## (A) Lesson Context

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|---|---|---|---|
| BIG PICTURE of this UNIT: | • How do we analyze and then work with a data set that shows both increase and decrease<br>• What is a parabola and what key features do they have that makes them useful in modeling applications<br>• How do I use graphs, data tables and algebra to analyze quadratic equations? | | |
| CONTEXT of this LESSON: | Where we've been<br><br>In Lesson 6, you learned about factoring quadratic expressions | Where we are<br><br>We algebraically solve for zeroes/x-intercepts in quad fcns using factoring | Where we are heading<br><br>How can I use graphs and equations to make predictions from quadratic data sets & quadratic models and quadratic equations |

## (B) Lesson Objectives:

a. Review the algebraic skills of factoring
b. Understand what solving a quadratic equation in the form of $ax^2 + bx + c = 0$ means in terms of graphs
c. Use the skills of factoring to solve quadratic equations

## (C) Understanding Terms

To SOLVE an equation means:

*If* $f(x) = 2x + 5$, solve $f(x) = 0$ — Verify GRAPHICALLY

*If* $f(x) = x - 3$, solve $f(x) = 0$ — Verify GRAPHICALLY

*If* $f(x) = (2x + 5)(x - 3)$, solve $f(x) = 0$ — Verify GRAPHICALLY

*If* $f(x) = 2x^2 - x - 15$, solve $f(x) = 0$ — Verify GRAPHICALLY

*If* $f(x) = 2^x - 8$, solve $f(x) = 0$ — Verify GRAPHICALLY

## (C) Practice with Solving by Factoring

### More Quadratic Equations – Solve by Factoring

**Solve by factoring:**

1. $x^2 - 2x - 24 = 0$
2. $t^2 - 3t - 18 = 0$
3. $z^2 - 6z + 5 = 0$
4. $x^2 - 7v + 6 = 0$
5. $x^2 + 9x + 8 = 0$
6. $v^2 + 10v + 9 = 0$
7. $x^2 - 10x + 25 = 0$
8. $y^2 - 12y + 36 = 0$
9. $10y^2 + 5y = 0$
10. $8x^2 - 12x = 0$
11. $x^2 - 21 = 4x$
12. $t^2 - 20 = t$
13. $2x^2 + x = 10$
14. $4x^2 + 12x = -9$
15. $5r^2 = 12 + 11r$
16. $3x^2 = x + 2$
17. $5y^2 = 11y - 2$
18. $16v^2 - 8v + 1 = 0$
19. $25y^2 - 10y + 1 = 0$
20. $x^2 - 1 = 0$
21. $t^2 - 25 = 0$
22. $16y^2 - 1 = 0$
23. $4z^2 - 25 = 0$
24. $x + 28 = x(x - 2)$
25. $y + 12 = y(y - 3)$
26. $x^2 - 4x - 4 = 3x^2 - 5x - 3$
27. $x^2 + 2x + 3 = (2x - 1)(x + 5)$
29. $x^2 + x - 1 = (2x + 1)(x + 2)$

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## (D)Changing from Factored Form to Standard Form

You are now given pairs of zeroes/x-intercepts ➔ you must write an equation of a parabola that has these zeroes, both in factored form and in standard form

| | | |
|---|---|---|
| Zeroes are x = -3 and x = 5 | Zeroes are x = 4 and x = 9 | Zeroes are x = -1 and x = 11 |
| Zeroes are x = -3 and x = 2.5 | Zeroes are x = -3 and x = -3 | Zeroes are x = 5 and x = -5 |
| $x_1 = \frac{2}{3}$ and $x_2 = -\frac{1}{2}$ | $x_1 = \frac{5}{7}$ and $x_2 = -\frac{4}{3}$ | $x_1 = 0.05$ and $x_2 = -0.20$ |

## (E) Solving Quadratic Equations ➔ Application Problems

**11.** A model rocket is shot into the air and its path is approximated by $h = -5t^2 + 30t$, where $h$ is the height of the rocket above the ground in metres and $t$ is the elapsed time in seconds.
**(a)** When will the rocket hit the ground?
**(b)** What is the maximum height of the rocket?

**12.** A baseball is thrown from the top of a building and falls to the ground below. Its path is approximated by the relation $h = -5t^2 + 5t + 30$, where $h$ is the height above ground in metres and $t$ is the elapsed time in seconds.
**(a)** How tall is the building?
**(b)** When will the ball hit the ground?
**(c)** When does the ball reach its maximum height?
**(d)** How high above the building is the ball at its maximum height?

**13.** **Application:** A small company that manufactures snowboards uses the relation $P = 162x - 81x^2$ to model its profit. In the model, $x$ represents the number of snowboards in thousands, and $P$ represents the profit in thousands of dollars.
**(a)** What is the maximum profit the company can earn?
**(b)** How many snowboards must it produce to earn this profit?
**(c)** The company breaks even when there is neither a profit nor a loss. What are the break-even points for the company?

**14.** A computer software company models the profit on its latest game using the relation $P = -2x^2 + 28x - 90$, where $x$ is the number of games it produces in hundred thousands and $P$ is the profit in millions of dollars.
**(a)** What is the maximum profit the company can earn?
**(b)** How many games must it produce to earn this profit?
**(c)** What are the break-even points for the company?

**18.** **Thinking, Inquiry, Problem Solving:** Soundz Inc. makes CD players. Last year, accountants modelled the company's profit by $P = -5x^2 + 60x - 135$. Over the course of the year, in an effort to become more efficient, Soundz Inc. restructured its operation, eliminating some employees and reducing costs. This year, accountants are using $P = -7x^2 + 70x - 63$ to project the company's profit. In both models, $P$ is the profit in hundreds of thousands of dollars and $x$ is the number of CD players made, in hundreds of thousands. Was Soundz Inc.'s restructuring effective? Justify your answer.

## (F) Practice – Graphing & Word Problem Context

Apply to Problems ➔ Mr. S. can sell 500 apples per week when he charges 50 cents per apple. Through market research, his wife (being smarter than Mr. S of course) knows that for every price increase of 2 cents per apple, he will sell 10 less apples.

i. Determine an equation that can you used to model Mr. S.'s expected revenues.

ii. What price should he charge to maximize his revenues?

iii. What is his maximum revenue?

iv. How many price increments are required such that his business has NO revenue?

Apply to Problems ➔ The profits of a company in its first 13 months of operations are modelled by the quadratic function $P(m) = -0.25m^2 + 3m - 5$ where m is the number of months (and m = 1 represents January) and P(m) is measured in billions of pesos. (CALC INACTIVE)

a. Determine when the company "breaks even".

b. Determine in which month the company maximizes its profits.

c. What are the company's maximum profits?

d. Solve and interpret P(m) < 0 given that the domain is $D: \{m \in Z \mid 0 \le m \le 13\}$

e. For what values of m are the profits DECREASING? Explain how you determined your answer.

f. Solve P(m) = -12 and interpret