IM3 Lesson 4.7: Rational Functions

BIG PICTURE of this UNIT:	• What is a Polynomial and how do they look? • What are the attributes of a Polynomial? • How do I work with Polynomials?		
CONTEXT of this LESSON:	Where we've been We have discussed the basics: degree, type, and operations (+, –, x)	Where we are We have solidified the basics of polynomials and their graphs. Now we have a small adventure into rational functions	Where we are heading What are the key attributes of a rational function, linear over linear?

(A) Lesson Objectives:

a. Warm Up Synthetic Division Problem/Polynomial Graph Analysis
b. Work on and attempt to develp an understanding of KEY Vocabulary.
c. Begin to analyze the the attributes of a rational functions linear/linear and it's effect on the graph.
d. Observations and patterns in the graphs of rationals
e. Extention to Quadratic/linear and linear over quadratic.

(B) Some Context

Lets try and get a grasp on what a rationa function looks like and how we can use our knowledge of reciprocal functions to develop the concepts of linear/linear rationals.
Warm Up.

Completely factor one of the following polynomials

$x^3 + 2x^2 - 7x + 4$ $\qquad$ $x^4 + 3x^3 - 7x^2 - 15x + 18$

Lesson 4.7: Rational Functions

Rational functions is a function taking the below form. In this course g(x) and h(x) will be restricted to linear functions of the form ax + b. So we will be working with functions that look a bit like…

$$f(x)=\frac{g(x)}{h(x)}=\frac{ax+b}{cx+d}$$

$$ex.$$

$$f(x)=\frac{2x-1}{x+5}$$

It is a much needed fact that h(x)≠0.

Rational Functions Investigation # 1

$$f(x)=\frac{1}{x-h}+k$$

Using Desmos and your lovely brains open a new google doc between you and your partner and make a table similar to the one below. Please then fill it out…

Rational Function	Vertical Asymptote	Horizontal Asymptote	Screen Capture of Graph
$\frac{1}{x}$			
$\frac{1}{x-2}$			
$\frac{1}{x+3}+1$			
$\frac{1}{x}-4$			
$\frac{1}{x-1}+5$			
$f(x)=\frac{1}{x-M}+N$			

Rational Functions Investigation # 2

$$f(x)=\frac{ax+b}{cx+d}$$

Using Desmos and your lovely brains fill out the below table.

Rational Function	Record a, b, c, d	Vertical Asymptote	Horizontal Asymptote	Screen Capture of Graph	
$\frac{x}{x+3}$					
$\frac{2x+1}{2x+3}$					
$\frac{2x}{x+3}$					
$\frac{3x-1}{x+3}$					
$\frac{4x+2}{2x-8}$					
$f(x)=\frac{ax+b}{cx+d}$					

Discussion Time:…

Rational Functions Investigation # 3

Using Desmos and your lovely brains fill out the below table. However this is more complex now. You will have a linear/quadratic or a quadratic/linear

Rational Function Linear/Quadratic	Simplified	Vertical Asymptote/s	Horizontal Asymptote/s	Screen Capture of Graph
$\frac{x}{x^2+3x}$				
$\frac{x+1}{x^2+6x+5}$				
$\frac{x-4}{x^2-16}$				
$\frac{3x-1}{x^2+7x+10}$				
$\frac{x+2}{x^2+4x+4}$				

$\frac{x^2+3x+2}{x}$				
$\frac{x^2-6x-9}{x-3}$				
$\frac{x^2-4}{x-2}$				
$\frac{x^2-7x-18}{2x-18}$				
$\frac{x^2+7x-8}{x+1}$				

AP/HL Extension: More complex Rational Functions

Section 1: What value of x will make each function undefined?

1. $\frac{3x^3 + 9}{2x - 8}$

2. $\frac{5x^3 - 95x^2}{x^2 - 81}$

3. $\frac{3x^2 + 16x - 21}{2x}$

4. $\frac{6x}{x^2 + 6x - 72}$

Section 2: Simplify each fraction.

5. $\frac{x^2 + 6x - 27}{x - 3}$

6. $\frac{6x^3 + 16x}{2x}$

7. $\frac{x^2 + 5x + 4}{x + 4}$

8. $\frac{x^2 - 10x + 24}{x^2 - 16}$

9. $\frac{x^2 - 5x - 24}{x^2 - 8x - 33}$

Section 3: What value of x, if any, will create a hole in the graph of the function?

10. $\frac{3x + 6}{x + 2}$

12. $\frac{x^2 - 20x + 75}{x^2 - 6x - 135}$

Hint: Holes occur when both the numerator and denominator equal zero.

11. $\frac{x^2 - x - 56}{x^2 - 4x - 32}$

13. $\frac{x^2 - 2x - 63}{x^2 - 16x + 48}$

Section 4: For what value of x, if any, is there a vertical asymptote in the graph of the function?

14. $\frac{x^2 + 8x + 2}{x - 17}$

16. $\frac{9x^2 + 36x}{x^2 - 3x - 28}$

Hint: Vertical asymptotes occur when just the denominator equals zero.

15. $\frac{4x^3 + 24x^2}{x - 6}$

17. $\frac{x^2 - 14x + 48}{x^2 - 64}$