Problem Solving with Quadratic Equations | Unit 5 Lesson 8

(A) Lesson Context

BIG PICTURE of this UNIT:	• How do we analyze and then work with a data set that shows both increase and decrease • What is a parabola and what key features do they have that makes them useful in modeling applications • How do I use graphs, data tables and algebra to analyze quadratic equations?		
CONTEXT of this LESSON:	Where we've been In Lesson 6, you were factoring quadratics in order to find the zeroes and other key features of a parabola	Where we are We will now solve quadratic equations, wherein we ultimately need to use the process of factoring	Where we are heading How can I use EQUATIONS to make predictions about parabolas and quadratic data sets & quadratic models

(B) Lesson Objectives:

a. Review & practice the algebraic skills of expanding and factoring
b. Use the skills of factoring and expanding in solving equations and contextual problems

(C) Solving Quadratic Equations

Example 1: Given the quadratic relation $y = x^2 - 8x + 12$;

a. Sketch a graph of this parabola and label all key points/features.

b. Solve $0 = x^2 - 8x + 12$. What does your solution mean?

c. Solve $-3 = x^2 - 8x + 12$. What does your solution mean?

d. Solve $5 + 8x = x^2 + 12$. What does your solution mean?

Example 2: Given the quadratic relation $y = 6x^2 + x - 15$

a. Factor $y = 6x^2 + x - 15$

b. Solve $0 = 6x^2 + x - 15$

c. Solve $36 = 6x^2 + x - 15$

d. Graph $y = 6x^2 + x - 15$ and determine the zeroes. How does your GRAPH verify that your algebra in parts a,b,c has been done correctly?

e. Using your graphing calculator to find the vertex and then rewrite the equation in VERTEX FORM.

Example 3: Given the quadratic relation $y = 6x^2 + 5x - 4$

a. Factor $y = 6x^2 + 5x - 4$

b. Solve $0 = 6x^2 + 5x - 4$

c. Solve $65 = 6x^2 + 5x - 4$

d. Graph $y = 6x^2 + 5x - 4$ and determine the zeroes. How does your GRAPH verify that your algebra in parts a,b,c has been done correctly?

e. Using your graphing calculator to find the vertex and then rewrite the equation in VERTEX FORM.

Example 4: Given the quadratic relation $y = 2x^2 - 20x + 50$

a. Factor $y = 2x^2 - 20x + 50$

b. Solve $0 = 2x^2 - 20x + 50$

c. Solve $72 = 2x^2 - 20x + 50$

d. Graph $y = 2x^2 - 20x + 50$ and determine the zeroes. How does your GRAPH verify that your algebra in parts a,b,c has been done correctly?

e. Using your graphing calculator to find the vertex and then rewrite the equation in VERTEX FORM.

(D) Key to SOLVING EQUATIONS:

➔ STEP #1 ➔

➔ STEP #2 ➔

➔ STEP #3 ➔

(E) Modeling with Quadratic Equations

Mr Santowski runs a clothing business and models how his revenues on sales of denim jeans are related to price changes. He uses the quadratic equation $R = 300 + 20x - x^2$, where R represents his daily revenue in dollars and x represents an increase or decrease in price. (So x = +1 represents a price increase of 1 dollar and x = -2 represents a price decrease of 2 dollars)

a. Determine the price change that will result in maximum revenues. What is the maximum revenue

b. Factor the equation $R = 300 + 20x - x^2$.

c. Solve the equation $0 = 300 + 20x - x^2$ and interpret what the answers mean, given the context.

d. Solve the equation $300 = 300 + 20x - x^2$ and interpret what the answers mean, given the context.

e. Make a sketch of the relation.

f. Solve the equation $375 = 300 + 20x - x^2$ and interpret what the answers mean, given the context

g. Solve the equation $144 = 300 + 20x - x^2$ and interpret what the answers mean, given the context

1. The profits of a company in its first 13 months of operations are modelled by the quadratic function $P(m) = -0.25m^2 + 3m - 5$ where m is the number of months (and m = 1 represents January) and P(m) is measured in billions of pesos. (CALC INACTIVE)

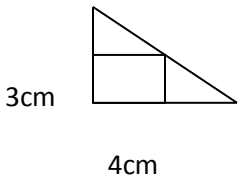
    a. Determine when the company "breaks even".
    b. Determine in which month the company maximizes its profits.
    c. What are the company's maximum profits?
    d. Solve and interpret P(m) < 0 given that the domain is $D: \{m \in Z \mid 0 \le m \le 13\}$
    e. For what values of m are the profits DECREASING? Explain how you determined your answer.
    f. Solve P(m) = -12 and interpret

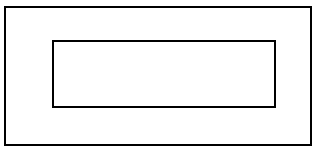
2. If the length of one side of a square is tripled and the length of an adjacent side is increased by 10, the resulting rectangle has an area that is 6 times the area of the original square. Find the length of a side of the original square.

3. The length of a rectangle is 7 units more than its width. If the width is doubled and the length is increased by 2, the area is increased by 42 square units. Find the dimensions of the original rectangle.

4. Among all rectangles that have a perimeter of 20 feet, find the dimensions of the one with the largest area.

5. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 4 cm if two sides of the rectangle lie along the legs as shown in the figure. (Hint: set the triangle with the right angle at the origin of a graph and write the equation of the line containing the hypotenuse)





6. A frame for a picture is 2½ cm wide. The picture enclosed inside the frame is 5 cm longer than it is high. If the area of the picture itself is 300 cm^2, what are the dimensions of the outer frame? (see diagram above)

7. A farmer has 3000 feet of fence available to enclose a rectangular field. Assuming that he uses all of his fence material, find the length of each of the sides of the rectangle which will maximize the area. What is the maximum area he can enclose?

8. A farmer with 4000 meters of fencing material wants to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest area that he can enclose? What will the dimensions be?

9. In a trapezoid, the smaller base is 3 more than the height, the larger base is 5 less than 3 times the height, and the area of the trapezoid is 45 square centimeters. Find, in centimeters, the height of the trapezoid.

11. A model rocket is shot into the air and its path is approximated by $h = -5t^2 + 30t$, where h is the height of the rocket above the ground in metres and t is the elapsed time in seconds.
(a) When will the rocket hit the ground?
(b) What is the maximum height of the rocket?

12. A baseball is thrown from the top of a building and falls to the ground below. Its path is approximated by the relation $h = -5t^2 + 5t + 30$, where h is the height above ground in metres and t is the elapsed time in seconds.
(a) How tall is the building?
(b) When will the ball hit the ground?
(c) When does the ball reach its maximum height?
(d) How high above the building is the ball at its maximum height?

13. **Application:** A small company that manufactures snowboards uses the relation $P = 162x - 81x^2$ to model its profit. In the model, x represents the number of snowboards in thousands, and P represents the profit in thousands of dollars.
(a) What is the maximum profit the company can earn?
(b) How many snowboards must it produce to earn this profit?
(c) The company breaks even when there is neither a profit nor a loss. What are the break-even points for the company?

14. A computer software company models the profit on its latest game using the relation $P = -2x^2 + 28x - 90$, where x is the number of games it produces in hundred thousands and P is the profit in millions of dollars.
(a) What is the maximum profit the company can earn?
(b) How many games must it produce to earn this profit?
(c) What are the break-even points for the company?

18. **Thinking, Inquiry, Problem Solving:** Soundz Inc. makes CD players. Last year, accountants modelled the company's profit by $P = -5x^2 + 60x - 135$. Over the course of the year, in an effort to become more efficient, Soundz Inc. restructured its operation, eliminating some employees and reducing costs. This year, accountants are using $P = -7x^2 + 70x - 63$ to project the company's profit. In both models, P is the profit in hundreds of thousands of dollars and x is the number of CD players made, in hundreds of thousands. Was Soundz Inc.'s restructuring effective? Justify your answer.

Now work through Q4,8,12,13 in Nelson 10, Chapter 3.9

http://mrsantowski.tripod.com/2013IntegratedMath2/Homework/Nelson10_Chap39_Prob_Solving_with_Quad_Equations.pdf