

Rational functions of the form $y = \frac{ax+b}{cx+d}$

→ Every rational function of the form $y = \frac{ax+b}{cx+d}$ has a graph called a hyperbola.

The graph of any rational function $y = \frac{ax+b}{cx+d}$ has a vertical and a horizontal asymptote.

Investigation – graphing rational functions 2

a Use your GDC to show sketches of

$$y = \frac{x}{x+3}, y = \frac{x+1}{x+3}, y = \frac{2x}{x+3} \text{ and } y = \frac{2x-1}{x+3}$$

b Copy and complete the table.

Rational function	Vertical asymptote	Horizontal asymptote	Domain	Range
$y = \frac{x}{x+3}$				
$y = \frac{x+1}{x+3}$				
$y = \frac{2x}{x+3}$				
$y = \frac{2x-1}{x+3}$				

c What do you notice about the horizontal asymptotes?

d What do you notice about the domain and the value of the vertical asymptote?

→ The vertical asymptote occurs at the x -value that makes the denominator zero.

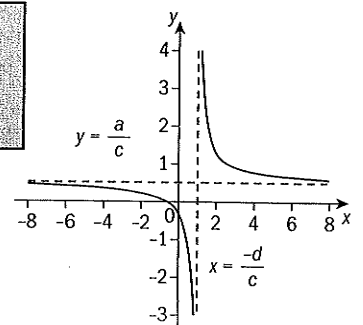
→ The horizontal asymptote is the line $y = \frac{a}{c}$.

To find the horizontal asymptote rearrange the equation to make x the subject.

$$\begin{aligned} y &= \frac{ax+b}{cx+d} \\ y(cx+d) &= ax+b \\ cyx - ax &= b - dy \\ x &= \frac{b-dy}{cy-a} \end{aligned}$$

The horizontal asymptote occurs when the denominator is zero, that is, when

$$cy = a \text{ or } y = \frac{a}{c}$$



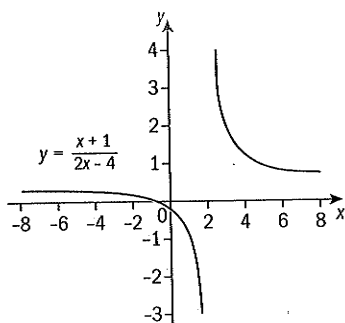
Example 5

For the function $y = \frac{x+1}{2x-4}$

- a sketch the graph
- b find the vertical and horizontal asymptotes
- c state the domain and range.

Answers

a



b Vertical asymptote $x = 2$

Horizontal asymptote $y = \frac{1}{2}$

c Domain $x \in \mathbb{R}, x \neq 2$

Range $y \in \mathbb{R}, y \neq \frac{1}{2}$

When $2x - 4 = 0, x = 2$

$a = 1, c = 2, y = \frac{a}{c}$

Exercise 5D

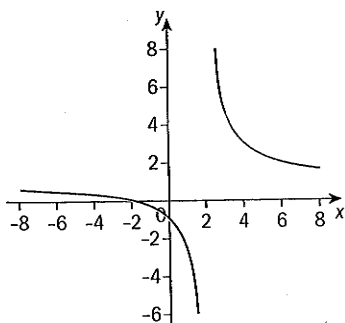
- 1 Identify the horizontal and vertical asymptotes of these functions and then state the domain and range.

a $y = \frac{x+2}{x-3}$ b $y = \frac{2x+2}{3x-1}$ c $y = \frac{-3x+2}{-4x-5}$ d $y = \frac{34x-2}{16x+4}$

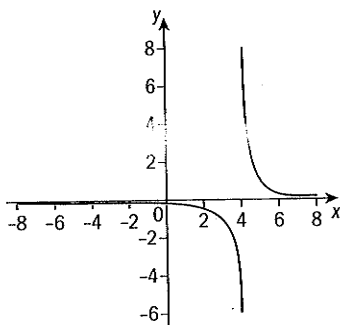
- 2 Match the function with the graph.

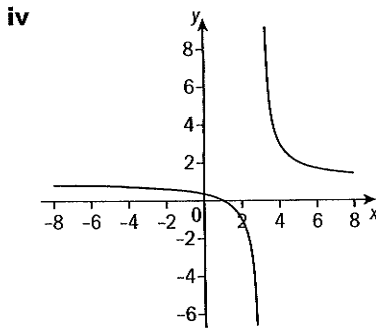
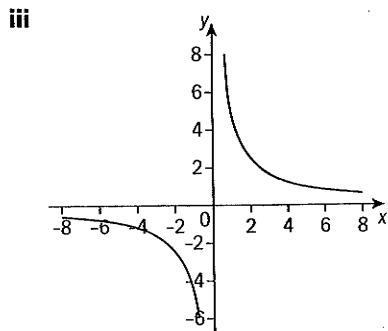
a $y = \frac{5}{x}$ b $y = \frac{x+2}{x-2}$ c $y = \frac{x-1}{x-3}$ d $y = \frac{1}{x-4}$

i



ii





3 Sketch each function using your GDC and state the domain and range.

a $y = \frac{x+2}{x+3}$

b $y = \frac{x}{4x+3}$

c $y = \frac{x-7}{3x-8}$

d $y = \frac{9x+1}{3x-2}$

e $y = \frac{-3x+10}{4x-12}$

f $y = \frac{5x+2}{4x}$

g $y = \frac{3x}{2x-4}$

h $y = \frac{7x}{-x-15}$

i $y = \frac{14x-4}{2x-1}$

Check your answer by using your GDC to graph the function.

4 Write a rational function that has a vertical asymptote at $x = -4$ and a horizontal asymptote at $y = 3$

5 Chris and Lee design T-shirts for surfers and set up a T-shirt printing business in their garage. It will cost \$450 to set up the equipment and they estimate that it will cost \$5.50 to print each T-shirt.

- Write a linear function $C(x)$ giving the total cost of producing x T-shirts. Remember to take the set-up cost into account.
- Write a rational function $A(x)$ giving the **average cost** per T-shirt of producing x of them.
- What is the domain of $A(x)$ in the context of the problem? Explain.
- Write down the vertical asymptote of $A(x)$.
- Find the horizontal asymptote for $A(x)$. What meaning does this value have in the context of the problem?



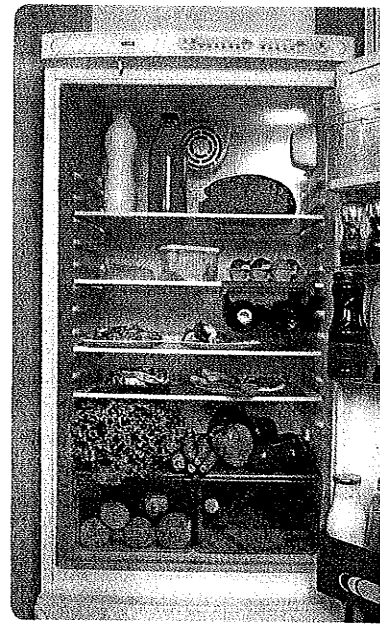
Sketch the function.

EXAM-STYLE QUESTION

- 6 Young's rule is a way of calculating doses of medicine for children over the age of two, based on the adult dose. 'Take the age of the child in years and divide by their age plus 12. Multiply this number by the adult dose.' This is modeled by the function $c = \frac{at}{t+12}$ where c is the child's dose, a is the adult dose in mg and t is the age of the child in years.



- a Make a table of values for ages 2 to 12 with an adult dose of 100 mg.
 - b Use your values from a to draw a graph of the function.
 - c Use the graph to estimate the dose for a $7\frac{1}{2}$ -year old.
 - d Write down the equation of the horizontal asymptote.
 - e What does the value of the horizontal asymptote mean for Young's rule?
7. The average cost of electricity per year for a refrigerator is \$92.
- a A new refrigerator costs \$550. Determine the total annual cost for a refrigerator that lasts for 15 years. Assume costs include purchase and electricity.
 - b Develop a function that gives the annual cost of a refrigerator as a function of the number of years you own the refrigerator.
 - c Sketch a graph of that function. What is an appropriate window? Label the scale.
 - d Since this is a rational function, determine its asymptotes.
 - e Explain the meaning of the horizontal asymptote in terms of the refrigerator.
 - f A company offers a refrigerator that costs \$1200, but says that it will last at least twenty years. Is this refrigerator worth the difference in cost?

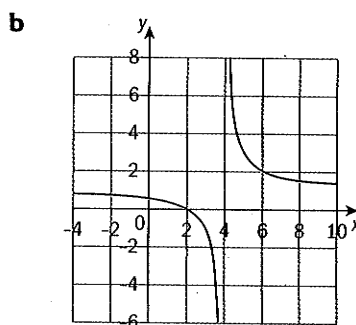
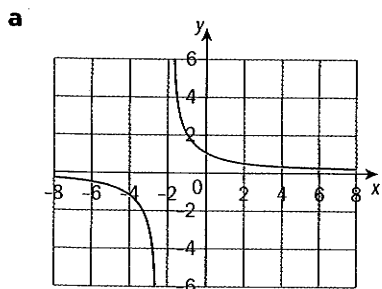


Review exercise

EXAM-STYLE QUESTION

- 1 Match the function with the graph.

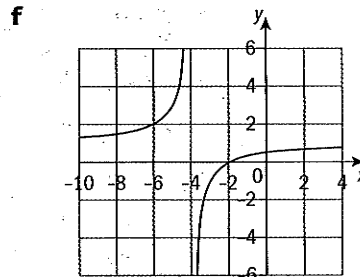
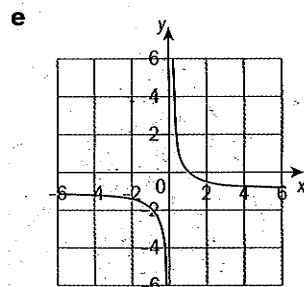
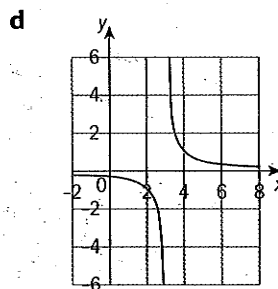
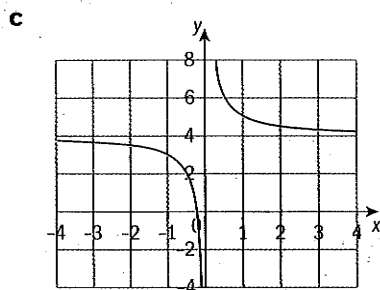
i $f(x) = \frac{2}{x+2}$ ii $f(x) = \frac{1}{x-3}$ iii $f(x) = \frac{4x+1}{x}$
 iv $f(x) = \frac{1-x}{x}$ v $f(x) = \frac{x-2}{x-4}$ vi $f(x) = \frac{x+2}{x+4}$



Extension material on CD:
Worksheet 5 - Continued
fractions and asymptotes



EXAM-STYLE QUESTIONS



2 Given **a** $f(x) = \frac{5}{x}$ **b** $f(x) = \frac{1}{x+1}$ **c** $f(x) = \frac{x+3}{3-x}$

- i Sketch the function.
- ii Determine the vertical and horizontal asymptotes of the function.
- iii Find the domain and range of the function.

3 For each of these functions, write down the asymptotes, domain and range.

