Name: _____ Date: _____ Period: _____

Inverse Functions Worksheet

Find a table of values for each function and its inverse.

1. a. $f(x) = 3x + 1$

Function	
x	f(x)

Inverse	
x	$f^{-1}(x)$

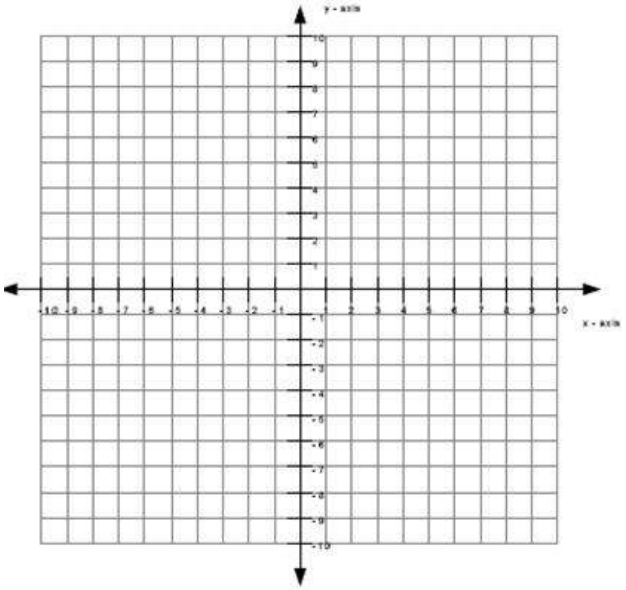
b. $f(x) = (2 - x)^2$

Function	
x	f(x)

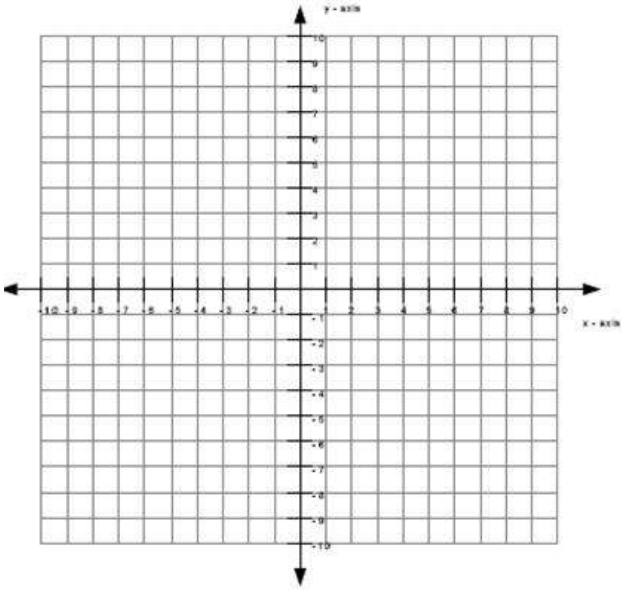
Inverse	
x	$f^{-1}(x)$

2. Graph each function, its inverse, and their line of symmetry. Label the function and its inverse on each graph.

a. $f(x) = \frac{1}{2}x + 1$



b. $f(x) = (x - 2)^2 + 3$



3. Find the domain and range of the each function and the domain and range of its inverse in problems 2 (a-b) above.

a. $f(x) = \frac{1}{2}x + 1$

$f(x)$ Domain: _____ Range: _____

$f^{-1}(x)$ Domain: _____ Range: _____

b. $f(x) = (x - 2)^2 + 3$

$f(x)$ Domain: _____ Range: _____

$f^{-1}(x)$ Domain: _____ Range: _____

4. For each function in problems 2 and 3 (a-b) above, identify whether its inverse is or is not a function. Explain your answer in complete sentences:

   a. Is the inverse of $f(x) = \frac{1}{2}x + 1$ a function? Explain.

   b. Is the inverse of $f(x) = (x-2)^2 + 3$ a function? Explain.

5. Let's apply our knowledge of functions and their inverses to a real world problem:

   To make a long-distance call, your phone company charges $1.50 to make the connection, and an additional $0.10 for every minute that you are on the line once connected.

   a. Write an equation for the price of a long-distance call, *p*, in terms of the length of the call in minutes, *m*:

   b. When you get the phone bill, you see that your sister made a long-distance call that cost $2.75. How long was she on the phone?

   c. Think about how you solved part (b). Write an equation to determine *m* in terms of *p*. (That is, how do you calculate the length of a call based on its price?)

6. Find the inverse of each function below using the Flip and Find method.

   a. $f(x) = 3x + 4$

   b. $f(x) = (2x-3)^2 - 1$

   c. $f(x) = \frac{x+5}{-5}$

   d. $f(x) = \sqrt{(x-5)}$

Inverse Functions

The function $g(x)$ are inverses of each other $f(x)$ if $g(f(x)) = x$ and $g(f(x)) = x$.

The inverse of the function $f(x)$ is indicated with the notation $f^{-1}(x)$, read *f* inverse (this notation does **not** mean $\frac{1}{f(x)}$).

I. Model Problems

In this example we will find the inverse of a discrete function for a given as a list of ordered pairs.

Example 1: If $f = \{(3,2),\ (4,-6),(-2,11),(5,5)\}$ ***find*** $f^{-1}(x)$.

When finding the inverse exchange *x* and *y*. The ordered pairs (*x*,*y*) become (*y*,*x*).

$f = \{(3,2),\ (4,-6),(-2,11),(5,5)\}$

$f^{-1} = \{(2,3),\ (-6,4),(11,-2),(5,5)\}$

Answer: $f^{-1} = \{(2,3),\ (-6,4),(11,-2),(5,5)\}$

In these examples we will find the inverse of functions given as an equation.

Example 2: If $f(x) = 3x + 10$ ***find*** $f^{-1}(x)$.

Write function in terms of *y*.

$$f(x) = 3x + 10$$
$$y = 3x + 10$$

When finding the inverse exchange *x* and *y*.

$$x = 3y + 10$$

Solve for *y*.

$$-10 \qquad -10$$
$$\frac{x-10}{3} = \frac{3y}{x}$$
$$\frac{x-10}{3} = y$$

Rewrite as $f^{-1}(x)$.

$$f^{-1}(x) = \frac{x-10}{3}$$

Answer: $f^{-1}(x) = \frac{x-10}{3}$

Example 3: If $f(x) = \sqrt{x+12}$ ***find*** $f^{-1}(x)$.

Write function in terms of *y*.

$$f(x) = \sqrt{x+12}$$
$$y = \sqrt{x+12}$$

When finding the inverse exchange *x* and *y*.

$$x = \sqrt{y+12}$$

Solve for *y*. Square both sides of the equation.

$$x^2 = \left(\sqrt{y+12}\right)^2$$
$$x^2 = y + 12$$
$$-12 \qquad -12$$
$$x^2 - 12 = y$$

Rewrite as $f^{-1}(x)$.

$$f^{-1}(x) = x^2 - 12$$

Answer: $f^{-1}(x) = x^2 - 12$

II. Practice Problems

Solve.

1. Is $g(x) = \frac{1}{2}x - 2$ the inverse of $f(x) = 2x + 4$? Justify your answer.
2. Is $g(x) = 4x + 24$ the inverse of $f(x) = \frac{1}{4}x + 6$? Justify your answer.
3. Is $h(x) = x^2 - 2$ the inverse of $g(x) = \sqrt{x+2}$? Justify your answer.
4. Is $h(x) = x^2$ the inverse of $g(x) = \sqrt{x}$? Justify your answer.

Find the inverse of the given function.

5. $f = \{(1,3), (2,-5), (3,6)\}$
6. $g = \{(-4,1), (-3,2), (0,0), (1,10)\}$
7. $h = \{(-1,-1), (0,0), (3,3), (6,6)\}$
8.

x	y
−3	−2
−1	2
0	4
1	6
3	8

9.

x	y
−3	0
1	2
6	3
13	4
22	5

10. $f(x) = 3x - 7$
11. $g(x) = -4x + 5$
12. $h(x) = \frac{2}{5}x + 6$
13. $f(x) = \frac{3x+4}{7}$
14. $g(x) = \frac{1}{4}x + 6$
15. $g(x) = -3x - 10$
16. $f(x) = \sqrt{x-4}$
17. $g(x) = \sqrt{2x+8}$
18. $h(x) = \sqrt{3x} - 6$
19. $f(x) = 4\sqrt{x}$
20. Graph the inverse of $f(x) = 4x - 12$.

Calculating the inverse of a quadratic function algebraically

Given a quadratic function $f(x) = ax^2 \pm bx \pm c$ we can use a process called **Completing the Square** to find the inverse.

We use the fact that $(u+v)^2 = u^2 + 2uv + v^2$ and $(u-v)^2 = u^2 - 2uv + v^2$; we let $u = \sqrt{a}x$ and $v = \frac{b}{2\sqrt{a}}$. Then we can rewrite the function $ax^2 + bx + c$ as $u^2 + 2uv + v^2 - v^2 + c$, since $u^2 + 2uv + v^2 - v^2 = ax^2 + bx$.

Examples

$$f(x) = x^2 + 6x - 4$$

1. $y = x^2 + 6x + 4$

2.
$$\begin{aligned} x &= y^2 + 6y - 4 \\ &= (y^2 + 6y + 9) - 9 + 4 \\ &= (y+3)^2 - 5 \end{aligned}$$

3.
$$\begin{aligned} (y+3)^2 &= x + 5 \\ y + 3 &= \pm\sqrt{x+5} \\ y &= \pm\sqrt{x+5} - 3 \end{aligned}$$

4. $f^{-1}(x) = \pm\sqrt{x+5} - 3$

$$f(x) = 4x^2 - 8x - 5$$

1. $y = 4x^2 - 8x - 5$

2.
$$\begin{aligned} x &= 4y^2 - 8y - 5 \\ &= (4y^2 - 8y + 4) - 4 - 5 \\ &= (2y-2)^2 - 9 \end{aligned}$$

3.
$$\begin{aligned} (2y-2)^2 &= x + 9 \\ 2y - 2 &= \pm\sqrt{x+9} \\ 2y &= \pm\sqrt{\tfrac{x+9}{2}} \\ y &= \pm\sqrt{\tfrac{x+9}{2}} + 2 \end{aligned}$$

4. $f^{-1}(x) = \pm\sqrt{\frac{x+9}{2}} + 2$

Practice

$f(x) = x^2 - 8x + 12$

1.

2.

3.

4. $f^{-1}(x) =$

$f(x) = 9x^2 + 12x + 7$

1.

2.

3.

4. $f^{-1}(x) =$

To be handed in:

For each of the following functions, calculate the inverse function:

1. $f(x) = x^2 + 6x + 1$

2. $f(x) = x^2 - 12x - 5$

3. $f(x) = x^2 - 4x + 12$

4. $f(x) = x^2 + 10x + 15$