

## (A) Lesson Context

|                           |                                                                                                                                                                                                                                                                                                                                    |                                                                                                                  |                                                                                                                                                             |
|---------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|
| BIG PICTURE of this UNIT: | <ul style="list-style-type: none"> <li>How can I analyze growth or decay patterns in data sets &amp; contextual problems?</li> <li>How can I algebraically &amp; graphically summarize growth or decay patterns?</li> <li>How can I compare &amp; contrast linear and exponential models for growth and decay problems.</li> </ul> |                                                                                                                  |                                                                                                                                                             |
| CONTEXT of this LESSON:   | <p>Where we've been</p> <p>In Lessons 1 &amp; 2 you generated &amp; analyzed data from a variety of activities</p>                                                                                                                                                                                                                 | <p>Where we are</p> <p>How do we work with equations &amp; situations that model growth &amp; decay patterns</p> | <p>Where we are heading</p> <p>How can I use equations that will help me make predictions about scenarios which feature exponential growth &amp; decay?</p> |

## (A) Lesson Objectives:

- Write exponential equations to model real world applications
- Make predictions/extrapolations through numeric or algebraic analysis
- Use multiple representations to solve the exponential equations that arise from real world applications

**(B) Review** → An Exponential equation has the form  $Y = C(a)^x$  or  $Y = C(1 + r)^x$ , where  $C$  = initial value,  $a$  is the growth factor/common ratio. (It turns out that  $a = 1 + r$ , where  $r$  is the decimal value of % increase given).

For the following equations, (i) decide if they can be used to model growth or decay and (ii) determine the rate at which the change happens.

|                          |  |  |
|--------------------------|--|--|
| $Y = 200(1.15)^x$        |  |  |
| $Y = 400(0.85)^x$        |  |  |
| $Y = 100(2)^x$           |  |  |
| $Y = 100(\frac{1}{2})^x$ |  |  |
| $Y = 200(1.05)^x$        |  |  |
| $Y = 400(1.75)^x$        |  |  |
| $Y = 100(0.75)^x$        |  |  |
| $Y = 100(0.995)^x$       |  |  |
| $Y = 1,000(0.30)^x$      |  |  |
| $Y = 2500(1.5)^x$        |  |  |

**(C) Opening Exploration →** Mr Santowski has been given a new job contract. He will earn \$40,000 per year and get a raise of 6% of his previous years' salary (i.e his salary grows by 6% per year)

- a) Define the variables that you will be using to model this problem.
- b) Graph the function on your TI-84
- c) Write an equation for Mr. S's salary.
- d) What does the y-intercept represent?
- e) What would my salary be in 8 years?
- f) After how many years would my salary be \$70,000?
- g) What assumption are you making as you answer Qd,e?
- h) I would like Mr. S's salary to be modelled with a linear relation. HOW would you change the original info so that a linear model can be used?

**(D) Opening Exploration →** Mr Santowski has purchased a new car. It cost \$50,000 but its value depreciates at a rate of 12% raise per year for the next 6 years

- a) Define the variables that you will be using to model this problem.
- b) Graph the function on your TI-84.
- c) Write an equation for the value of Mr. S's car.
- d) What does the y-intercept represent?
- e) What would be the value of my car be in 8 years?
- f) After how many years would the value of my car be \$7,000?
- g) I would like the value of Mr. S's car to be modelled with a linear relation. HOW would you change the original info so that a linear model can be used?

## Lesson 3: Modeling with Exponential Functions | Unit 4 – Exponential Relations

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**(E) Examples:** For each question, show your equation and a sketch of your graph.

- a. A colony of 1,000 ants can increase by 15% in a month.
  - i. How many ants will be in the colony after 10 months?
  - ii. How long will it take to get 7,500 ants in the colony?
- b. A population of 10 hamsters will triple every year.
  - i. What will be the population after 4 years?
  - ii. How long will it take to get 1,500 hamsters?
  - iii. Determine the WEEKLY growth rate for the hamsters.
- c. A baby weighing 7 pounds at birth may increase in weight every month according to the function  $W(m) = 7(1.11)^m$ .
  - i. How much will the baby weigh after 1 year?
  - ii. When will the baby weigh 18 pounds?
  - iii. Determine the monthly rate of growth for this infant.
  - iv. Determine the approximate DAILY rate of growth for this infant.
- d. A deposit of \$1500 in an account pays interest on the balance annually and the account balance is modeled by the function  $B(t) = 1500(1.0725)^t$ .
  - i. Determine the yearly rate of increase of the account balance.
  - ii. What is the account balance after 8 years?
  - iii. When will the value of the account be double its original value?

**(F) Examples :** For each question, show your equation and a sketch of your graph

- a. A colony of 100,000 ants is infected by a virus and its monthly population is modeled by the following function:  
 $P(m) = 100000(0.88)^m$ .
- How many ants will be in the colony after 10 months?
  - How long will it take to get 25,000 ants in the colony?
  - Determine the monthly rate of decrease of ant population.
  - Determine the DAILY death rate for the ant colony.
- b. A sample of 100 g radioactive plutonium-238 has a half-life of 87.7 years, so it will exponentially decay every year.
- Determine the YEARLY decay rate for plutonium.
  - What amount will remain after 400 years?
  - How long will it take to eliminate 95% of the plutonium?
- c. An investment of \$150,000 in an account loses value at a rate of 3.25% annually.
- What is the account balance after 5 years?
  - When will the value of the account be half its original value?

**(G) Homework Links:**

<http://frontenacss.limestone.on.ca/teachers/dcasey/0F7D449A-00870BC8.23/Exp%20Growth%20Decay%20Word%20Probs.pdf>