

Lesson 1: Introduction to Exponential Relations | Unit 4 – Exponential Relations

(A) Lesson Context

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> • How can I analyze growth or decay patterns in data sets & contextual problems? • How can I algebraically & graphically summarize growth or decay patterns? • How can I compare & contrast linear and exponential models for growth and decay problems. 		
CONTEXT of this LESSON:	<p>Where we've been</p> <p>In Gr. 8, you studied exponents and graphs of exponential relations</p>	<p>Where we are</p> <p>What patterns/relationships exist in data sets that exhibit growth & decay patterns</p>	<p>Where we are heading</p> <p>How can I develop equations that will help me make predictions about scenarios which feature exponential growth & decay?</p>

(B) Lesson Objectives:

- Generate data through various hands-on activities
- Analyze the data to look for patterns in the data that was generated
- Make predictions/extrapolations through numeric or algebraic analysis

(C) **Number Patterns in Data Sets** - You are given the following data sets. For EACH data set, you will:

- write out the pattern that you observe in the data set, that you can use to make predictions about the terms that follow
- Record the next 6 numbers in the data set
- Write an equation (or develop an alternative plan) that will allow you predict/calculate the 25th number in your data set

Data Set #1 → {1,2,4,8,16,32,64,...} → and as a data table →		<table border="1"> <tr> <td>X</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> </tr> <tr> <td>y</td> <td>1</td> <td>2</td> <td>4</td> <td>8</td> <td>16</td> <td>32</td> <td>64</td> </tr> </table>	X	1	2	3	4	5	6	7	y	1	2	4	8	16	32	64
X	1	2	3	4	5	6	7											
y	1	2	4	8	16	32	64											
Describe the pattern in words	List the next 6 numbers in the data set, given the pattern you determined	Formula/equation/method for determining the 25 th number in your data set																

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Data Set #2 → {10,20,40,80,160,320,640,...} → as a data table →

X	1	2	3	4	5	6	7
y	10	20	40	80	160	320	640

Describe the pattern in words

List the next 6 numbers in the data set, given the pattern you determined

Formula/equation/method for determining the 25th number in your data set

Data Set #3: $\left\{ \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \dots \right\}$ or as a data table

X	1	2	3	4	5	6	7
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

Describe the pattern in words

List the next 6 numbers in the data set, given the pattern you determined

Formula/equation/method for determining the 25th number in your data set

Data Set #4

Year	1825	1850	1875	1900	1925	1950	1975
Population (in thousands)	200	252	318	401	504	635	800

Describe the pattern in words

List the next 6 numbers in the data set, given the pattern you determined

Formula/equation/method for determining the 25th number in your data set

(D) PAPER FOLDING: Getting to the Moon

In this simulation activity, you will predict how many times you fold a piece of paper in order to get a tall enough piece of folded paper that reaches to the moon.

PREDICTION: how many times can you fold a piece of A4 paper, so that the resulting height of the folded piece of paper reaches to the moon? _____.

ACTIVITY: Follow these steps and answer the questions asked.

- a. In trial #0, you simply have 1 sheet of paper (data point of (0,1) is already recorded for you).
- b. For trial #1, you will fold your paper in half (so in other words, you now have folded the original sheet for the first time). In our simulation, place 2 full sheets in a stack on your table, one on top of the other.
- c. For trial #2, you will fold your paper in half again (so in other words, you now have folded the original sheet for the second time). In our simulation, place another 2 full sheets on your stack, one on top of the other. How many sheets do you now have? _____.
- d. For trial #3, you will fold your paper in half again (so in other words, you now have folded the original sheet for the third time). In our simulation, place another 4 full sheets on your table, one on top of the other. How many sheets do you now have? _____.
- e. For trial #4, you will fold your paper in half again (so in other words, you now have folded the original sheet for the fourth time). In our simulation, place another 8 full sheets on your table, one on top of the other. How many sheets do you now have? _____.
- f. For trial #5, you will fold your paper in half again (so in other words, you now have folded the original sheet for the fifth time). In our simulation, place another 16 full sheets on your stack, one on top of the other. How many sheets do you now have? _____.
- g. For trial #6, you will fold your paper in half again (so in other words, you now have folded the original sheet for the sixth time). In our simulation, place another 32 full sheets on your table, one on top of the other. How many sheets do you now have? _____.
- h. For trial #7, you will fold your paper in half again (so in other words, you now have folded the original sheet for the seventh time). In our simulation, place 64 full sheets on your table, one on top of the other. How many sheets do you now have? _____.
- i. For trial #8, you will fold your paper in half again (so in other words, you now have folded the original sheet for the eighth time). In our simulation, place another 128 full sheets on your table, one on top of the other. How many sheets do you now have? _____.

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j. You now need some data/information from the internet. What data/information do you need?

k. Record the information: _____ & _____.

# of folds	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Sheets of Paper	1														
Height															

l. You now need to make some calculations →

m. So make your final prediction → how many times do you need to fold a sheet of A4 paper in order to get a height equal to the earth-moon distance?

(E) CAC Payment Options

Mr. Rutherford is offering Mr. Santowski & Mr. Smith new contract options for the New Year. Here are the terms of the contracts being offered:

OPTION A → Here is Mr. Smith's payment option: Get paid \$5,000 US per day for each day in the month of January.

OPTION B → Here is Mr. Santowski payment option:

1. Get paid 1 piastre on the first day of January.
2. But then on the 2nd of January, return the 1 piastre and get paid double yesterday's wage, so get 2 piastres for having worked 2 days.
3. Now, on the 3rd of January, return the 2 piastres and get paid double yesterday's wage of 2 piastres, making it a total of 4 piastres pay for these three days.
4. Alas, on the 4th of January, return the 4 piastres and get paid double yesterday's wage of these 4 piastres, making it a total of 8 piastres pay for these four days.
5. Oh, woe is me. On the 5th of January, I return the 8 piastres, but get paid double yesterday's wage of these 8 piastres, making it a total of 16 piastres pay for these five days.

- a. Which option would you choose and why?
- b. Are the salaries ever equal? If so when? If not why not?
- c. How much does each Math teacher get paid by the end of January? Convert to a common currency & show your work.

(F) Grains of Rice Challenge → Legend of the Ambalappuzha Paal Payasam

There is a well-known story of the man who invented chess. The local ruler was so pleased with the invention that he offered the inventor a great reward in gold. The inventor suggested an alternative reward: he would get one grain of rice on the first square of the chess board, two grains on the second square, four on the third, eight on the fourth, etc., doubling the number of grains each time. The ruler saw that this must be a much better deal for him, and accepted. The board has 64 squares.

- a. How many total grains of rice did the ruler have to pay the inventor? Show your work.
- b. If these grains of rice were lined up end to end, how far would the line go? Show your work and internet data/information you needed to come up with an estimate.
- c. If these grains of rice were used to cover up the land in India, how deep would the pile be? Show your work and internet data/information you needed to come up with an estimate.

The Legend of the Ambalappuzha Paal Payasam is an alternate version of the same story. Check it out!

