f(x) + 4$



In questions 20–23, specify a sequence of transformations to perform on the graph of x^2 to obtain the graph of the given function.

20 $h: x \mapsto (x - 3)^2 + 5$

21 $h: x \mapsto -x^2 + 2$

22 $h: x \mapsto \frac{1}{2}(x + 4)^2$

23 $f: x \mapsto [3(x - 1)]^2 - 6$

24 Using your GDC, for each function $f(x)$ in questions 24–26 sketch the graph of a) $f(|x|)$ and c) $f(|x|)$. Clearly label any intercepts or asymptotes.

a) $f(x) = 4$
b) $f(x) = (x - 4)(x + 2)$
c) $f(x) = x^2$

27 $f(x) = 1$ and $g: x \mapsto x^2 + 2x$. The function $(f \circ g)(x)$ is defined for all $x \in \mathbb{R}$ except the interval $]a, b[$.

a) Find the values of a and b.
b) Find the range of $f \circ g$.

28 g and h are defined as $g(x) = 2x - 7$ and $h(x) = 3(2 - x)$.

a) Find $(g \circ h)(x)$
b) Find $(h \circ g)(x)$

3 Consider the functions $f(x) = 5x - 2$ and $g(x) = \frac{4 - x}{3}$.

a) Find g^{-1}.

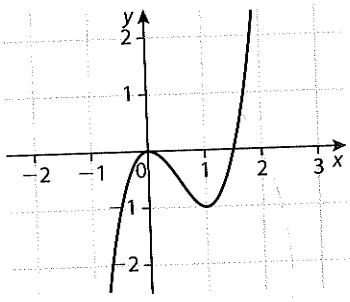
b) Solve the equation $(f \circ g^{-1})(x) = 8$.

4 The functions g and h are defined by $g: x \mapsto x - 3$ and $h: x \mapsto 2x$.

a) Find an expression for $(g \circ h)(x)$.

b) Show that $g^{-1}(14) + h^{-1}(14) = 24$.

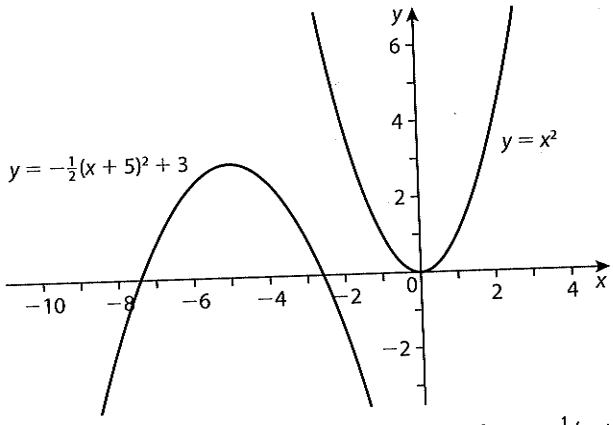
5 The diagram right shows the graph of $y = f(x)$. It has maximum and minimum points at (0, 0) and (1, −1), respectively.



a) Copy the diagram and, on the same diagram, draw the graph of $y = f(x + 1) - \frac{1}{2}$.

b) What are the coordinates of the minimum and maximum points of $y = f(x + 1) - \frac{1}{2}$?

6 The diagram shows parts of the graphs of $y = x^2$ and $y = -\frac{1}{2}(x + 5)^2 + 3$.



The graph of $y = x^2$ may be transformed into the graph of $y = -\frac{1}{2}(x + 5)^2 + 3$ by these transformations.

A reflection in the line $y = 0$, followed by
a vertical stretch by scale factor k, followed by
a horizontal translation of p units, followed by
a vertical translation of q units.

Write down the value of

a) k **b)** p **c)** q.

7 The function f is defined by $f(x) = \frac{4}{\sqrt{16 - x^2}}$, for $-4 < x < 4$.

a) Without using a GDC, sketch the graph of f.

b) Write down the equation of each vertical asymptote.

c) Write down the range of the function f.

8 Let $g: x \mapsto \frac{1}{x}$, $x \neq 0$.

a) Without using a GDC, sketch the graph of g.

The graph of g is transformed to the graph of h by a translation of 4 units to the left and 2 units down.

b) Find an expression for the function h.

c) **(i)** Find the x- and y-intercepts of h.
(ii) Write down the equations of the asymptotes of h.
(iii) Sketch the graph of h.

9 Consider $f(x) = \sqrt{x+3}$.

a) Find:
(i) $f(8)$ **(ii)** $f(46)$ **(iii)** $f(-3)$

b) Find the values of x for which f is undefined.

c) Let $g: x \mapsto x^2 - 5$. Find $(g \circ f)(x)$.

10 Let $g(x) = \dfrac{x-8}{2}$ and $h(x) = x^2 - 1$.

a) Find $g^{-1}(-2)$.

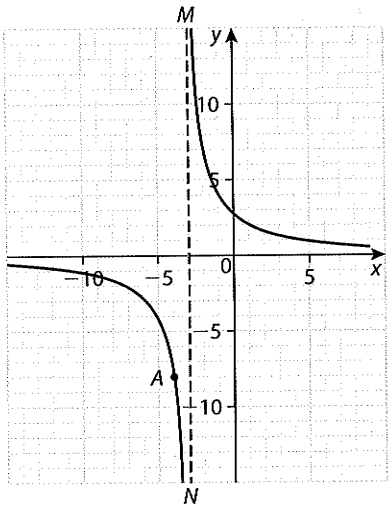
b) Find an expression for $(g^{-1} \circ h)(x)$.

c) Solve $(g^{-1} \circ h)(x) = 22$.

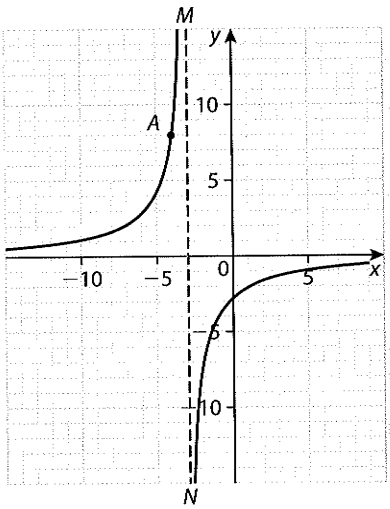
11 Given the functions $f: x \mapsto 3x - 1$ and $g: x \mapsto \dfrac{4}{x}$, find the following:

a) f^{-1} **b)** $f \circ g$ **c)** $(f \circ g)^{-1}$ **d)** $g \circ g$

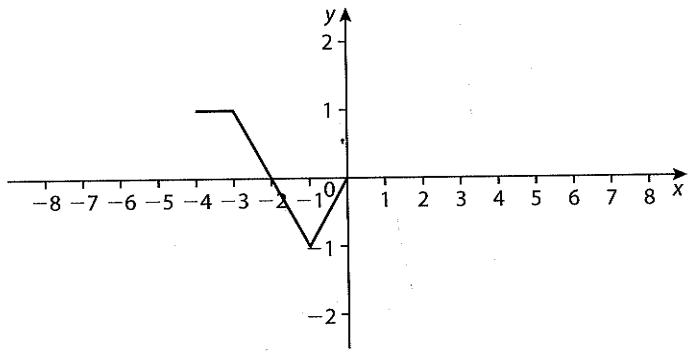
12 **a)** The diagram shows part of the graph of the function $h(x) = \dfrac{a}{x-b}$. The curve passes through the point $A(-4, -8)$. The vertical line (MN) is an asymptote. Find the value of: **(i)** a **(ii)** b.



The graph of $h(x)$ is transformed as shown in the diagram right. The point A is transformed to $A'(-4, 8)$. Give the geometric description of the transformation.



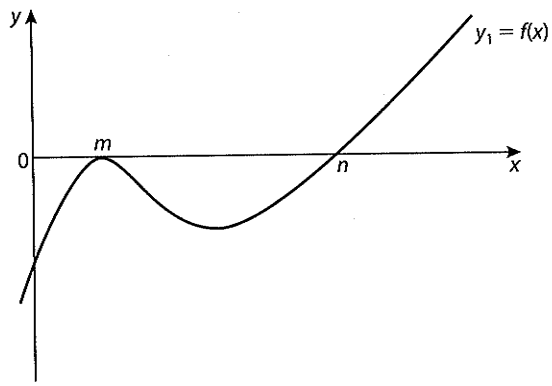
13 The graph of $y = f(x)$ is shown in the diagram.



a) Make two copies of the coordinate system as shown in the diagram but without the graph of $y = f(x)$. On the first diagram sketch a graph of $y = 2f(x)$, and on the second diagram sketch a graph of $y = f(x - 4)$.

b) The point $A(-3, 1)$ is on the graph of $y = f(x)$. The point A' is the corresponding point on the graph of $y = -f(x) - 1$. Find the coordinates of A'.

14 The diagram below shows the graph of $y_1 = f(x)$. The x-axis is a tangent to $f(x)$ at $x = m$ and $f(x)$ crosses the x-axis at $x = n$.



On the same diagram, sketch the graph of $y_2 = f(x - k)$, where $0 < k < n - m$ and indicate the coordinates of the points of intersection of y_2 with the x-axis.

15 Given functions $f : x \mapsto x + 1$ and $g : x \mapsto x^3$, find the function $(f \circ g)^{-1}$.

16 If $f(x) = \dfrac{x}{x+1}$ for $x \neq -1$ and $g(x) = (f \circ f)(x)$, find

a) $g(x)$

b) $(g \circ g)(2)$.

17 Let $f : x \mapsto \sqrt{\dfrac{1}{x^2} - 2}$. Find

a) the set of real values of x for which f is real and finite;

b) the range of f.

18 The function $f : x \mapsto \dfrac{2x+1}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$. Find the inverse function, f^{-1}, clearly stating its domain.

19 The one-to-one function f is defined on the domain $x > 0$ by $f(x) = \dfrac{2x - 1}{x + 2}$.

a) State the range, A, of f.

b) Obtain an expression for $f^{-1}(x)$, for $x \in A$.

20 The function f is defined by $f : x \mapsto x^3$.

Find an expression for $g(x)$ in terms of x in each of the following cases

a) $(f \circ g)(x) = x + 1$;

b) $(g \circ f)(x) = x + 1$.

21 **a)** Find the largest set S of values of x such that the function $f(x) = \dfrac{1}{\sqrt{3 - x^2}}$ takes real values.

b) Find the range of the function f defined on the domain S.

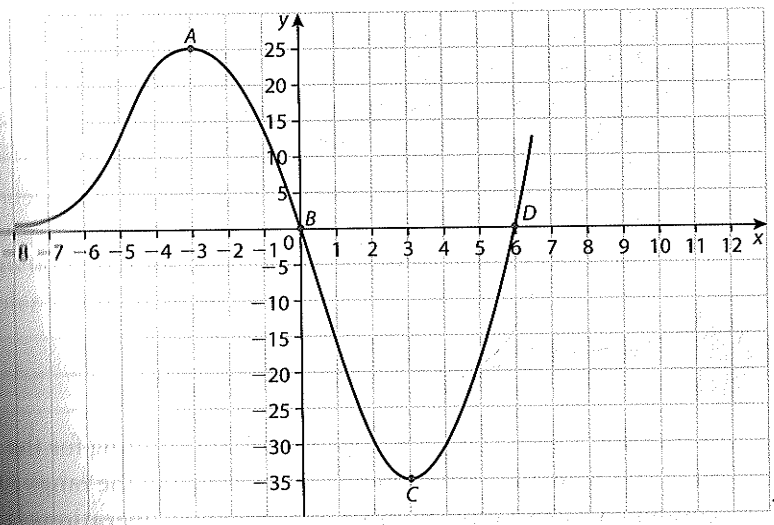
22 Let f and g be two functions. Given that $(f \circ g)(x) = \dfrac{x + 1}{2}$ and $g(x) = 2x - 1$, find $f(x - 3)$.

23 The diagram below shows the graph of $y = f(x)$ which passes through the points A, B, C and D.

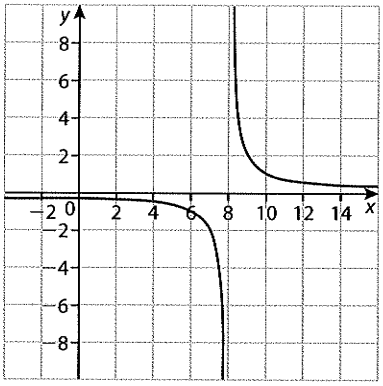
Sketch, indicating clearly the images of A, B, C and D, the graphs of

a) $y = f(x - 4)$;

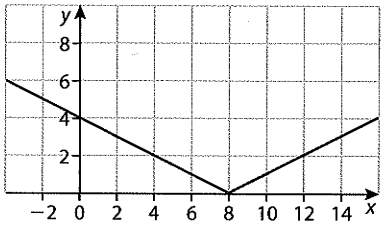
b) $y = f(-3x)$.



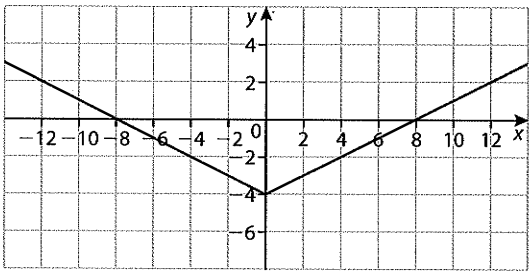
24 a)



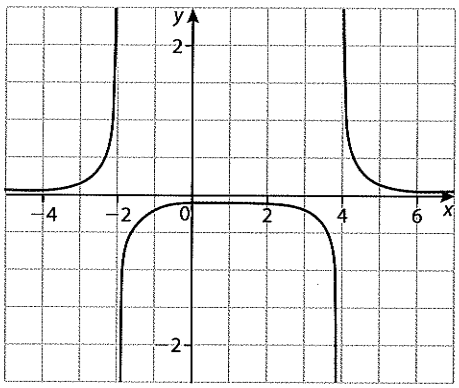
b)



c)



25 a)



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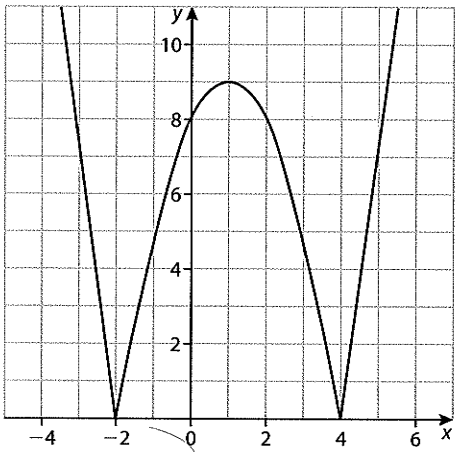
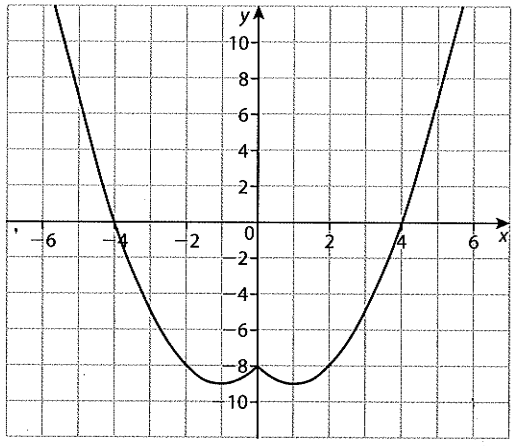
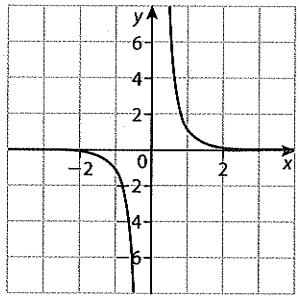


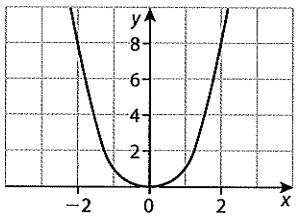
c)



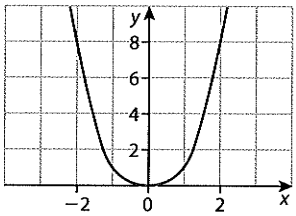
26 a)



b)



c)



Practice questions

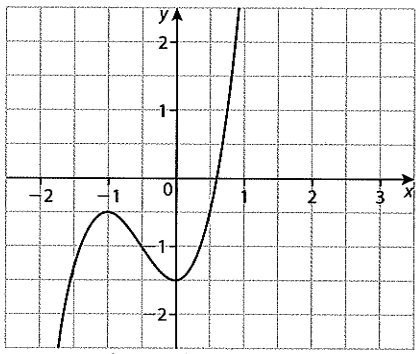
1 a) $a = -3, b = 1$ b) range: $y \geqslant 0$

2 a) 5 b) 3

3 a) $g^{-1}(x) = -3x + 4$ b) $x = \frac{2}{3}$

4 a) $(g \circ h)(x) = 2x - 3$ b) 24

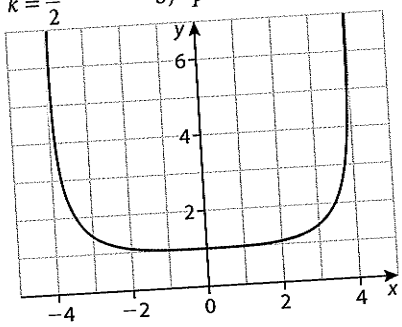
5 a)



maximum at $\left(-1, -\frac{1}{2}\right)$; minimum at $\left(0, -\frac{3}{2}\right)$

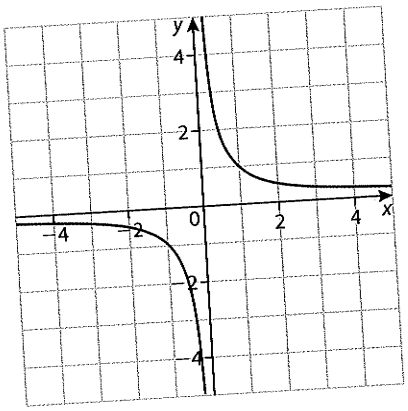
6 a) $k = \frac{1}{2}$ b) $p = -5$ c) $q = 3$

7 a)



b) $x = 4, x = -4$ c) range: $y \geqslant 1$

8 a)

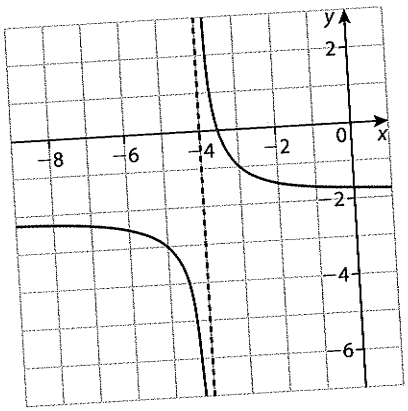


b) $h(x) = \frac{1}{x+4} - 2$

c) (i) x-intercept: $\left(-\frac{7}{2}, 0\right)$; y-intercept: $\left(0, -\frac{7}{4}\right)$

(ii) Vertical asymptote: $x = -4$; horizontal asymptote: $y = -2$

(iii)



9 a) (i) $\sqrt{11}$ (ii) 7 (iii) 0

b) $x < -3$

c) $(g \circ f)(x) = x - 2$

10 a) 4 b) $(g^{-1} \circ h)(x) = 2x^2 + 6$ c) $x = \pm 2\sqrt{2}$

11 a) $f^{-1}(x) = \frac{1}{3}x + \frac{1}{3}$

b) $(f \circ g)(x) = \frac{12}{x} - 1$

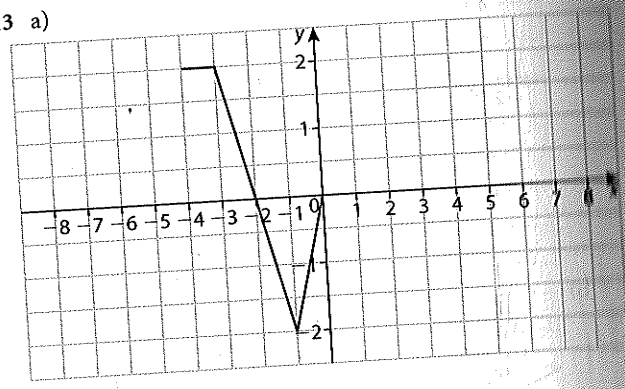
c) $(f \circ g)^{-1}(x) = \frac{12}{x+1}$

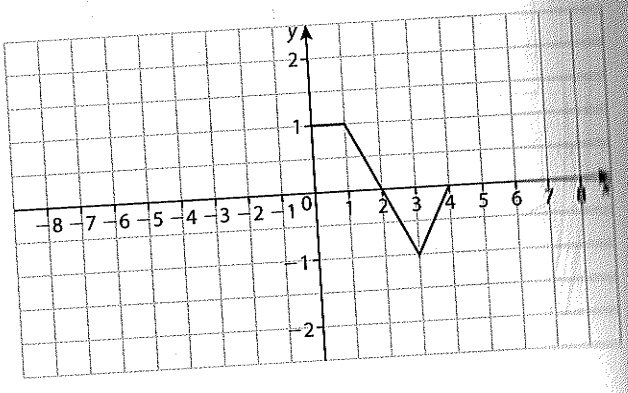
d) $(g \circ g)(x) = x$

12 a) (i) $a = 8$ (ii) $b = -3$

b) Reflection over x-axis

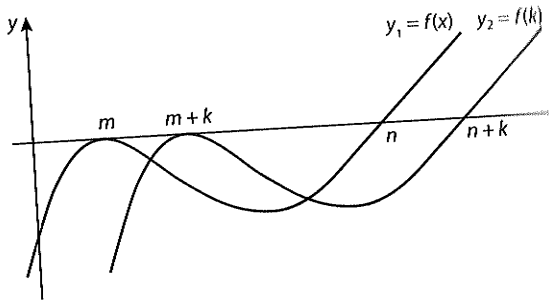
13 a)





b) $A'(-3, -2)$

14



15 $(f \circ g^{-1})(x) = \sqrt[3]{x} + 1$

16 a) $g(x) = \frac{x}{2x+1}$ b) $\frac{2}{9}$

17 a) $-\frac{\sqrt{2}}{2} \leqslant x \leqslant \frac{\sqrt{2}}{2}, x \neq 0$

b) $f(x) \geqslant 0$

18 $f^{-1}(x) = \frac{x+1}{x-2}, x \neq 2$

19 a) $-\frac{1}{2} < A < 2$ b) $f^{-1}(x) = \frac{-2x-1}{x-2}$

20 a) $g(x) = \sqrt[3]{x+1}$ b) $g(x) = \sqrt[3]{x} + 1$

21 a) $S = \{x : -\sqrt{3} < x < \sqrt{3}\}$ b) $f(x) \geqslant \frac{\sqrt{3}}{3}$

22 $\frac{x}{4}$

23 a) $A(1, 25), B(4, 0), C(7, -35), D(10, 0)$

b) $A(-1, -25), B(0, 0), C(1, 35), D(2, 0)$