(A) Lesson Context

BIG PICTURE of this UNIT:	• How can I analyze growth or decay patterns in data sets & contextual problems? • How can I algebraically & graphically summarize growth or decay patterns? • How can I compare & contrast linear and exponential models for growth and decay problems.		
CONTEXT of this LESSON:	Where we've been In Lessons 3 & 4, you looked at how exponential equations can be used to model real world scenarios	Where we are How can we solve exponential equations that arise when we model growth & decay patterns	Where we are heading How can I use algebra, data tables, graphs & equations to make predictions about scenarios which feature exponential growth & decay?

(A) Lesson Objectives

a. Use algebraic strategies to solve Exponential equations
b. Use multiple representations to verify algebraic solutions
c. Apply Exponential Equations to real world applications

(B) Intro to Algebraic Strategies - Solve for the variable:

1. $y = 3^3$

2. $27 = x \cdot 3^2$

3. $27 = x^3$

4. $27 = 3^x$

5. $y = 17^4$

6. $500 = x \cdot 4^4$

7. $76 = x^9$

8. $56 = 5^x$

(C) Exponential Equations – Solving algebraically in context: $y = a(1 + r)^x$

Solving for y:

1. You deposit $1600 in a bank account. Find the balance after 3 years if the account pays 2.5% annual interest compounded quarterly.
2. The population of HS students at CAC can be modeled with an exponential function. The number of students continues to decline at an annual rate of 11%. If there were 350 students present in 2013, how many HS students would be predicted to be at CAC in 2020?

Solving for a:

3. In 8 years, you want the money you invest to reach $10,000. The account pays 8% annual interest compound monthly. How much money do you need to invest?
4. The population of HS students at CAC since the year 2000 can be modeled with an exponential function. The number of students continues to decline at an annual rate of 11%. There are currently 320 HS students at CAC. How many where present in 2000?

Solving for r:

5. After investing $2000 for 15 years, you now have $8,000. What interest rate does the investment pay annually?
6. The population of HS students at CAC can be modeled with an exponential function. If there were 370 students present in 2011 and 315 students in 2014, what is the annual rate of decrease of student population in HS at CAC?
7. The value of land in New Cairo grows exponentially. Five years ago, 10 hectares of land cost 0.75 million LE and today, the same 10 hectares cost 2.5 million LE. Determine the annual rate of increase of the land.

Solving for t:

8. You buy a new computer for $2100. The computer decreases in value by 50% annually. When will the computer be worth $600?
9. The population of HS students at CAC since the year 2000 can be modeled with an exponential function. The number of students continues to decline at an annual rate of 11%. How long would it take for the student population to decline from 350 students to 250 students?
10. The value of land in New Cairo grows exponentially. Today 10 hectares of land cost 2.5 million LE and the value of the land is increasing at an annual rate of 17.5%. How long will it take for the land value to be 4.0 million LE?

(D)Classwork

1. Diego decided to invest his $500 tax refund rather than spending it. He found a bank that would pay him 4% interest, compounded quarterly. If he deposits the entire $500 and does not deposit or withdraw any other amount, how long will it take him to double his money in the account?

2. William wants to have a total of $4000 in two years so that he can put a hot tub on his deck. He finds an account that pays 5% interest compounded monthly. How much should William put into this account so that he'll have $4000 at the end of two years?

3. The annual consumption of pork per person was about 35 lb in 1997 and about 20 lb in 2007. Assuming consumption is decreasing according to the exponential-decay model:
   a. Find the value of r, the rate of growth. Write the corresponding exponential equation.
   b. Estimate the consumption of pork in 2010.
   c. In what year (theoretically) will the consumption of pork be 10 lb per person?

4. Kelly plans to put her graduation money into an account and leave it there for 4 years while she goes to college. She receives $750 in graduation money that she puts it into an account that earns 4.25% interest compounded semi-annually. How much will be in Kelly's account at the end of four years?

5. ABC Bank is offering to double your money! They say that if you invest with them at 6% interest compounded quarterly they will double your money. If you invest $1500 in the account, how long will it take to double your money.

6. At what rate, converted semiannually, will $600 amount to $900 in 8 years?

7. The number of wolves in the wild in the northern section of the Cataragas county is decreasing at the rate of 3.5% per year. Your environmental studies class has counted 80 wolves in the area. After how many years will this population of 80 wolves drop below 15 wolves if this rate of decrease continues?

8. Jane bought a Saturn Vue in 2002 for $20,000. In 2007, the salvage value of her Vue was $15,000.
   a. Find the value of r, the rate of growth. Write the corresponding exponential equation.
      b. What is the salvage value in 2010?
      c. In what year (theoretically) will the salvage value of the Vue be half of what Jane paid for it?

Extension Questions: QUESTION #1

Mr Smith's wife has just learned that she is pregnant! Mr. Smith wants to know when his new baby will arrive and decides to do some research. On the Internet, he finds the following article:

Hormone Levels for Pregnant Women

When a woman becomes pregnant, the hormone HCG (human chlorionic gonadotropin) is produced in order to enable the baby to develop.

During the first few weeks of pregnancy, the level of HCG hormone grows exponentially, starting with the day the embryo is implanted in the womb. However, the rate of growth varies with each pregnancy. Therefore, doctors cannot use just a single test to determine how long a woman has been pregnant. Commonly, the HCG levels are measured two days apart to look for this rate of growth.

A woman who is not pregnant will often have an HCG level of between 0 and 5 mIU (milli-international units) per ml (milliliter).

Then Smith remembered that his wife was tested for HCG during her last two doctor visits.

1. On March 21, her HCG level was 200 mIU/ml, while two days later, her HCG level was 392 mIU/ml.. Assuming that the model for HCG levels is of the form $y = ab^x$, what equation models the growth of HCG for his wife's pregnancy?

2. Assume her HCG level was 5 mIU on the day of implantation. How many days after implantation was his wife's first doctor visit? March 21? What day did the baby most likely become implanted?

3. Smith also learned that a baby is born approximately 37 weeks after implantation. What day can Smith expect to become a father?

QUESTION #2: ***<u>Doses of Medicine</u>***

Medicine in the body decays in an exponential way. Mr. Smith is taking some medication. On Monday Mr. Smith took the pills. On Tuesday he had 15 mg of medicine left in his body. On Friday he had 6.328125 mg left in his body.

1. Create an exponential equation modeling this situation. (Remember… starting should be on Monday)

2. When the amount of medicine in Mr. Smith's body drops below 4 mg, he needs to take another pill. When does Mr. Smith need to take more medicine?

Answer the following questions that deal with the doubling concept. Recall that the formula $y = Ca^x$ which can now be rewritten as $y = C(2)^{\frac{t}{D}}$. In these two formulas, recall what the variables really mean:

$y = Ca^x$	$y = C(2)^{\frac{t}{D}}$
y ➔ C ➔ a ➔ x ➔	y ➔ C ➔ t ➔ D ➔

1. A dish has 212 bacteria in it. The population of bacteria will double every 2 days. How many bacteria will be present in . . .
   a) 8 days b) 11 days c) 4 hours d) 2 months

2. An experiment starts off with *X* bacteria. This population of bacteria will double every 7 days and grows to 11,888 in 32 days. How many bacteria where present at the start of the experiment?

3. A bacteria culture grows according to the formula: $y = 12000(2)^{\frac{t}{4}}$ where t is in hours. How many bacteria are present:
   (a) at the beginning of the experiment?
   (b) after 12 hours?
   (c) after 19 days?
   (d) What is the doubling time of the bacteria?

4. A bacteria culture starts with 3000 bacteria. After 3 hours there are 48 000 bacteria present. What is the length of the doubling period?

5. Mr S. makes an initial investment of $15,000. This initial investment will double every 9 years. What is the value of this investment in . . .
   a) 20 years b) 6 years c) What is the yearly rate of increase of this investment?

Answer the following questions that deal with the doubling concept. Recall that the formula $y = Ca^x$ which can now be rewritten as $y = C\left(\frac{1}{2}\right)^{\frac{t}{H}}$. In these two formulas, recall what the variables really mean:

$y = Ca^x$	$y = C\left(\frac{1}{2}\right)^{\frac{t}{H}}$
y ➔ C ➔ a ➔ x ➔	y ➔ C ➔ t ➔ H ➔

6. Iodine-131 is a radioactive isotope of iodine that has a half-life of 8 days. A science lab initially has 200 grams of iodine-131. How much iodine-131 will be present in . . .

   a) 8 days　　b) 20 days　　c) 1 year　　d) 2 months

7. A medical experiment starts off with *X* grams of a radioactive chemical called Mathematus. This chemical will decay in half every 15 seconds and in the course of the experiment, will decay to 9.765 g in 2 minutes. How much Mathematus was present at the start of the experiment?

8. A chemical decays according to the formula: $y = 12000\left(\frac{1}{2}\right)^{\frac{t}{25}}$ where t is in time in hours and y is amount of chemical left, measured in grams. What amount of chemical is present:

   (e) at the beginning of the experiment?
   (f) after 100 hours?
   (g) after 19 days?
   (h) What is the half-life of the chemical?

9. A block of dry ice is losing its mass at a rate of 12.5% per hour. At 1 PM it weighed 50 pounds. What was its weight at 5 PM? What was the approximate half-life of the block of dry ice under these conditions?