Exponential and Logarithmic Functions

MOST of the functions we have considered so far have been polynomial or rational functions, with a few others involving roots of polynomial or rational functions. Functions that can be expressed in terms of addition, subtraction, multiplication, division, and the taking of roots of variables and constants are called *algebraic functions.*

In Chapter 5 we introduce and investigate the properties of *exponential functions* and *logarithmic functions.* These functions are not algebraic; they belong to the class of *transcendental functions.* Exponential and logarithmic functions are used to model a variety of real-world phenomena: growth of populations of people, animals, and bacteria; radioactive decay; epidemics; absorption of light as it passes through air, water, or glass; magnitudes of sounds and earthquakes. We consider applications in these areas plus many more in the sections that follow.

CHAPTER 5

SECTIONS

5-2 Exponential Models

- Mathematical Modeling
- Data Analysis and Regression
- A Comparison of Exponential Growth Phenomena

In Section 5-2 we use exponential functions to model a wide variety of real-world phenomena, including growth of populations of people, animals, and bacteria; radioactive decay; spread of epidemics; propagation of rumors; light intensity; atmospheric pressure; and electric circuits. The regression techniques introduced in Chapters 2 and 3 to construct linear and quadratic models are extended to construct exponential models.

Mathematical Modeling

Populations tend to grow exponentially and at different rates. A convenient and easily understood measure of growth rate is the **doubling time**—that is, the time it takes for a population to double. Over short periods the **doubling time growth model** is often used to model population growth:

$$P = P_0 2^{t/d}$$

where
$$\begin{aligned} P &= \text{Population at time } t \\ P_0 &= \text{Population at time } t = 0 \\ d &= \text{Doubling time} \end{aligned}$$

Note that when $t = d$,

$$P = P_0 2^{d/d} = P_0 2$$

and the population is double the original, as it should be. We use this model to solve a population growth problem in Example 1.

EXAMPLE 1 Population Growth

Nicaragua has a population of approximately 6 million and it is estimated that the population will double in 36 years. If population growth continues at the same rate, what will be the population:

(A) 15 years from now? (B) 40 years from now?

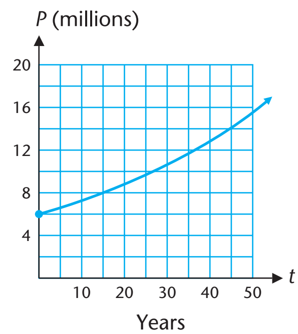
SOLUTIONS

We use the doubling time growth model:

$$P = P_0 2^{t/d}$$

Substituting $P_0 = 6$ and $d = 36$, we obtain

$$P = 6(2^{t/36}) \qquad \text{Figure 1}$$



› **Figure 1**
$P = 6(2^{t/36})$.

(A) Find P when $t = 15$ years:

$$P = 6(2^{15/36}) \approx 8 \text{ million}$$

(B) Find P when $t = 40$ years:

$$P = 6(2^{40/36}) \approx 13 \text{ million}$$

MATCHED PROBLEM 1

The bacterium *Escherichia coli* (*E. coli*) is found naturally in the intestines of many mammals. In a particular laboratory experiment, the doubling time for *E. coli* is found to be 25 minutes. If the experiment starts with a population of 1,000 *E. coli* and there is no change in the doubling time, how many bacteria will be present:

(A) In 10 minutes? (B) In 5 hours?

Write answers to three significant digits.

››› EXPLORE-DISCUSS 1

The doubling time growth model would *not* be expected to give accurate results over long periods. According to the doubling time growth model of Example 1, what was the population of Nicaragua 500 years ago when it was settled as a Spanish colony? What will the population of Nicaragua be 200 years from now? Explain why these results are unrealistic. Discuss factors that affect human populations that are not taken into account by the doubling time growth model.

As an alternative to the doubling time growth model, we can use the equation

$$y = ce^{kt}$$

where y = Population at time t
c = Population at time 0
k = Relative growth rate

The **relative growth rate** k has the following interpretation: Suppose that $y = ce^{kt}$ models the population growth of a country, where y is the number of persons and t is time in years. If the relative growth rate is $k = 0.03$, then at any time t, the population is growing at a rate of $0.03y$ persons (that is, 3% of the population) per year. Example 2 illustrates this approach.

EXAMPLE 2 Medicine—Bacteria Growth

Cholera, an intestinal disease, is caused by a cholera bacterium that multiplies exponentially by cell division as modeled by

$$N = N_0e^{1.386t}$$

where N is the number of bacteria present after t hours and N_0 is the number of bacteria present at $t = 0$. If we start with 1 bacterium, how many bacteria will be present in

(A) 5 hours? (B) 12 hours?

Compute the answers to three significant digits.

SOLUTIONS

(A) Use $N_0 = 1$ and $t = 5$:

$$\begin{aligned} N &= N_0e^{1.386t} && \text{Let } N_0 = 1 \text{ and } t = 5. \\ &= e^{1.386(5)} && \text{Calculate to three significant digits.} \\ &= 1{,}020 \end{aligned}$$

(B) Use $N_0 = 1$ and $t = 12$:

$$\begin{aligned} N &= N_0 e^{1.386t} && \text{Let } N_0 = 1 \text{ and } t = 12. \\ &= e^{1.386(12)} && \text{Calculate to three significant digits.} \\ &= 16{,}700{,}000 \end{aligned}$$

MATCHED PROBLEM 2

Repeat Example 2 if $N = N_0 e^{0.783t}$ and all other information remains the same.

Exponential functions can also be used to model radioactive decay, which is sometimes referred to as *negative growth.* Radioactive materials are used extensively in medical diagnosis and therapy, as power sources in satellites, and as power sources in many countries. If we start with an amount A_0 of a particular radioactive isotope, the amount declines exponentially in time. The rate of decay varies from isotope to isotope. A convenient and easily understood measure of the rate of decay is the **half-life** of the isotope—that is, the time it takes for half of a particular material to decay. We use the following **half-life decay model:**

$$\begin{aligned} A &= A_0 \left(\tfrac{1}{2}\right)^{t/h} \\ &= A_0 2^{-t/h} \end{aligned}$$

where A = Amount at time t

A_0 = Amount at time $t = 0$

h = Half-life

Note that when $t = h$,

$$A = A_0 2^{-h/h} = A_0 2^{-1} = \frac{A_0}{2}$$

and the amount of isotope is half the original amount, as it should be.

EXAMPLE 3 Radioactive Decay

The radioactive isotope gallium 67 (^{67}Ga), used in the diagnosis of malignant tumors, has a biological half-life of 46.5 hours. If we start with 100 milligrams of the isotope, how many milligrams will be left after

(A) 24 hours? (B) 1 week?

Compute answers to three significant digits.

SOLUTIONS

We use the half-life decay model:

$$A = A_0\left(\tfrac{1}{2}\right)^{t/h} = A_0 2^{-t/h}$$

Using $A_0 = 100$ and $h = 46.5$, we obtain

$$A = 100(2^{-t/46.5}) \qquad \text{Figure 2}$$

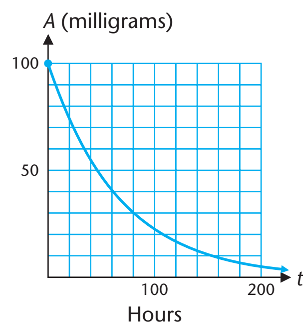


Figure 2
$A = 100(2^{-t/46.5})$.

(A) Find A when $t = 24$ hours:

$$\begin{aligned} A &= 100(2^{-24/46.5}) \qquad \text{Calculate to three significant digits.} \\ &= 69.9 \text{ milligrams} \end{aligned}$$

(B) Find A when $t = 168$ hours
(1 week = 168 hours):

$$\begin{aligned} A &= 100(2^{-168/46.5}) \qquad \text{Calculate to three significant digits.} \\ &= 8.17 \text{ milligrams} \end{aligned}$$

MATCHED PROBLEM 3

Radioactive gold 198 (^{198}Au), used in imaging the structure of the liver, has a half-life of 2.67 days. If we start with 50 milligrams of the isotope, how many milligrams will be left after:

(A) $\frac{1}{2}$ day? (B) 1 week?

Compute answers to three significant digits.

As an alternative to the half-life decay model, we can use the equation $y = ce^{-kt}$, where c and k are positive constants, to model radioactive decay. Example 4 illustrates this approach.

EXAMPLE 4 Carbon-14 Dating

Cosmic-ray bombardment of the atmosphere produces neutrons, which in turn react with nitrogen to produce radioactive carbon-14. Radioactive carbon-14 enters all living tissues through carbon dioxide, which is first absorbed by plants. As long as a plant or animal is alive, carbon-14 is maintained in the living organism at a constant level. Once the organism dies, however, carbon-14 decays according to the equation

$$A = A_0 e^{-0.000124t}$$

where A is the amount of carbon-14 present after t years and A_0 is the amount present at time $t = 0$. If 1,000 milligrams of carbon-14 are present at the start, how many milligrams will be present in

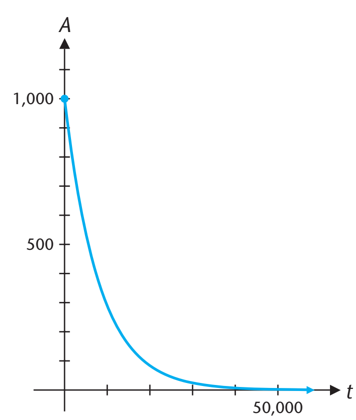
(A) 10,000 years? (B) 50,000 years?

Compute answers to three significant digits.

SOLUTIONS

Substituting $A_0 = 1{,}000$ in the decay equation, we have

$$A = 1{,}000e^{-0.000124t}$$ Figure 3



› Figure 3

(A) Solve for A when $t = 10{,}000$:

$$\begin{aligned} A &= 1{,}000e^{-0.000124(10{,}000)} \\ &= 289 \text{ milligrams} \end{aligned}$$ Calculate to three significant digits.

(B) Solve for A when $t = 50{,}000$:

$$\begin{aligned} A &= 1{,}000e^{-0.000124(50{,}000)} \\ &= 2.03 \text{ milligrams} \end{aligned}$$ Calculate to three significant digits.

More will be said about carbon-14 dating in Exercise 5-5, where we will be interested in solving for t after being given information about A and A_0.

MATCHED PROBLEM 4

Referring to Example 4, how many milligrams of carbon-14 would have to be present at the beginning to have 10 milligrams present after 20,000 years? Compute the answer to four significant digits.

We can model phenomena such as learning curves, for which growth has an upper bound, by the equation $y = c(1 - e^{-kt})$, where c and k are positive constants. Example 5 illustrates such limited growth.

EXAMPLE 5

Learning Curve

People assigned to assemble circuit boards for a computer manufacturing company undergo on-the-job training. From past experience, it was found that the learning curve for the average employee is given by

$$N = 40(1 - e^{-0.12t})$$

where N is the number of boards assembled per day after t days of training (Fig. 4).

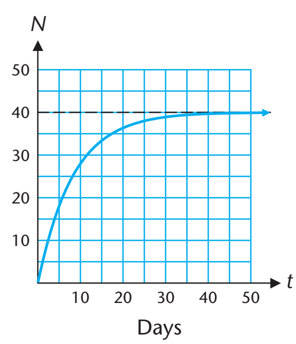


Figure 4
$N = 40(1 - e^{-0.12t})$.

(A) How many boards can an average employee produce after 3 days of training? After 5 days of training? Round answers to the nearest integer.

(B) Does N approach a limiting value as t increases without bound? Explain.

SOLUTION

(A) When $t = 3$,

$$N = 40(1 - e^{-0.12(3)}) = 12 \quad \text{Rounded to nearest integer}$$

so the average employee can produce 12 boards after 3 days of training. Similarly, when $t = 5$,

$$N = 40(1 - e^{-0.12(5)}) = 18 \quad \text{Rounded to nearest integer}$$

Because $e^{-0.12t}$ approaches 0 as t increases without bound,

$$N = 40(1 - e^{-0.12t}) \rightarrow 40(1 - 0) = 40$$

So the limiting value of N is 40 boards per day. (Note the horizontal asymptote with equation $N = 40$ that is indicated by the dashed line in Fig. 4.)

MATCHED PROBLEM 5

A company is trying to expose as many people as possible to a new product through television advertising in a large metropolitan area with 2 million potential viewers. A model for the number of people N, in millions, who are aware of the product after t days of advertising was found to be

$$N = 2(1 - e^{-0.037t})$$

(A) How many viewers are aware of the product after 2 days? After 10 days? Express answers as integers, rounded to three significant digits.

(B) Does N approach a limiting value as t increases without bound? Explain.

We can model phenomena such as the spread of an epidemic or the propagation of a rumor by the *logistic equation*.

$$y = \frac{M}{(1 + ce^{-kt})}$$

where M, c, and k are positive constants. Logistic growth, illustrated in Example 6, approaches a limiting value as t increases without bound.

EXAMPLE 6 Logistic Growth in an Epidemic

A community of 1,000 individuals is assumed to be homogeneously mixed. One individual who has just returned from another community has influenza. Assume the community has not had influenza shots and all are susceptible. The spread of the disease in the community is predicted to be given by the logistic curve

$$N(t) = \frac{1{,}000}{1 + 999e^{-0.3t}}$$

where N is the number of people who have contracted influenza after t days (Fig. 5).

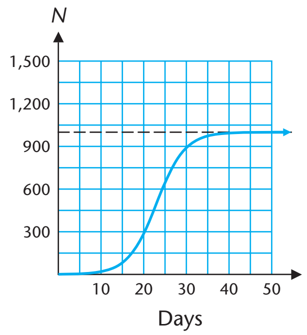


Figure 5
$N = \dfrac{1{,}000}{1 + 999e^{-0.3t}}$.

(A) How many people have contracted influenza after 10 days? After 20 days? Round answers to the nearest integer?

(B) Does N approach a limiting value as t increases without bound? Explain.

SOLUTIONS

(A) When $t = 10$,

$$N = \frac{1{,}000}{1 + 999e^{-0.3(10)}} = 20 \quad \text{Rounded to nearest integer}$$

so 20 people have contracted influenza after 10 days. Similarly, when $t = 20$,

$$N = \frac{1{,}000}{1 + 999e^{-0.3(20)}} = 288 \quad \text{Rounded to nearest integer}$$

so 288 people have contracted influenza after 20 days.

(B) Because $e^{-0.3t}$ approaches 0 as t increases without bound,

$$N = \frac{1{,}000}{1 + 999e^{-0.3t}} \rightarrow \frac{1{,}000}{1 + 999(0)} = 1{,}000$$

So the limiting value is 1,000 individuals (all in the community will eventually contract influenza). (Note the horizontal asymptote with equation $N = 1{,}000$ that is indicated by the dashed line in Fig. 5.)

MATCHED PROBLEM 6

A group of 400 parents, relatives, and friends are waiting anxiously at Kennedy Airport for a charter flight returning students after a year in Europe. It is stormy and the plane is late. A particular parent thought he had heard that the plane's radio had gone out and related this news to some friends, who in turn passed it on to others. The propagation of this rumor is predicted to be given by

$$N(t) = \frac{400}{1 + 399e^{-0.4t}}$$

where N is the number of people who have heard the rumor after t minutes.

(A) How many people have heard the rumor after 10 minutes? After 20 minutes? Round answers to the nearest integer.

(B) Does N approach a limiting value as t increases without bound? Explain.

Data Analysis and Regression

We use exponential regression to fit a function of the form $y = ab^x$ to a set of data points, and logistic regression to fit a function of the form

$$y = \frac{c}{1 + ae^{-bx}}$$

to a set of data points. The techniques are similar to those introduced in Chapters 2 and 3 for linear and quadratic functions.

EXAMPLE 7 Infectious Diseases

The U.S. Department of Health and Human Services published the data in Table 1.

Table 1 Reported Cases of Infectious Diseases

Year	Mumps	Rubella
1970	104,953	56,552
1980	8,576	3,904
1990	5,292	1,125
1995	906	128
2000	323	152

An exponential model for the data on mumps is given by

$$N = 91{,}400(0.835)^t$$

where N is the number of reported cases of mumps and t is time in years with $t = 0$ representing 1970.

(A) Use the model to predict the number of reported cases of mumps in 2010.

(B) Compare the actual number of cases of mumps reported in 1980 to the number given by the model.

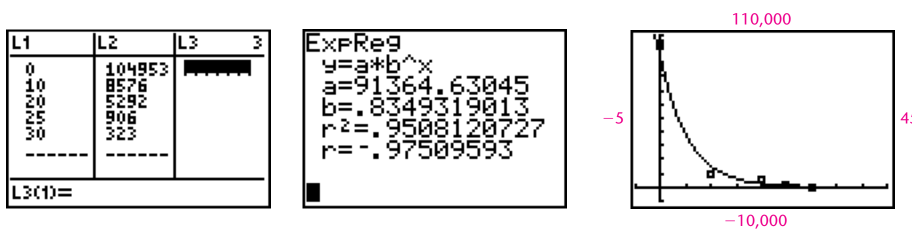
SOLUTIONS

(A) The year 2010 is represented by $t = 40$. Evaluating $N = 91{,}400(0.835)^t$ at $t = 40$ gives a prediction of 67 cases of mumps in 2010.

(B) The year 1980 is represented by $t = 10$. Evaluating $N = 91{,}400(0.835)^t$ at $t = 10$ gives 15,060 cases in 1980. The actual number of cases reported in 1980 was 8,576, nearly 6,500 less than the number given by the model.

Technology Connections

Figure 6 shows the details of constructing the exponential model of Example 7 on a graphing calculator.



(a) Entering the data (b) Finding the model (c) Graphing the data and the model

Figure 6

MATCHED PROBLEM 7

An exponential model for the data on rubella in Table 1 is given by

$$N = 44{,}500(0.815)^t$$

where N is the number of reported cases of rubella and t is time in years with $t = 0$ representing 1970.

(A) Use the model to predict the number of reported cases of rubella in 2010.

(B) Compare the actual number of cases of rubella reported in 1980 to the number given by the model.

EXAMPLE 8 AIDS Cases and Deaths

The U.S. Department of Health and Human Services published the data in Table 2.

Table 2 Acquired Immunodeficiency Syndrome (AIDS) Cases and Deaths in the United States

Year	Cases Diagnosed to Date	Known Deaths to Date
1985	23,185	12,648
1988	107,755	62,468
1991	261,259	159,294
1994	493,713	296,507
1997	672,970	406,179
2000	774,467	447,648
2003	929,985	524,060

A logistic model for the data on AIDS cases is given by

$$N = \frac{948{,}000}{1 + 17.8e^{-0.317t}}$$

where N is the number of AIDS cases diagnosed by year t with $t = 0$ representing 1985.

(A) Use the model to predict the number of AIDS cases diagnosed by 2010.

(B) Compare the actual number of AIDS cases diagnosed by 2003 to the number given by the model.

SOLUTIONS

(A) The year 2010 is represented by $t = 25$. Evaluating

$$N = \frac{948{,}000}{1 + 17.8e^{-0.317t}}$$

at $t = 25$ gives a prediction of approximately 942,000 cases of AIDS diagnosed by 2010.

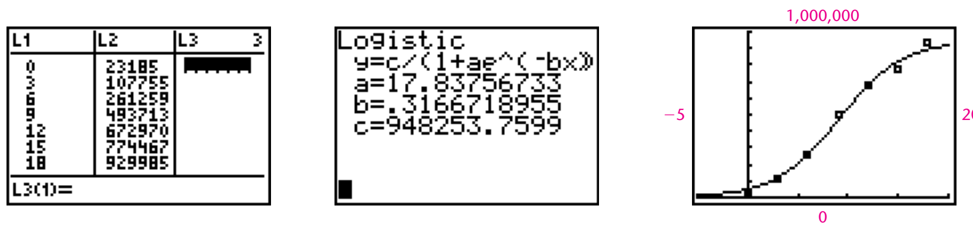
(B) The year 2003 is represented by $t = 18$. Evaluating

$$N = \frac{948{,}000}{1 + 17.8e^{-0.317t}}$$

at $t = 18$ gives 895,013 cases in 2003. The actual number of cases diagnosed by 2003 was 929,985, nearly 35,000 greater than the number given by the model.

Technology Connections

Figure 7 shows the details of constructing the logistic model of Example 7 on a graphing calculator.



(a) Entering the data (b) Finding the model (c) Graphing the data and the model

Figure 7

MATCHED PROBLEM 8

A logistic model for the data on deaths from AIDS in Table 2 is given by

$$N = \frac{520{,}000}{1 + 19.3e^{-0.353t}}$$

where N is the number of known deaths from AIDS by year t with $t = 0$ representing 1985.

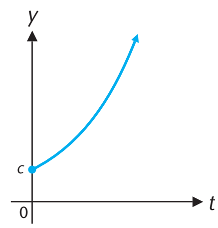
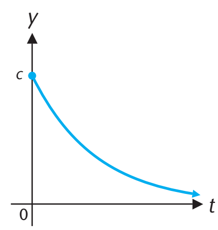
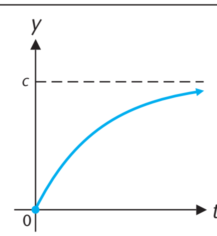
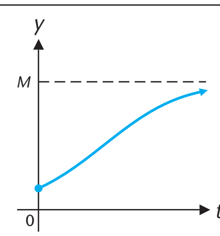
(A) Use the model to predict the number of known deaths from AIDS by 2010.

(B) Compare the actual number of known deaths from AIDS by 2003 to the number given by the model.

A Comparison of Exponential Growth Phenomena

The equations and graphs given in Table 3 compare the growth models discussed in Examples 1 through 8. Following each equation and graph is a short, incomplete list of areas in which the models are used. In the first case (unlimited growth), $y \to \infty$ as $t \to \infty$. In the other three cases (exponential decay, limited growth, and logistic growth), the graph approaches a horizontal asymptote as $t \to \infty$; these asymptotes ($y = 0$, $y = c$, and $y = M$, respectively) are easily deduced from the given equations. Table 3 only touches on a subject that you are likely to study in greater depth in the future.

Table 3 Exponential Growth and Decay

Description	Equation	Graph	Uses
Unlimited growth	$y = ce^{kt}$ $c, k > 0$		Short-term population growth (people, bacteria. etc.); growth of money at continuous compound interest
Exponential decay	$y = ce^{-kt}$ $c, k > 0$		Radioactive decay; light absorption in water, glass, and the like; atmospheric pressure; electric circuits
Limited growth	$y = c(1 - e^{-kt})$ $c, k > 0$		Learning skills; sales fads; company growth; electric circuits
Logistic growth	$y = \dfrac{M}{1 + ce^{-kt}}$ $c, k, M > 0$		Long-term population growth; epidemics; sales of new products; company growth

ANSWERS TO MATCHED PROBLEMS

1. (A) 1,320 bacteria (B) $4{,}100{,}100 = 4.10 \times 10^6$ bacteria
2. (A) 50 bacteria (B) 12,000 bacteria
3. (A) 43.9 milligrams (B) 8.12 milligrams
4. 119.4 milligrams
5. (A) 143,000 viewers; 619,000 viewers
   (B) N approaches an upper limit of 2 million, the number of potential viewers
6. (A) 48 individuals; 353 individuals
   (B) N approaches an upper limit of 400, the number of people in the entire group.
7. (A) 12 cases
   (B) The actual number of cases was 1,850 less than the number given by the model.
8. (A) 519,000 deaths
   (B) The actual number of known deaths was approximately 21,000 greater than the number given by the model.

5-2 Exercises

APPLICATIONS

1. GAMING A person bets on red and black on a roulette wheel using a *Martingale strategy.* That is, a \$2 bet is placed on red, and the bet is doubled each time until a win occurs. The process is then repeated. If black occurs n times in a row, then $L = 2^n$ dollars is lost on the nth bet. Graph this function for $1 \le n \le 10$. Although the function is defined only for positive integers, points on this type of graph are usually joined with a smooth curve as a visual aid.

2. BACTERIAL GROWTH If bacteria in a certain culture double every $\frac{1}{2}$ hour, write an equation that gives the number of bacteria N in the culture after t hours, assuming the culture has 100 bacteria at the start. Graph the equation for $0 \le t \le 5$.

3. POPULATION GROWTH Because of its short life span and frequent breeding, the fruit fly *Drosophila* is used in some genetic studies. Raymond Pearl of Johns Hopkins University, for example, studied 300 successive generations of descendants of a single pair of *Drosophila* flies. In a laboratory situation with ample food supply and space, the doubling time for a particular population is 2.4 days. If we start with 5 male and 5 female flies, how many flies should we expect to have in
(A) 1 week? (B) 2 weeks?

4. POPULATION GROWTH If Kenya has a population of about 34,000,000 people and a doubling time of 27 years and if the growth continues at the same rate, find the population in
(A) 10 years (B) 30 years

Compute answers to 2 significant digits.

5. INSECTICIDES The use of the insecticide DDT is no longer allowed in many countries because of its long-term adverse effects. If a farmer uses 25 pounds of active DDT, assuming its half-life is 12 years, how much will still be active after
(A) 5 years? (B) 20 years?

Compute answers to two significant digits.

6. RADIOACTIVE TRACERS The radioactive isotope technetium-99m (^{99m}Tc) is used in imaging the brain. The isotope has a half-life of 6 hours. If 12 milligrams are used, how much will be present after
(A) 3 hours? (B) 24 hours?

Compute answers to three significant digits.

7. POPULATION GROWTH If the world population is about 6.5 billion people now and if the population grows continuously at a relative growth rate of 1.14%, what will the population be in 10 years? Compute the answer to two significant digits.

8. POPULATION GROWTH If the population in Mexico is around 106 million people now and if the population grows continuously at a relative growth rate of 1.17%, what will the population be in 8 years? Compute the answer to three significant digits.

9. POPULATION GROWTH In 2005 the population of Russia was 143 million and the population of Nigeria was 129 million. If the populations of Russia and Nigeria grow continuously at relative growth rates of -0.37% and 2.56%, respectively, in what year will Nigeria have a greater population than Russia?

10. POPULATION GROWTH In 2005 the population of Germany was 82 million and the population of Egypt was 78 million. If the populations of Germany and Egypt grow continuously at relative growth rates of 0% and 1.78%, respectively, in what year will Egypt have a greater population than Germany?

11. SPACE SCIENCE Radioactive isotopes, as well as solar cells, are used to supply power to space vehicles. The isotopes gradually lose power because of radioactive decay. On a particular space vehicle the nuclear energy source has a power output of P watts after t days of use as given by

$$P = 75e^{-0.0035t}$$

Graph this function for $0 \le t \le 100$.

12. EARTH SCIENCE The atmospheric pressure P, in pounds per square inch, decreases exponentially with altitude h, in miles above sea level, as given by

$$P = 14.7e^{-0.21h}$$

Graph this function for $0 \le h \le 10$.

13. MARINE BIOLOGY Marine life is dependent upon the microscopic plant life that exists in the *photic zone,* a zone that goes to a depth where about 1% of the surface light still remains. Light intensity I relative to depth d, in feet, for one of the clearest bodies of water in the world, the Sargasso Sea in the West Indies, can be approximated by

$$I = I_0 e^{-0.00942d}$$

where I_0 is the intensity of light at the surface. To the nearest percent, what percentage of the surface light will reach a depth of
(A) 50 feet? (B) 100 feet?

14. MARINE BIOLOGY Refer to Problem 13. In some waters with a great deal of sediment, the photic zone may go down only 15 to 20 feet. In some murky harbors, the intensity of light d feet below the surface is given approximately by

$$I = I_0 e^{-0.23d}$$

What percentage of the surface light will reach a depth of
(A) 10 feet? (B) 20 feet?

15. AIDS EPIDEMIC The World Health Organization estimated that 39.4 million people worldwide were living with HIV in 2004. Assuming that number continues to increase at a relative growth rate of 3.2% compounded continuously, estimate the number of people living with HIV in
(A) 2010 (B) 2015

16. AIDS EPIDEMIC The World Health Organization estimated that there were 3.1 million deaths worldwide from HIV/AIDS during the year 2004. Assuming that number continues to increase at a relative growth rate of 4.3% compounded continuously, estimate the number of deaths from HIV/AIDS during the year
(A) 2008 (B) 2012

17. NEWTON'S LAW OF COOLING This law states that the rate at which an object cools is proportional to the difference in temperature between the object and its surrounding medium. The temperature T of the object t hours later is given by

$$T = T_m + (T_0 - T_m)e^{-kt}$$

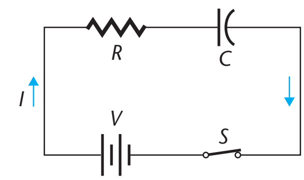
where T_m is the temperature of the surrounding medium and T_0 is the temperature of the object at $t = 0$. Suppose a bottle of wine at a room temperature of 72°F is placed in the refrigerator to cool before a dinner party. If the temperature of in the refrigerator is kept at 40°F and $k = 0.4$, find the temperature of the wine, to the nearest degree, after 3 hours. (In Exercise 5-5 we will find out how to determine k.)

18. NEWTON'S LAW OF COOLING Refer to Problem 17. What is the temperature, to the nearest degree, of the wine after 5 hours in the refrigerator?

19. PHOTOGRAPHY An electronic flash unit for a camera is activated when a capacitor is discharged through a filament of wire. After the flash is triggered, and the capacitor is discharged, the circuit (see the figure) is connected and the battery pack generates a current to recharge the capacitor. The time it takes for the capacitor to recharge is called the *recycle time*. For a particular flash unit using a 12-volt battery pack, the charge q, in coulombs, on the capacitor t seconds after recharging has started is given by

$$q = 0.0009(1 - e^{-0.2t})$$

Find the value that q approaches as t increases without bound and interpret.



20. MEDICINE An electronic heart pacemaker uses the same type of circuit as the flash unit in Problem 19, but it is designed so that the capacitor discharges 72 times a minute. For a particular pacemaker, the charge on the capacitor t seconds after it starts recharging is given by

$$q = 0.000\,008(1 - e^{-2t})$$

Find the value that q approaches as t increases without bound and interpret.

21. WILDLIFE MANAGEMENT A herd of 20 white-tailed deer is introduced to a coastal island where there had been no deer before. Their population is predicted to increase according to the logistic curve

$$N = \frac{100}{1 + 4e^{-0.14t}}$$

where N is the number of deer expected in the herd after t years.
(A) How many deer will be present after 2 years? After 6 years? Round answers to the nearest integer.
(B) How many years will it take for the herd to grow to 50 deer? Round answer to the nearest integer.
(C) Does N approach *a* limiting value as t increases without bound? Explain.

22. TRAINING A trainee is hired by a computer manufacturing company to learn to test a particular model of a personal computer after it comes off the assembly line. The learning curve for an average trainee is given by

$$N = \frac{200}{4 + 21e^{-0.1t}}$$

(A) How many computers can an average trainee be expected to test after 3 days of training? After 6 days? Round answers to the nearest integer.
(B) How many days will it take until an average trainee can test 30 computers per day? Round answer to the nearest integer.
(C) Does N approach a limiting value as t increases without bound? Explain.