(A) Lesson Context

BIG PICTURE of this UNIT:	• How do we analyze and then work with a data set that shows both increase and decrease • What is a parabola and what key features do they have that makes them useful in modeling applications • How do I use graphs, data tables and algebra to analyze quadratic equations?		
CONTEXT of this LESSON:	Where we've been In Lesson 1, you looked for number patterns & graphed in data from a variety of activities	Where we are How can we use the graphing calculator to graph scatter plots and use the GDC to determine the quadratic equations	Where we are heading How can I use graphs of quadratic relations to make predictions from quadratic data sets & quadratic models and quadratic equations

(B) Lesson Objectives:

a. Prepare scatter plots of quadratic data on the graphing calculator
b. Use the graphing calculator to determine the regression equations of the data sets
c. Introduce key features of the graphs of quadratic relations (the graphs are called parabolas)

http://mrsantowski.tripod.com/2012IntegratedMath2/HW/Nelson10_Chap36_Quad_Regression.pdf

(C) Example #1 – Number Patterns from Lesson 1

Given the number pattern .....2,4,8,14,22,32,44, ..., we will create a data table as:

X	*0*	*1*	*2*	*3*	*4*	*5*	*6*
y	2	4	8	14	22	32	44

The quadratic equation from the TI-84 is:

Use the TI-84 to evaluate y(3.5)

Given the number pattern .....16,15,12,7,0,-9,-20 .....

we will create a data table as:

X	*4*	*5*	*6*	*7*	*8*	*9*	*10*
y	16	15	12	7	0	-9	-20

The quadratic equation from the TI-84 is:

Use the TI-84 to solve y(x) = 5

(D) Example #2 – Contextual Data Sets from Lesson #1 & Contextual Analysis

(A) This data set shows the relationship between the profit, P, in millions of Euros and the number years producing, n, a specific type of model (say a Toyota Land Cruiser) since 2000.

n	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
p(n)	-40	-18	0	14	24	30	32	30	24	14	0	-18

(a) Explain what the point (4,24) means in the context of the question.

(b) USE the TI-84 TO determine the equation. This form of the equation is called STANDARD FORM.

(c) Write the equation using FUNCTION NOTATION.

(d) Explain what the statement P(9) = 14 means

(e) Evaluate p(3.5) using your TI-84

(f) Solve 25 = p(n) using your TI-84

(g) What does the term PROFITS for a business mean?

(h) Why might the profits be decreasing after year 6 (2006)?

(i) When should Toyota stop producing Land Cruisers? Explain why.

(j) What TOTAL profit did Toyota make from its Land Cruisers in the first 10 years of production?

(B) This data set shows the relationship between the operational costs, C in millions of dollars, for a large dairy farm and the month, m, of the year since January (where January is m = 0)

m	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
C(m)	150	117	90	69	54	45	42	45	54	69	90	117	150

(a) What was the equation from the TI-84?

(b) Why would the relationship between operational costs and months be quadratic? (Non-math based reasons)

(c) Why might costs for the dairy farm be lowest in July (m = 6)?

(d) The manager of the dairy farm adds some new farm equipment in an effort to control her costs. The new equation that models the relationship between costs and months is given by $C = 2m^2 – 24m + 122$. Explain why you believe that her efforts to control costs were good or not good (explanation must be based upon the graphs you draw.)

(E) Example #3 – Contextual Data Sets from Lesson #1

The Paymore Shoe company introduced a new line of neon green high heel running shoes. The table below shows the number of pairs of shoes sold at one store over an 11 month period.

Month	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***
Shoes sold	56	60	62	62	60	56	50	42	32	20	6

(a) Determine the equation using the TI-84 that can be used to model the relationship between the sales of shoes and the month.

(F) Example #4 – Contextual Data Sets from Lesson #1

A ball is tossed straight up in the air. Its height is recorded every quarter second and the data set is recorded below

Time (s)	***0***	***0.25***	***0.50***	***0.75***	***1.00***	***1.25***	***1.50***	***1.75***	***2.00***
Height (m)	1.5	3.5	4.9	5.7	5.7	5.2	4.1	2.4	0.1

(a) Draw a scatter plot on your calculator.

(b) Determine the equation that models the relationship between the height of the ball, in meters, and the time in flight, seconds.

(c) Determine the maximum height of the ball and state at what time the maximum height is reached.

(d) How long is the ball in flight?

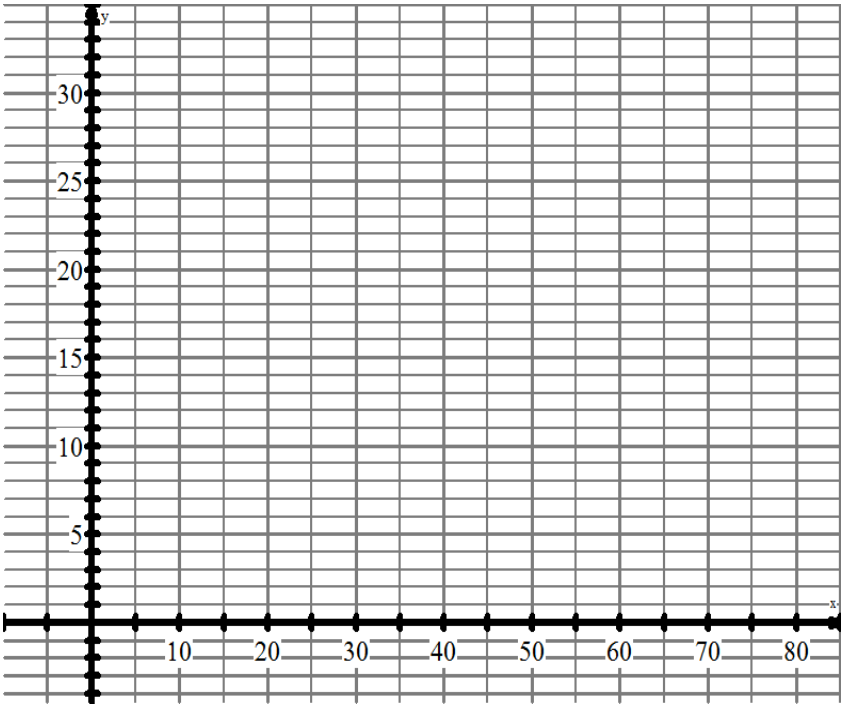
(e) State the domain and range for this relationship.

Car owners who wonder why they are unable to get the gas mileage advertised for their make and model of car should examine their driving habits. Many cars achieve the best fuel economy when driven at approximately at a certain speed. Data reflected fuel economy at various speeds for a particular make of car is provided.

1. What does the data in the table below tell you about the relationship between average speed and fuel economy?

Speed (miles per hour)	***Fuel Economy (miles per gallon)***
15	22.3
20	25.5
25	27.5
30	29.0
35	28.8
40	30.0
45	29.9
50	30.2
55	30.4
60	28.8
65	27.4
70	25.3

2. Create a scatter plot for the data in the table above. Sketch the scatter plot in the space below.



3. Determine a quadratic regression equation for the data. Record the equation below and sketch the graph of the regression equation on the scatter plot above. Write the equation in function form, as F(s) =

4. Describe the fit of the graph of the regression equation on the scatter plot.

5. At what speed should Mr S drive in order to OPTIMIZE his fuel economy?

6. Evaluate F(85)

7. Solve 20 = F(s)

PRACTICE EXERCISES – CONSOLIDATION LEVEL

1. Individual's Retirement Fund

The following table gives the average amount, in thousands of dollars, of an individual's retirement fund.

Year	Value
1985	9
1990	19
1995	45
2000	101
2005	196

(a) Use this information to construct a quadratic regression to represent the model rounding all constants to 3 decimal places.

Use x = 1 for 1985, x = 2 for 1986, ...

(b) To the nearest thousand dollars, what will the fund be worth in 2010?

2. Sales of TV Antennas

The total sales, *S*, of TV antennas for various years from 1980 to 1995 are shown in the table below, where *t = 0* represents the year 1980. Sales are shown in millions of dollars.

t	*S*
2	76.3
5	82.2
7	84.6
13	80.9
15	77.3

(a) Determine the quadratic regression equation that models this data.

[*Round coefficients to the nearest thousandth.]*

(b) Using the regression equation found, determine in what year sales reached their *maximum.*

(c) Use the regression equation to estimate the total sales of TV antennas for 2008. [*Round the answer to the nearest tenth of a million.*]

3. Average Cost of new Sedan

The following table give the average cost, to the nearest hundred, of a new 4-door sedan.

Year	Value
1991	$12,800
1994	$15,500
1997	$19,200
2000	$24,300
2003	$30,100

(a) Use this information to construct a quadratic regression to represent the model, rounding all constants to 3 decimal places.

Use x = 1 for 1991, x = 2 for 1992, ...

(b) Using this regression model, estimate during which year the average cost of a new 4- door sedan reached 37,000.

4. Sales of new T-Shirt

Sales of a new T-shirt style are shown in the table below. These sales were recorded at two-month intervals for one year and the values for sales, S, of the new T-shirt style are given in thousands of dollars.

(a) Write a regression equation with coefficients rounded to the nearest hundredth.

m	*S*
2	56
4	64
6	62
8	53
10	43
12	18

(b) Using this regression equation, estimate, to the nearest thousand dollars,

sales for month 11 of this year.

EXTENSION/CONNECTION OR ADVANCED QUESTIONS/PROBLEMS

Maple sap production vs. tree age

Tree age (in years)	Sap production (in ml)
7	200
50	350
10	370
17	380
35	480
8	280
27	420
40	430
12	320
45	360
22	480
42	390
30	430
37	450

Linear regression

1. Use your calculator to graph a scatterplot of the data. Sketch it above, making sure to properly label your graph.
2. Derive a linear model for the data, rounding to three places. Write it below.
3. Use the linear model to predict the sap production from a 20-year-old maple tree.
4. What is the value of the correlation coefficient?
5. In general, what do correlation coefficient values indicate?
6. What does this value tell us about this linear model in particular?
7. What is the value of the coefficient of determination, r^2 ?
8. Specifically, what does this tell us about how variability in age accounts for variability in sap production in our linear model?

Quadratic regression

9. Derive a quadratic model for the data, rounding to three places. Write it below.
10. Use the quadratic model to predict the sap production from a 20-year-old maple tree.
11. What is the value of the coefficient of determination, r^2 ?
12. Specifically, what does this tell us about how variability in age accounts for variability in sap production in our quadratic model?
13. Compare the results of the linear model versus the quadratic model. Which is the better predictive model?