Unit 2 Lesson 8 - Exploring Composition of Functions

(A) Lesson Objectives

a. Introduce the composition of functions and how it can be represented numerically, algebraically and graphically.
b. Understand that graphical transformations are formed through composite functions

(B) Contextual Perspective

(A) Composition of Functions – An Example

- The following example will illustrate one ways of understand the composition of functions
- Andrew earns a daily wage of $20/h plus $15/d for travel expenses.
- We can demonstrate this with a table of values

Hours Worked	Daily Earnings
2	
3	
4	
5	
6	
7	
9	

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(A) Composition of Functions – An Example

- The following example will illustrate one ways of understand the composition of functions
- However, Andrew also pays union fees at 2.5% of his daily earnings, which we can write as the equation Fees = 0.025 x (daily earnings)
- We can also demonstrate with a table of values

Hours Worked	Daily Earnings	Union Fees Paid
2		
3		
4		
5		
6		
7		
9		

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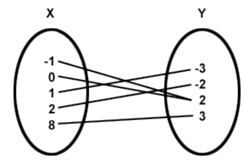
What we see is that the one function value (daily earnings or E (which is a function of hours)) is being substituted into the second function (Fees = 0.025 x daily earnings) in order to generate the final value for the union fees.

We can generate a direct formula for the union fees by substituting the earnings function into the Fees function as follows: Fees = 0.025(20h + 15).

Hence, the Fees function is called a composed function as Fees(daily earnings) = 0.025 x daily earnings

(C) <u>Introduction to Composite Functions</u>.

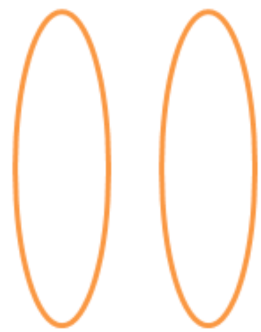


<u>Review</u> - We have explored the multiple representations of function:

- Literal
- Numerical Data (table-of-values and mapping diagrams)
- Graphical
- Algebraic

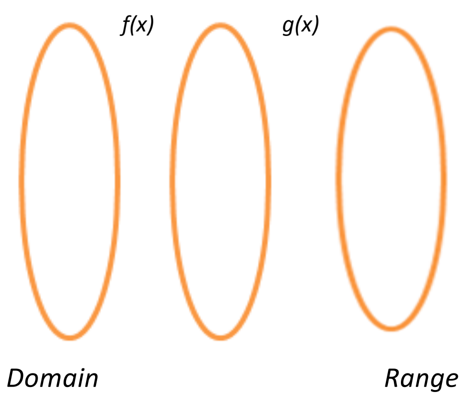
1. Let's revisit functions from a mapping perspective. Complete the mapping diagram for the function ***f(x) = x***^{2} , where the Domain = {-3,-2,-1,0, 1, 2, 3}

   What is the Range of ***f(x)*** ____________________



2. **Sometimes functions undergo <u>more than one</u> mapping.**

   Let's consider these two functions: ***f(x) = x***^{2} *and* ***g(x) = x - 1*** **,** what would this mapping look like below. Let the Domain = {-3,-2,-1,0, 1, 2, 3}



Determine the Range ______________________________

If our data is mapped via f(x) and then mapped by g(x), the question then remains:

"What is the new equation of the twice mapped data that will allow us to get the same result in 1 step?"

3. **<u>Notation</u>**

   Given the functions f(x) and g(x) where x is mapped via f(x) FIRST and then mapped AGAIN via g(x), then this IDEA or CONCEPT is represented by the notation: $g(f(x))$ or $g \circ f(x)$

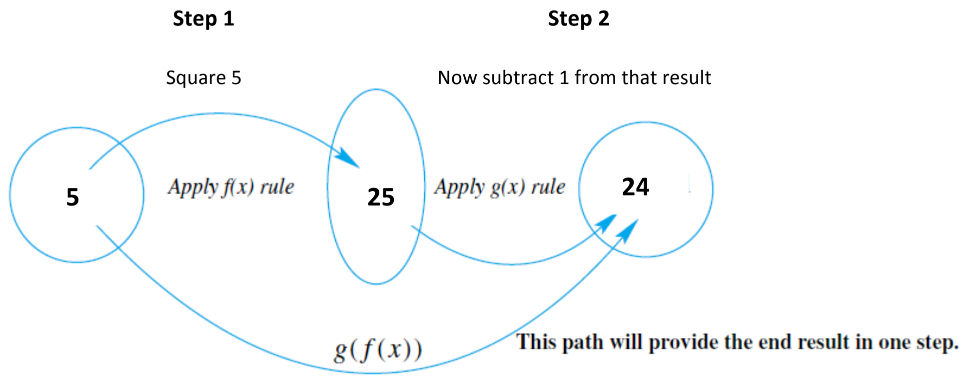
Consider the functions:

$f(x) = x^2$

$g(x) = x - 1$

Find the value of:

$g(f(x))$ or $g \circ f(x)$



4. PROCESS: Working with Composite Functions

If $f(x) = x^2$ and $g(x) = x + 1$ determine: *Model in a mapping diagram if needed*

$g \circ f(0)$	$g \circ f(3)$	$g \circ f(x)$	
$f \circ g(0)$	$f \circ g(3)$	$f \circ g(x)$	

If $f(x) = x^2$ and $g(x) = x - 2$ determine: *Model in a mapping diagram if needed*

$f \circ g(4)$	$f \circ g(1)$	$f \circ g(x)$	
$g \circ f(4)$	$g \circ f(1)$	$g \circ f(x)$	

Function Composition Worksheet

NAME ____________________________

For problems 1–4, use $f(x) = 2x^2 - x$ and $g(x) = x + 6$ to find the indicated values.

1. $(f \circ g)(2)$
2. $(g \circ f)(2)$
3. $(f \circ g)(x)$
4. $(g \circ f)(x)$

For problems 5-8, use $f(x) = \dfrac{2x+1}{3x-2}$ and $g(x) = 5x - 1$ to find the indicated values.

5. $(f \circ g)(2)$
6. $(g \circ f)(2)$
7. $(f \circ g)(x)$
8. $(g \circ f)(x)$

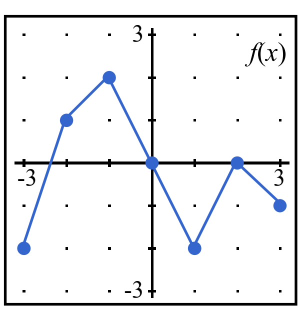
For problems 9–14, use the table definitions of $H(t)$ and $r(t)$ shown below to find the indicated value.

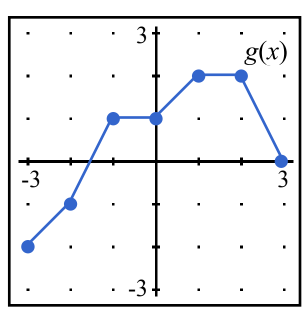
t	1.0	1.5	2.0	2.5	3.0	3.5
$H(t)$	2.8	2.6	2.5	2.0	1.0	2.2

t	2.0	2.2	2.4	2.6	2.8	3.0
$r(t)$	1.2	1.5	3.0	2.8	2.5	2.0

9. $(r \circ H)(2.5)$
10. $(r \circ H)(1.0)$
11. $(H \circ r)(2.2)$
12. $(H \circ r)(3.0)$
13. $(H \circ H)(2.0)$
14. $(r \circ r)(2.4)$

Problems 15-20 refer to the graphs of $f(x)$ and $g(x)$ shown. Find the indicated value.





15. $(f \circ g)(1)$
16. $(f \circ g)(-3)$
17. $(g \circ f)(1)$
18. $(g \circ f)(-1)$
19. $(f \circ f)(3)$
20. $(g \circ g)(0)$

5. **Observations:** What are the key things I have noticed about composite functions?

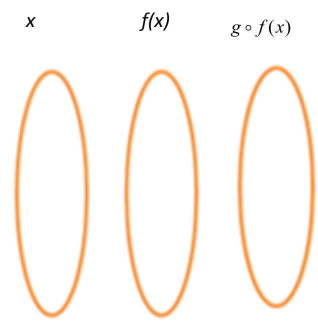
(C) Graphical Representations of Composite Functions

Consider the functions:

f(x) = x^2 and ***g(x) = x - 4***

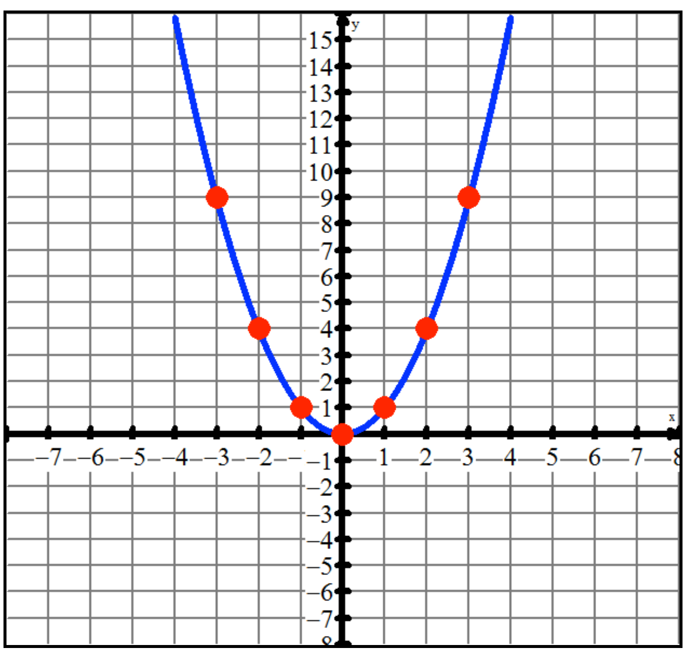
Determine $g \circ f(x)$ = ____________________

Mapping Diagram



Here is the graph of ***f(x) = x***^{2}

Graph the function $y = g \circ f(x)$ using your GDC



Graphical transformations

How has the graph of ***f(x) = x***^{2} transformed to the graph of $y = g \circ f(x)$?

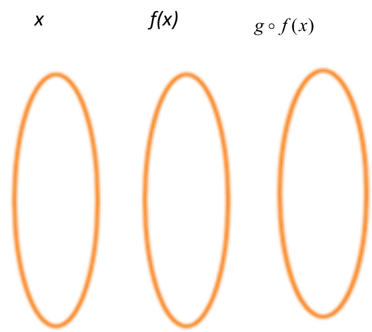
Explain graphical changes in detail.

Consider the functions:

f(x) = x² and ***g(x) = x + 3***

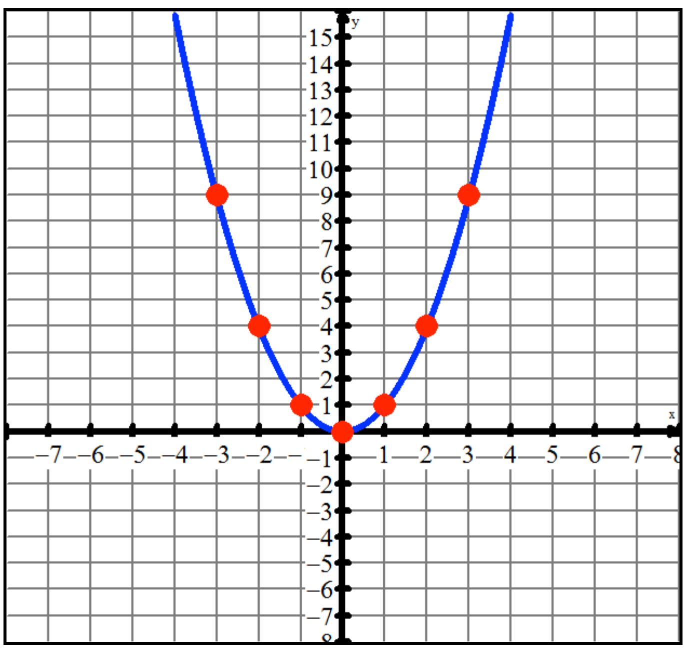
Determine $g \circ f(x)$ = ____________________

Mapping Diagram



Here is the graph of ***f(x) = x²***

Graph the function $y = g \circ f(x)$ using your GDC



Graphical transformations

How has the graph of ***f(x) = x²*** transformed to the graph of $y = g \circ f(x)$?

Explain graphical changes in detail.

<u>Check for Understanding:</u>

What do you think the graphical transformation would be for the function ***f(x) = x² - 5 ?*** Explain your thinking.

What do you think the 2 parent functions are that were composed to form ***f(x) = x² - 5 ?*** Justify your conjecture.

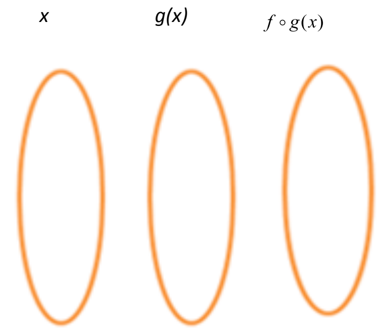
Unit 2 Lesson 8 - Exploring Composition of Functions

Consider the functions:

f(x) = x² and ***g(x) = x - 1***

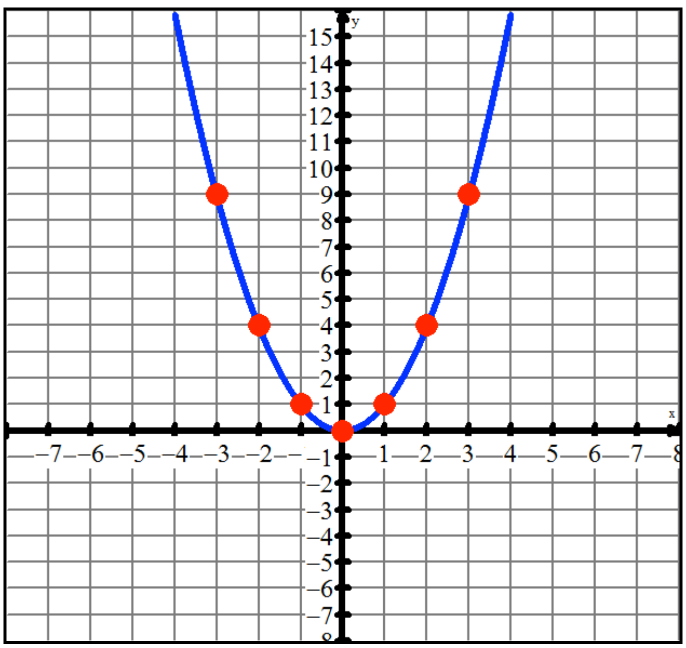
Determine $f \circ g(x)$ = ____________________

Mapping Diagram



Here is the graph of ***f(x) = x²***

Graph the function $y = f \circ g(x)$ using your GDC



Graphical transformations

How has the graph of ***f(x) = x²*** transformed to the graph of $y = f \circ g(x)$?

Explain graphical changes in detail.

Try this!

What do you think the graphical transformation would be for the function $y = (x - 6)^2$ ***?*** Explain your thinking.

What do you think the 2 parent functions are that were composed to form $y = (x - 6)^2$ ***?*** Justify your conjecture.

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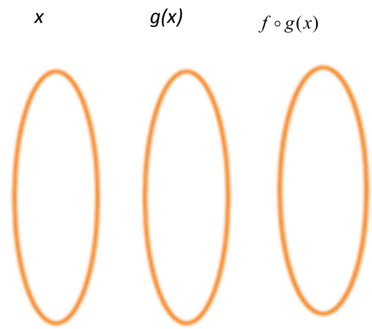
Consider the functions:

f(x) = x² and ***g(x) = x + 3***

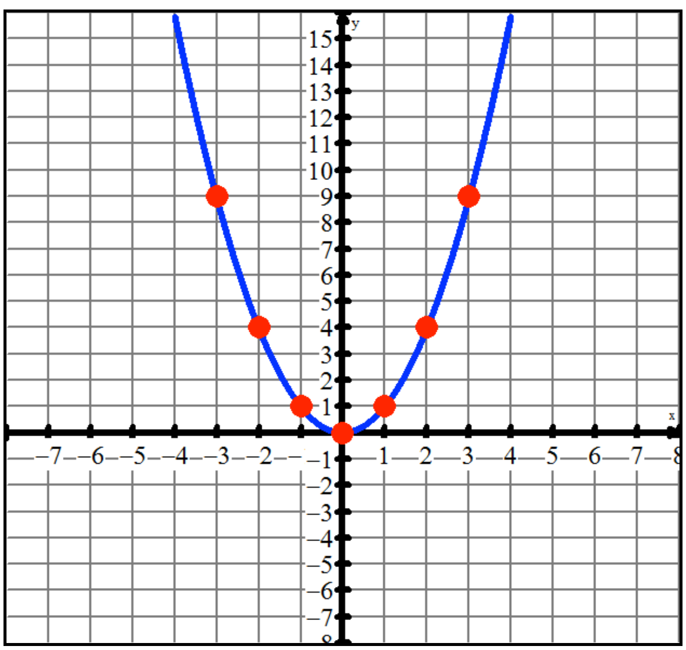
Determine $f \circ g(x)$ = ____________________

Mapping Diagram



Here is the graph of ***f(x) = x²***

Graph the function $y = f \circ g(x)$ using your GDC



Graphical transformations

How has the graph of ***f(x) = x²*** transformed to the graph of $y = f \circ g(x)$?

Explain graphical changes in detail.

<u>Try this!</u>

What do you think the graphical transformation would be for the function $y = (x + 8)^2$ ***?*** Explain your thinking.

What do you think the 2 parent functions are that were composed to form $y = (x + 8)^2$ ***?*** Justify your conjecture.

Unit 2 Lesson 8 - Exploring Composition of Functions

Understanding & Connections so Far - SUMMARY:

Describe the transformation $y = x^2$ has undergone to form $y = (x + h)^2$.

What are the parent functions that were composed to form $y = (x + h)^2$?

Describe the transformation $y = x^2$ has undergone to form $y = (x)^2 + k$.

What are the parent functions that were composed to form $y = (x)^2 + k$?

Describe the transformation $y = x^2$ has undergone to form $y = (x + h)^2 + k$.

What are the parent functions that were composed to form $y = (x + h)^2 + k$?

(D) Further Compositions

Consider the functions:

f(x) = x^2 and ***g(x) = 3x***

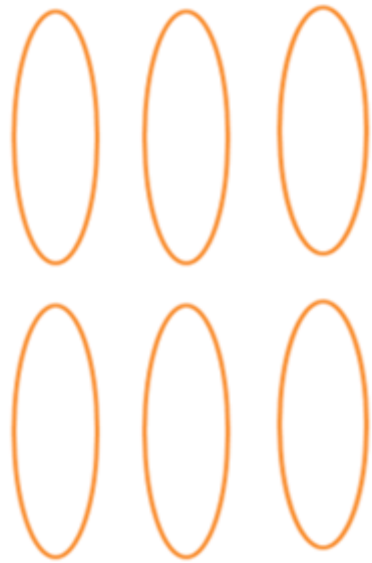
Determine $f \circ g(x)$ = ____________________

$f \circ g(2)$ = ____________________

Determine $g \circ f(x)$ = ____________________

$g \circ f(2)$ = ____________________

Mapping Diagram



Consider the functions:

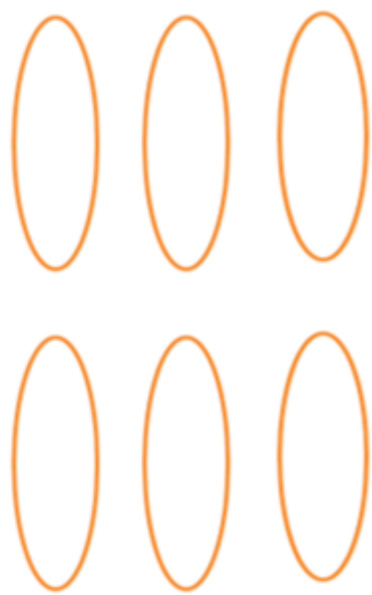
f(x) =x^2 and ***g(x) = -0.25x***

Determine $f \circ g(x)$ = ____________________

$f \circ g(2)$ = ____________________

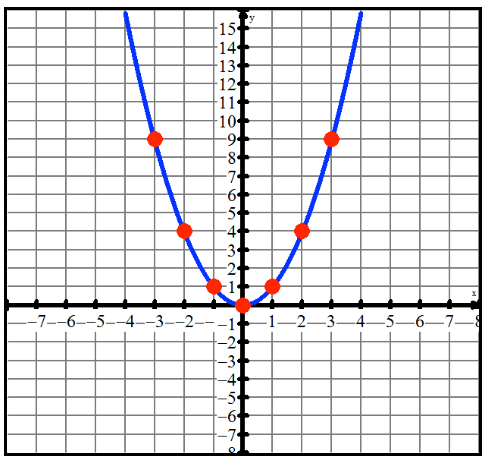
Determine $g \circ f(x)$ = ____________________

$g \circ f(2)$ = ____________________

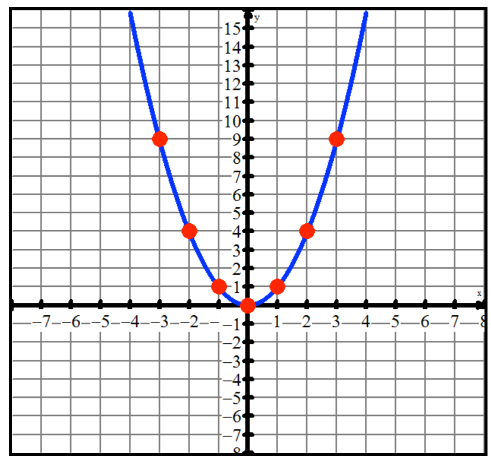


Extension:

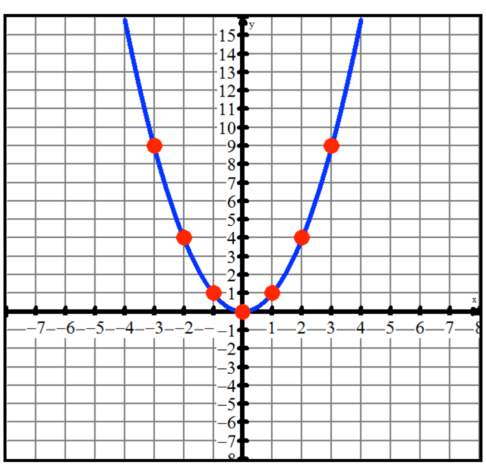
1. For the function $m(x) = 2x^2$ determine the parent functions that are composed to form ***m(x).*** Show this composition in a graph.



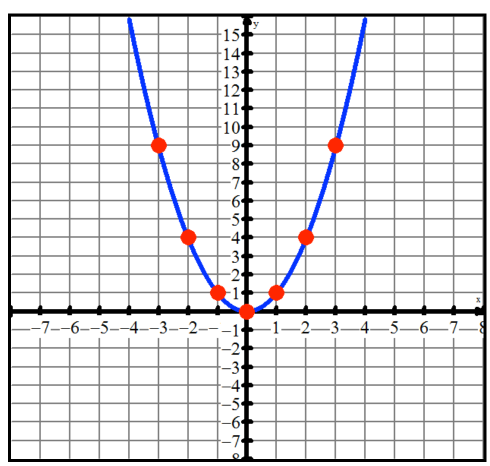
2. For the function $r(x) = -\frac{1}{2}x^2$ determine the two parent functions that are composed to form ***r(x).*** Show this composition in a graph.



3. For the function $h(x) = (2x)^2 + 1$ determine the parent functions that are composed to form $h(x)$. Show this composition in a graph.



4. Describe what transformations the function **p(x)** undergoes to form $y = -1/3(x-2)^2 + 3$. Show this composition in a graph.



Composition of Functions Worksheet Name ______________________________

I. Let $f(x) = 2x - 1$, $g(x) = 3x$, and $h(x) = x^2 + 1$. Compute the following:

1. $f(g(-3))$

2. $f(h(7))$

3. $g(h(24))$

4. $(h \circ f)(9)$

5. $(g \circ f)(0)$

6. $(h \circ g)(-4)$

7. $f(g(h(2)))$

8. $(h \circ g \circ f)(5)$

9. $g(f(h(-6)))$

II. Let $f(x) = 9 - x$, $g(x) = x^2 + x$, and $h(x) = x - 2$. Compute the following:

10. gof(3)

11. f(g(4))

12. hof(-6)

13. f(h(-3))

14. hog(11)

15. g(h(-9))

16. g(h(f(5)))

17. (hogof)(13)

18. f(g(h(-8)))