A. Lesson Context

BIG PICTURE of this UNIT:	• What is meant by the term FUNCTIONS and how do we work with them? • mastery with working with basics & applications of linear functions • mastery with working with basics & applications of linear systems • understanding basics of function concepts and apply them to lines & linear systems		
CONTEXT of this LESSON:	Where we've been In Lesson 8, you practiced with writing equations to various word problems & practiced HOW to solve the systems	Where we are We will analyze systems graphically & analytically to classify a systems in terms of the number of solutions	Where we are heading How do we apply the concept of "functions" to linear relations.

(A) Lesson Objectives:

a. Investigate the numbers of solutions that linear systems can have
b. Use multiple representations in solving linear systems

(B) Investigation #1 – Number of Solutions

1. On the same grid, use DESMOS to graph the lines y = -½x + 3 and x + 2y = -10.
2. How are the graphs related?

   The two lines are parallel (same slope & different y-intercept)

3. How are the equations related? If equation #2 is rearranged, we get y = -½x – 5, so the coefficients of the second equation are double the first equation, although they gave diffeernt constant values

Algebraic Verification (Use the substitution method)

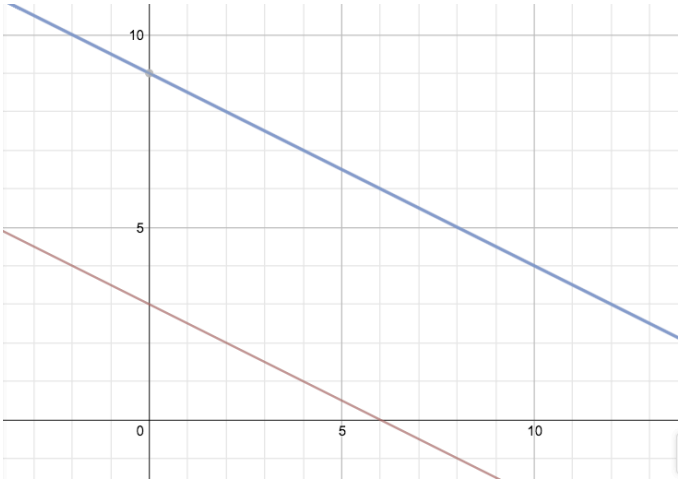
$$x+2\left(-\frac{1}{2}x+3\right)=-10$$

$$x-x+6=-10$$

$$0x=-16$$

Since 0x is NOT equal to -16, we are unable to find a value for x that makes our equation of 0x = -16 true. This means we are unable to find an x coordinate of an intersection point, as the two lines do NOT have a common intersection point (as they are parallel)

Graphic Verification using DESMOS (sketch)



1. On the same grid, use DESMOS to graph the lines -x + 3y = 9 and 2x – 6y = 18.

2. How are the graphs related?

   The two lines are parallel (same slope & different y-intercept)

3. How are the equations related?

   If we multiply the first equation by -2, we get the same coefficients as we have in eqn #2, except for a different constant value

Algebraic Verification (using Elimination method

$-x+3y=9$
$2x-6y=18$

$2(-x+3y=9)$
$2x-6y=18$

$-2x+6y=18$
$2x-6y=18$

$0x+0y=36$
$0=36$

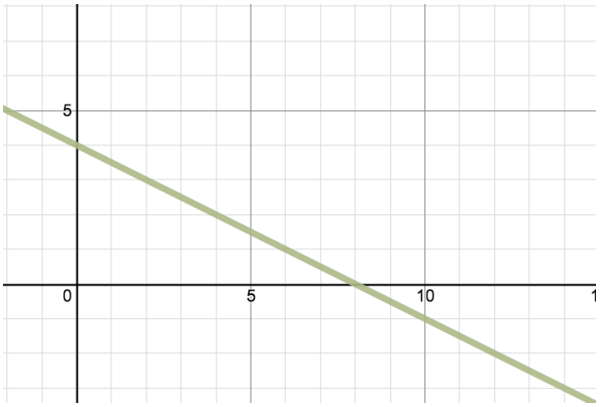
Since 0y (or 0x) is NOT equal to 36, we are unable to find a value for y (or x) that makes our equation of 0x = 366 true. This means we are unable to find an x coordinate of an intersection point, as the two lines do NOT have a common intersection point (as they are parallel)

Graphic Verification (Sketch from DESMOS)

(C) Investigation #2 – Number of Solutions

1. On the same grid, use DESMOS to graph the lines x + 2y = 8 and 2x + 4y = 16. Sketch the resultant graph on the grid included.

Graphic Verification – Sketch from DESMOS



2. How are the graphs related?

   The two lines overlap or coincide or lie one on top of the other

3. How are the equations related?

   The second equation is DOUBLE the first equation

4. On the same graph, use DESMOS to graph -8x – 16y = -64

5. How are the graphs & the three equations related?

   All three lines overlap & equations 2 & 3 are simply multiples of the first equation

6. PREDICT the appearance of the graph of ***k***x + 2***k***y = 8***k***, where ***k*** is any real number. Explain your prediction.

   The new line would overlap as the new equation is still simply a multiple of the first equation

7. Use the ELIMINATION METHOD to solve the linear system $\begin{matrix} x+2y=8 \\ 2x+4y=16 \end{matrix}$. Explain the "problem" that comes up with the algebraic solution.

$$-2(x+2y=8) \Rightarrow -2x-4y=-16$$
$$2x+4y=16 \Rightarrow \underline{2x+4y=16}$$
$$0x+0y=0$$

$\therefore x \in R$ as x (or y) could be any real number

Here we have the idea that ANY value for y will work in the equation 0y = 0, so we have an infinite number of values of y that will work ➔ why? ➔ because the two lines meet at EVERY SINGLE VALUE for y as the lines overlap

8. Use the SUBSTITUTION METHOD to solve the linear system $\begin{matrix} x+2y=8 \\ 2x+4y=16 \end{matrix}$. Explain the "problem" that comes up with the algebraic solution.

$x + 2y = 8$ ==> $x = 8 - 2y$

$2(8 - 2y) + 4y = 16$

$16 - 4y + 4y = 16$

$16 + 0y = 16$

$0y = 0$

Here we have the idea that ANY value for y will work in the equation 0y = 0, so we have an infinite number of values of y that will work ➔ why? ➔ because the two lines meet at EVERY SINGLE VALUE for y as the lines overlap

(D)CHALLENGE PROBLEM (AP/HL):

You are given the linear system defined by $\begin{matrix} 2x + By = 9 \\ -3x + 4y = C \end{matrix}$. For what value(s) of B and C will this system have:

a. Infinite solutions

b. No solutions

c. One unique value

(E) Word Problems

CALC INACTIVE. Mr. S. has $18,000 savings in 2 accounts. My total interest earned for the year was $930. One account earns me 6% annual interest and the other account earns me 3% annual interest. How much do I have in each account? Provide an ALGEBRAIC SOLUTION. Verify your solution ALGEBRAICALLY.

Let x represent the amount of money invested at 3%
Let y represent the amount of money invested at 6%

x + y = 18,000
0.03x + 0.06y = 930

(5000, 13000)

CALC INACTIVE. Mr. S travelled 1930 miles by car and plane. He drove to the airport at an average speed of 60 mph and the plane averaged 350 mph. The total trip took 8 hours. How long did it take to get to the airport? Provide an ALGEBRAIC SOLUTION. Verify your solution ALGEBRAICALLY.

Let x represent the time spent in the car
Let y represent the time spent in the plane

x + y = 8
60x + 350y = 1930

(3,5)