(A) Lesson Context

BIG PICTURE of this UNIT:	• How do algebraically & graphically work with growth and decay applications? • What are logarithms and how do we invert or undo an exponential function? • How do we work with simple algebraic and graphic situations involving the use of logarithms (or inversing exponentials?)		
CONTEXT of this LESSON:	Where we've been We have seen simple algebra skills (exponent laws & solving exponential eqns) in IM2 as well as graphs of simple exp fcns in the form of $y = AB^x$	Where we are Review fundamental algebra & graphic skills associated with exponential functions & their applications	Where we are heading How do work with the mathematically model $f(x) = AB^{k(x+c)} + d$?

(B) Lesson Objectives:

a. Review KEY algebra skills associated with Exponential Functions
b. Review the application of exponential models to assorted real world contexts.

(C) Fast Five - EXPLORATION: How do we work with exponents in the form of a/b?

Part 1 ➔ use your TI-84 to help answer these questions ➔ try to figure out HOW to come up with the answer without the use of the TI-84

$8^{\frac{1}{3}}$ $8^{\frac{2}{3}}$ $8^{\frac{4}{3}}$ $8^{\frac{5}{3}}$

$16^{\frac{3}{4}}$ $25^{\frac{3}{2}}$ $27^{\frac{2}{3}}$ $\left(\frac{1}{8}\right)^{\frac{2}{3}}$

Part 2 ➔ do NOT use a calculator as you work these out

$4^{\frac{3}{2}}$ $27^{\frac{4}{3}}$ $32^{\frac{3}{5}}$ $9^{-\frac{3}{2}}$

$\left(\frac{16}{81}\right)^{\frac{3}{4}}$ $\left(\frac{1}{4}\right)^{-\frac{3}{2}}$ $(-8)^{-\frac{2}{3}}$ $\left(\frac{-343}{1000}\right)^{\frac{2}{3}}$

(D)Part 1 – Exponential Algebra: Getting Ready (work through Nelson 12, Chap 2 Getting Ready Exercises on page 84)

Getting Ready

In this chapter, you will be working with models, functions, and equations of exponential and logarithmic functions. These exercises will help you warm up for the work ahead.

1. Write each power with a positive exponent.
   (a) 3^{-4} (b) 7^{-2} (c) a^{-8} (d) x^{-5}

2. Evaluate.
   (a) 2^8 (b) 3^5 (c) $(-8)^3$ (d) -4^2
   (e) -6^{-2} (f) $350(1.02)^3$ (g) $(-2)^{-5}$ (h) $(-1)^{-100}$
   (i) $\left(\frac{2}{3}\right)^2$ (j) $\left(\frac{3}{4}\right)^{-3}$ (k) $5^{-1} + \left(\frac{-4}{9}\right)^2$ (l) $\frac{7}{8} - 4^{-2}$

3. Evaluate to two decimal places where necessary.
   (a) $4^{\frac{1}{2}}$ (b) $(-8)^{\frac{1}{3}}$ (c) $\left(\frac{1}{16}\right)^{\frac{1}{2}}$ (d) $216^{-\frac{1}{3}}$
   (e) $2^{\frac{1}{2}}$ (f) $\left(\frac{5}{6}\right)^{\frac{1}{3}}$ (g) $200^{-\frac{2}{5}}$ (h) $(17.4)^{-\frac{3}{7}}$

4. Write each expression as a power of 2.
   (a) $2^4 \times 2^7$ (b) $2^6 \div 2^{-3}$ (c) $4^2 \div 2^m$ (d) 8^n

5. Evaluate.
   (a) $49^{\frac{1}{2}} + 16^{\frac{1}{4}}$ (b) $16^{\frac{3}{2}} + 16^{-0.5}$
   (c) $\left(\frac{1}{8}\right)^{\frac{1}{3}} - \sqrt[3]{\frac{27}{125}} + 4(8^{-\frac{2}{3}})$ (d) $(2^2 \times 5)^{-1}$
   (e) $\left(\frac{3^{-1}}{2^{-1}}\right)^{-2}$ (f) $(5^0 \times 5^4 \div 5^3)^{-4}$

6. Evaluate each expression for $x = 2$ and $y = -2$.
   (a) $\frac{4x^{-2}y^3}{x^3}$ (b) $(3x^2y^{-3})^{-2}$

7. Simplify.
   (a) $\frac{x^4y^2}{x^2y}$ (b) $\left(\frac{-1}{a}\right)^2(a^4b^2)^5$ (c) $(-d^3)^4\left(\frac{c}{d}\right)^6$ (d) $\left(\frac{x^5}{y^2}\right)^4\left(\frac{y^3}{x}\right)^2$

8. For each equation, make a table of values and sketch the graph.
   (a) $y = 2^x$ (b) $y = 3^x$ (c) $y = (0.5)^x$

9. Solve.
   (a) $y = 1.07^3$ (b) $\left(\frac{m^7}{m^6}\right)^2 = 196$ (c) $(p \times p^2)^3 = 3^9$
   (d) $c^{\frac{2}{3}} = 64$ (e) $3a^{\frac{4}{5}} = 48$ (f) $70 = 35a^2$

Part 1 – Exponential Algebra: Getting Ready (work through Nelson 12, Chap 2 Getting Ready Exercises on page 85)

10. Solve each equation by guessing and checking.

(a) $4^x = 60$ **(b)** $6^x = 10$ **(c)** $2.5^x = 200$ **(d)** $1.05^x = 2$

11. Write the first differences and the second differences for each relation. Is the relation linear, quadratic, or other? Write an equation for the relation.

(a)

x	0	1	2	3	4	5
y	0	2	4	6	8	10

(b)

x	0	1	2	3	4	5
y	−1	2	5	8	11	14

(c)

x	0	1	2	3	4	5
y	2	6	18	54	162	486

(d)

x	0	1	2	3	4	5
y	1	2	4	8	16	32

12. Solve each quadratic equation by factoring or by using the quadratic formula.

(a) $x^2 + 2x - 15 = 0$ **(b)** $n^2 + 7n - 30 = 0$

(c) $2y^2 + 9y + 4 = 0$ **(d)** $m^2 - 4 = 0$

(e) $17x + 5x^2 = -6$ **(f)** $0 = -4.9t^2 + 20.58t + 0.491$

13. Solve for x by making the bases equal and using the exponent laws. If the bases cannot be made equal, guess and check to solve for x, to two decimal places, if necessary.

(a) $3^x = 27$ **(b)** $2^x = \frac{1}{4}$ **(c)** $5^{2x-1} = \frac{1}{125}$

(d) $4(2)^x = 32$ **(e)** $8^x = 16\sqrt[3]{2}$ **(f)** $9^{-x-2} = \left(\frac{1}{27}\right)^{x+3}$

14. For each geometric sequence, determine the common ratio, the general term, and t_8.

(a) $6, -12, 24, -48, \ldots$ **(b)** $1, \frac{-2}{3}, \frac{4}{9}, \frac{-8}{27}, \ldots$

15. Determine the amount that each investment is worth. Use $A = P(1 + i)^n$.

(a) \$1000 invested at 8%/a, compounded annually, for 20 years

(b) \$5340 invested at 2%/a, compounded annually, for 5 years

16. A bacteria culture doubles in size every 15 min. How long will it take for a culture of 20 bacteria to grow to a population of 163 840?

17. Thorium-227 has a half-life of 18.4 days. How much time will a 50-mg sample take to decompose to 12.5 mg?

PART 2 – Modeling with Exponential Functions in the form of $f(x)=A(B)^x$

a. **Example 1**: The number of a certain bacteria doubles every hour. The initial population in a sample of this bacteria is 36.

1. Determine the number of bacteria after 8 hours.
2. Determine an exponential model for *N*, the number of bacteria after *t* hours i.e. N(t) = ???

b. **Example 2**: The number of bacteria in a different culture doubles every 2 hours. The initial population is again 36.

1. Determine the number of bacteria after 8 hours.
2. Determine an exponential model for *N*, the number of bacteria after *t* hours i.e. N(t) = ???
3. Determine the growth rate PER HOUR.

c. **Example 3**: The population of a small town appears to be increasing exponentially. Town planners need a model for predicting the future population. In the year 2000, the population was 35,000, while in the year 2010, the population grew to 57,010.

1. PREDICT: What will be the town's population in 2030?
2. Create an **exponential** algebraic model for the town's population growth.
3. Check your population model by using the fact that the town's population was 72,825 in 2015.
4. CALCULATE: What will be the town's population in 2030?

d. **Example 4**: Three years ago, the fish population in Loon Lake was 2500. Due to the effects of acid rain, there are now about 1950 fish in the lake. Assume that the decline of the fish population is exponential. Find the predicted fish population 5 years from now.

e. **Example 5**: What is the average annual rate of inflation if a loaf of bread cost $1.19 in 1991 but costs $1.50 in 2001?

(E) PART 3 – Exponential Functions in OTHER Forms

(post scan from Nelson 12, Chap 2.3 on page 107

2.3 Various Forms of Exponential Functions

SETTING THE STAGE

Now you have seen several examples of exponential growth. In grade 11, you used exponential growth formulas to describe compound interest, growth of bacterial populations, and geometric sequences. Here are some of these formulas:

Compound Interest:

$A = P(1 + i)^n$, where A is the amount, representing a combination of the principal and compounded interest
P is the original principal invested
i is the interest rate per compounding period
n is the number of compounding periods

Doubling period is the time required for a quantity to grow to twice its original amount.

Exponential Growth (involving a doubling period):

$M = c(2)^{\frac{t}{D}}$, where M is the total amount or number
c is the initial amount or number
2 is the growth factor
t is the time
D is the doubling period

Half-life is the time required for a material to decay to one-half of its original mass or quantity.

Exponential Decay (involving half-life):

$M = c\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where M is the remaining mass of the decayed material
c is the original mass of the material
$\frac{1}{2}$ is the decay factor
t is the time
h is the half-life

Geometric Sequence:

$t_n = a(r)^{n-1}$, where t_n is the nth term of the sequence
a is the first term
r is the common ratio
n is the number of terms

Exponential Function:

$y = c(a)^x$, where y is the total amount or number
c is the initial amount or number
a is the growth factor or decay rate
x is the number of growth periods or decay periods

(F) PART 3 – Exponential Functions in OTHER Forms

a. Example 1: One bacterium divides into two bacteria every 5 days.

1. Initially, there are 15 bacteria. How many bacteria will there be in 30 days?
2. What is the approximate growth rate PER DAY?

b. Example 2: Mr S invested $14,000 in an account that pays interest at 6% /a, compounded quarterly. How much money will Mr S have in 20 years time?

c. A sequence of numbers is listed (this sequence is called a GEOMETRIC SEQUENCE as the numbers all differ by a CONSTANT RATIO) ➔ {9,27,81,243,729, …..}

1. What is the first term of this sequence?
2. What is the common ratio in this sequence of numbers?
3. What is the 12 term?
4. What formula can we write to model this sequence of numbers?
5. How many terms must be listed such that the term value does NOT exceed 7,500,000?

d. Example 3: Archaeologists use carbon-14 dating to estimate the age of relics. It is known that the half life of carbon-14 is 5370 years. On an expedition, Mr S finds some wood and pottery and analyzes the amount of carbon-14 present. He finds that the samples contain about 35% of the C-14 that living organisms contain. How old are the wood & pottery samples?

e. EXTENSION to Example 3 ➔ Explain WHY the radioactive C-14 method only works for specimen that are less than 50,000 years old.

(G)Compound Interest

Problem Set 36: Compound Interest

Calculate the final amount (accumulated value) in each case:

a) $7,000 invested for 8 years at 9% per annum compounded annually.
b) $6,350 invested for 11 ½ years at 8% per annum compounded semi-annually.
c) $9,000 invested for 7 years at 10% per annum compounded quarterly.
d) $15,000 invested for 15 years at 12% per annum compounded monthly.
e) $10,000 invested for 8 years at 9% per annum compounded weekly.
f) $14,000 invested for 50 years at 4 ¾% per annum compounded daily.
g) $14 million invested for 1 month at 4% per annum compounded monthly.
h) $1 million for 1 month @ 4%/a c. m.

Answers:
a) $13,947.94 b) $15,650.94 c) $17,968.46 d) $89,937.03 e) $20,531.55 f) $150,490.93
g) $14,046,666.67 h) $1,003,333.33

Problem Set 37: Present Value

1. Mrs. Fangrad borrowed $9,500 for 3 years at 11.6% per annum, compounded quarterly.
   a) How much money did she owe at the end of 3 years?
   b) How much interest did she pay for the loan?
2. What is the Present Value of each amount?
   a) $9,000 in 5 years, invested at 11% per annum, compounded semi-annually.
   b) $50,000 in 9 months, invested at 11% per annum, compounded quarterly.
   c) $100,000 in 3 years, invested at 3% per annum, compounded monthly.
   d) $78,840 in 9 years, invested at 4.8% per annum, compounded annually.
   e) $250,000 in a year, invested at 8.75% per annum, compounded quarterly.
3. Sue wants to provide for her niece's education. How much should she invest on the day her niece is born to have $22.000 on her 18th birthday, if the money earns 7% per annum, compounded quarterly?
4. Samantha wants to have $40,000 available for a down payment on a house in 10 years. How much should she invest now at $6\frac{1}{4}$% per annum, compounded semi-annually?

Answers:
1 a) $13,387.77 b) $3,887.77 2 a) $55,268.88 b) $46,091.89 c) $91,403.38 d) $51,700.54
e) $229,270.89 3. $22,000 4. $21,616.27

Problem Set 38: Rates

1. $4,000 doubles in 5 years. Calculate the annual rate of interest, compounded annually.
2. $5,500 is invested for $7\frac{1}{2}$ years and accumulates to $11,434,10. Calculate the annual rate of interest compounded semi-annually.
3. $7,000 is tripled in 11 years. Calculate the annual rate of interest, compounded quarterly.
4. $3,850 is invested for 12 years and accumulates to $14,325.77. Calculate the annual rate of interest compounded monthly.
5. $1 million is invested for 20a and accumulates to $5 million. Calculate the annual rate of interest compounded weekly.
6. $5 is invested for 100 years and accumulates to $20,000. Calculate the annual rate of interest compounded daily.
7. $5,000 is invested for 9 years at $8\frac{1}{2}$% per annum compounded daily. If an investment of $4,000 yields the same accumulated value in 8 years, calculate the annual interest rate compounded annually.

Answers:
1. 14.9% 2. 10.0% 3. 10.1% 4. 11.0% 5. 8.1% 6. 8.3% 7. 13.1%

Problem Set 39: Rates

1. How long will it take for $3,000 to accumulate to $7,000 at each rate?
   a) 7% per annum compounded annually
   b) 9% per annum compounded semi-annually
   c) 8% per annum compounded quarterly
   d) 9% per annum compounded monthly
   e) $7\frac{3}{4}$% per annum compounded minutely
2. How long will it take for $2,700 to triple at 8% per annum compounded daily?
3. At what annual rate compounded semi-annually will $2,700 triple twice as fast as in #2?
4. At a certain rate of simple interest and a given amount of time, $500 will accumulate to $750. Calculate the accumulated value if $500 is invested at twice the rate of simple interest and three times as long.

Answers:
1 a) 12.52a or 12a27w b) 9.62a or 9a226d c) 10.697a or 10a255d d) 9.450a or 9a164d e) 10.933a or 10a11m
2. 13.734a or 13a38w 3. 16.7% 4. $2,000