A. Lesson Context

BIG PICTURE of this UNIT:	• How & why do we build NEW knowledge in Mathematics? • What NEW IDEAS & NEW CONCEPTS can we now explore with specific references to QUADRATIC FUNCTIONS? • How can we extend our knowledge of FUNCTIONS, given our BASIC understanding of Functions?		
CONTEXT of this LESSON:	Where we've been In Lesson 2, you reviewed a second method for solving quadratic eqns: completing the square & reviewed the idea of an INVERSE	Where we are NOW we will focus on addressing the idea of Inverses of Functions ➔ what ARE inverses of functions & how do we work with inverses of fcns	Where we are heading How do we extend our knowledge & skills of the algebra of quadratic fcns, and build in new ideas & concepts involving fcns.

B. Lesson Objectives

a. Introduce the concept of inverse functions through an exploration
b. Introduce the inverse function notations
c. Apply the concept of inverse linear & quadratic & exponential functions through a variety of representations

C. FAST FIVE: Skills Review

(a) Solve for x if $-3 = 2x + 5$	(a) Solve for x if $y = 2x + 5$
(b) Solve for x if $3x - 2(2) = 5$	(b) Solve for x if $3x - 2y = 5$
(c) Solve for x if $8 - 5 = 3(x - 2)$	(c) Solve for x if $y - 5 = 3(x - 2)$
(d) Solve for x if $3(x + 2)^2 + 5 = 32$	(d) Solve for x if $3(x + 2)^2 + 5 = y$

(e) Solve for x if $-\frac{1}{2}(x-1)^2 - 2 = -4$	(e) Solve for x if $-\frac{1}{2}(x-1)^2 - 2 = y$

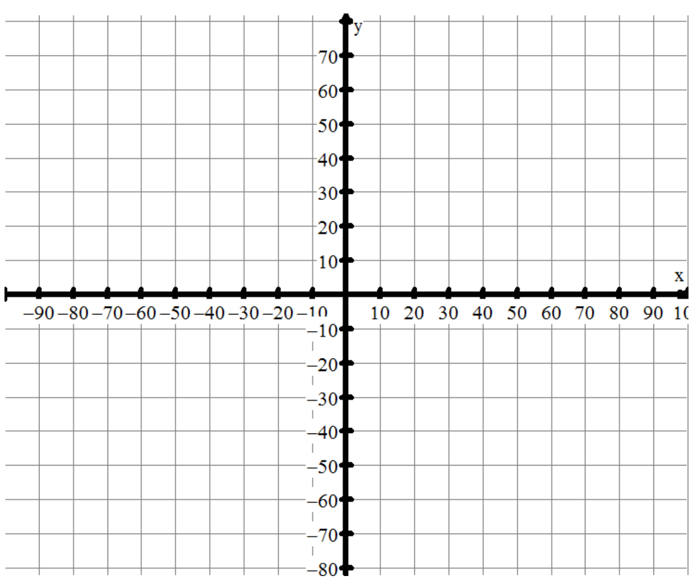
D. Opening Exploration – PART 1

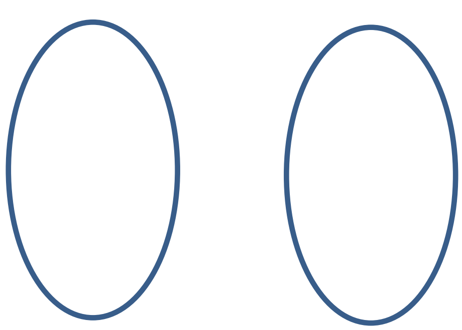
For American tourists visiting Canada, temperature data might seem a bit unusual. So a simplified "rule of thumb" for converting a temperature in degrees Celsius into degrees Fahrenheit is to double the Celcius temperature and then add 30.

(a) Copy and complete the table using the visitor's rule.

°C	-10	0	10	20	30	40
°F						

(b) Graph the relation.





(c) What is the independent variable?

(d) What is the dependent variable?

(e) Does our "temperature conversion rule" define a function? Explain.

(f) Let f represent the rule. What ordered pair, (0,????), belongs to f ?

(g) Let x represent the temperature in degrees Celsius. Write the equation for this rule in function notation.

(h) In the table, f (10) = 50, which corresponds to a point on the graph of y = f (x). What is the x-coordinate of this point? What is its y -coordinate?

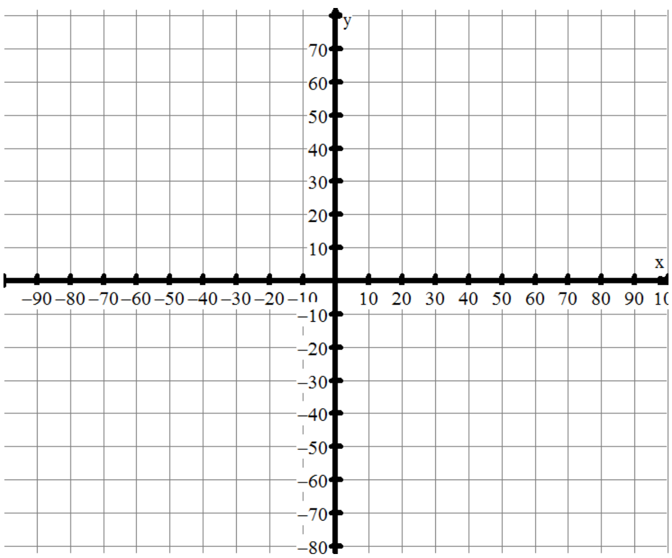
E. Opening Exercise – PART 2

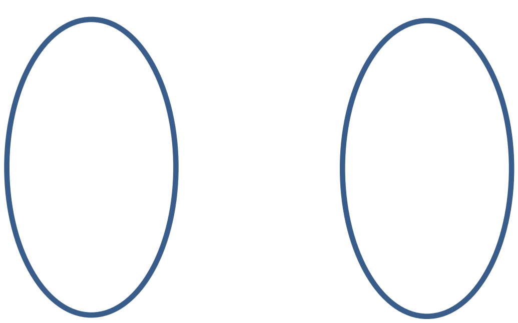
For CANADIAN tourists visiting THE US, temperature data might seem a bit unusual. So a simplified "rule of thumb" for converting a temperature in degrees Celsius into degrees Fahrenheit is …………

(a) Copy and complete the table using the visitor's rule.

°F	50	60	70	80	90	100
°C						

(b) Graph the relation.





(c) What is the independent variable?

(d) What is the dependent variable?

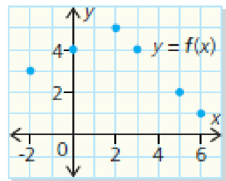
(e) Does our "temperature conversion rule" define a function? Explain.

(f) Let f represent the rule. What ordered pair, (0,????), belongs to f ?

(g) Let x represent the temperature in degrees Celsius. Write the equation for this rule in function notation.

(h) In the table, f (50) = 10, which corresponds to a point on the graph of y = f (x). What is the x-coordinate of this point? What is its y -coordinate?

F. Examples – Working with Mappings & Relations



The graph of $y = f(x)$ is shown.

i. State the domain and range of f.

ii. Draw an arrow diagram for f^{-1}.

iii. Evaluate.
(a) $f(2)$ (b) $f(4)$ (c) $f^{-1}(1)$ (d) $f^{-1}(4)$

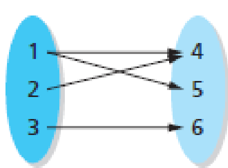
iv. Graph $y = f^{-1}(x)$.

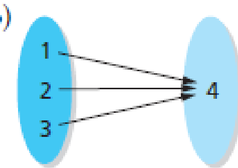
v. Is f^{-1} a function? Explain.

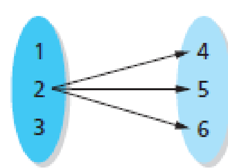
vi. State the domain and range of f^{-1}.

1. For each set of ordered pairs,
i. graph the relationship and its inverse
ii. is the relationship a function? Is the inverse a function? Explain.
(a) $\{(0, 1), (1, 3), (2, 5), (3, 7)\}$ (b) $\{(0, 3), (1, 3), (2, 3), (3, 3)\}$
(c) $\{(1, 1), (1, 2), (1, 3), (1, 4)\}$

2. For each of the following,
i. draw an arrow diagram for the inverse relationship
ii. state whether or not each inverse defines a function, and justify your answer

(a)


(b)


(c)


G. Key Concepts & Notations

d. The **inverse** of a relation and a function maps each output of the original relation back onto the corresponding input value. The inverse is the “reverse” of the original relation, or function

e. f^{-1} is the name given for the inverse relation.

f. $f^{-1}(x)$ represents the expression for calculating the value of f^{-1}.

g. If $(a,b) \in f$, then $(b,a) \in f^{-1}$.

h. Given a table of values for a function, interchange the independent and dependent variables to get a table for the inverse relation.

i. The domain of *f* is the range of f^{-1} and then range of *f* is the domain of f^{-1}.

j. To determine the equation of the inverse in function notation, interchange *x* and *y* and solve for *y*.

H. Examples – Working with Linear Relations

Example 3

The table shows all of the ordered pairs belonging to function g.

x	y
1	5
2	7
3	9
4	11
5	13

(a) Determine $g(x)$.

(b) Write the table for the inverse relation.

(c) Evaluate $g(5)$.

(d) Evaluate $g^{-1}(5)$.

(e) What are the coordinates of the point that corresponds to $g^{-1}(5)$ on the graph of g^{-1}?

(f) What are the coordinates of the point on the graph of g that corresponds to $g^{-1}(5)$?

(g) Determine $g^{-1}(x)$.

Example 5

A relation is $h(x) = -4x + 6$, where $\{x \mid -2 \leq x \leq 3, x \in \mathrm{R}\}$.

(a) Sketch the graph of $y = h(x)$.

(b) Sketch the graph of $y = h^{-1}(x)$.

(c) State the domain and range of h.

(d) State the domain and range of h^{-1}.

(e) Are h and h^{-1} functions? Explain.

13. Communication: An electronics store pays its employees by commission. The relation $p(s) = 100 + 0.05s$ is used to find an employee's weekly pay, p, in dollars, where s represents the employee's weekly sales in dollars.

(a) Describe the function as a rule.

(b) Determine $p^{-1}(s)$.

(c) Describe the inverse function as a rule.

(d) Describe a situation where the employee might use the inverse function.

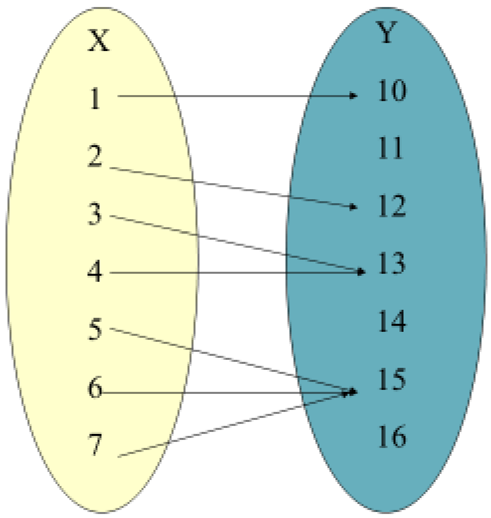
(e) State a reasonable domain and range for p^{-1}.

FURTHER EXAMPLES

(F) Examples

Determine:

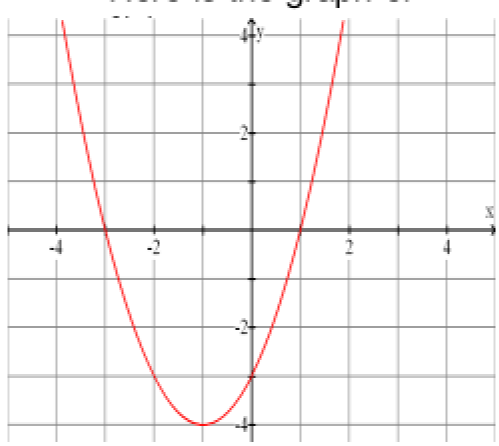
(a) Domain of f(x)
(b) Range of f(x)
(c) Mapping of inverse
(d) Domain of inverse
(e) Range of inverse
(f) Is the inverse a function?



(F) Examples

- Consider a graph of the following data:
- 1. State do domain and range of f
- 2. Evaluate f(-2), f(0), $f^{-1}(1)$, $f^{-1}(-2)$
- 3. Graph the inverse
- 4. Is the inverse a function?
- 5. State the domain and range of $f^{-1}(x)$

- Here is the graph of



FURTHER EXAMPLES

(F) Examples

- ex . If an object is dropped from a height of 80 m, its height above the ground in meters is given by $h(t) = -5t^2 + 80$
- 1. Graph the function
- 2. Find and graph the inverse
- 3. Is the inverse a function
- 4. What does the inverse represent?
- 5. After what time is the object 35 m above the ground?
- 6. How long does the object take to fall?

(F) Examples

- Ex. Determine the equation for the inverse of $y = 4x - 9$. Draw both graphs and find the D and R of each.
- Ex. Determine the equation for the inverse function of $y = 2x^2 + 4$. Draw both and find D and R of each.
- Ex. Determine the equation of the inverse function of $y = x^2 + 4x - 5$

ADVANCED EXAMPLES

(G) Composing with Inverses

- Now let $f(x) = x^2 + 2$.
- Determine the inverse of $y = f(x)$
- Graph both functions on a grid/graph in a square view window

- Graph the line $y = x$. What do you observe? Why?
- What transformation are we considering in this scenario?

- Now compose as follows $f \circ f^{-1}(x)$ and $f^{-1} \circ f(x)$. What do you notice?

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(G) Composing with Inverses

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