

Worksheet #1 *Factoring Polynomials: GCF and Factoring by Grouping*

Find the GCF (greatest common factor) of the expressions.

EX $5x^2y^2, 30x^3y$

The GCF is $5x^2y$

EX $2x(x + 5), 15(x + 5)$

The GCF is $(x + 5)$ **EXERCISES:** Find the GCF of the expressions.

1. $x^2, -x^6$

2. t^4, t^7

3. $2x^2, 12x$

4. $36x^4, 18x^3$

5. u^2v, u^3v^2

6. $x^6y^4, -xy$

7. $9y^8z^4, -12y^5z^4$

8. $-15x^6y^3, 45xy^3$

9. $14x^2, 1, 7x^4$

10. $5y^4, 10x^2y^2$

11. $28a^4b^2, 14a^3, 42a^2b^5$

12. $16x^2y, 12xy^2, 36x^2$

13. $2(x + 3), 3(x + 3)$

14. $14(x - 5), 3x(x - 5)$

15. $x(7x + 5), 7x + 5$

16. $x - 4, y(x - 4)$

Solutions:

1. x^2 3. $2x$ 5. u^2v 7. $3y^5z^4$ 9. 1 11. $14a^2$ 13. $(x + 3)$ 15. $(7x + 5)$

Factor out the GCF from each expression using the DISTRIBUTIVE PROPERTY (backwards).

EX $12x^3y^2 - 36x^2y + 6x$

$$\text{GCF is } 6x, \Rightarrow 6x(2x^3y^2 - 6xy + 1)$$

EXERCISES:

Factor out the GCF from each expression using the distributive property.

17. $3x + 3$

18. $5y - 5$

19. $6x + 36$

20. $4x - 28$

21. $8t - 16$

22. $4u - 12$

23. $25x - 10$

24. $14y - 7$

25. $24y^2 - 18$

26. $8x^3 + 12$

27. $x^2 + x$

28. $x^3 - x$

29. $25u^2 - 14u$

30. $36t^4 + 24t^2$

31. $2x^4 + 6x^3$

32. $9z^6 + 27z^4$

33. $27x^2 + 9y^2$

34. $12x^2 - 5x^3$

35. $12x^2 - 2x$

36. $12u^7 - 9u^5$

37. $10r^3 - 35r$

38. $-144a^8 + 24a^6$

39. $12x^2 + 16x - 8$

40. $9 - 3y - 15y^2$

41. $100 - 75z - 50z^2$

42. $42t^3 - 21t^2 + 7$

43. $9x^4 + 6x^3 + 18x^2$

44. $32a^5 - 2a^3 + 6a$

45. $5u^3 + 5u^2 + 5u$

46. $11y^3 - 22y^2 + 11y$

47. $16a^3b^3 + 24a^4b^3$

48. $9x^4y + 24x^2y$

49. $10ab + 10a^2b$

50. $21x^2y^5 + 35x^6y$

51. $15m^4n^3 - 25m^7n + 30m^4n^8$

52. $4xy + 8x^2y - 24x^4y^5$

Solutions:

17. $3(x + 1)$ 19. $6(x + 6)$ 21. $8(t - 2)$ 23. $5(5x - 2)$ 25. $6(4y^2 - 3)$ 27. $x(x + 1)$ 29. $u(25u - 14)$

31. $2x^3(x + 3)$ 33. $9(3x^2 + y^2)$ 35. $2x(6x - 1)$ 37. $5r(2r^2 - 7)$ 39. $4(3x^2 + 4x - 2)$ 41. $25(4 - 3z - 2z^2)$

43. $3x^2(3x^2 + 2x + 6)$ 45. $5u(u^2 + u + 1)$ 47. $8a^3b^3(2 + 3a)$ 49. $10ab(1 + a)$ 51. $5m^4n(3n^2 - 5m^3 + 6n^7)$

EX Factor out a positive GCF from this expression.

$$15 - 5x = 5(3 - x)$$

Factor out a negative GCF from this expression.

$$\begin{aligned} 15 - 5x &= -5(-3 + x) \\ &= -5(x - 3) \end{aligned}$$

BOTH ARE CORRECT!

EXERCISES:

Factor out the GCF from each expression using the distributive property.

Factor each TWICE...

1st: Factor out a **POSITIVE** GCF.

2nd: Factor out a **NEGATIVE** GCF.

53. $5 - 10x$

54. $3 - 6x$

55. $-10x - 3000$

56. $-3x^2 + 4$ **

57. $-x^2 + 5x + 10$ **

58. $-4x^2 - 8x + 20$

59. $4 + 12x - 2x^2$

60. $8 - 4x - 12x^2$

** The only positive common factor is one. For this problem, just do the the 2nd part by factoring out a negative one.

Solutions:

53. $5(1 - 2x)$, $-5(2x - 1)$ 55. $10(-x - 300)$, $-10(x + 300)$ 57. $-1(x^2 - 5x - 10)$ 59. $2(2 + 6x - x^2)$, $-2(x^2 - 6x - 2)$

Factor out the common binomial factor.

$$\begin{aligned} \text{EX} \quad x^2(3x + 1) - 3(3x + 1) \\ = (3x + 1)(x^2 - 3) \end{aligned}$$

$$\begin{aligned} \text{EX} \quad 7x(x - 3) - 4(3 - x) \\ = 7x(x - 3) + 4(x - 3) \\ = (x - 3)(7x + 4) \end{aligned}$$

Factor out a **negative one** to make this binomial $(x - 3)$.

EXERCISES:

Factor out the common binomial factor from each expression using the distributive property.

61. $x(x - 3) + 5(x - 3)$

62. $x(x + 6) + 3(x + 6)$

63. $y(q - 5) - 10(q - 5)$

64. $a^2(b + 2) - b(b + 2)$

65. $x^3(y + 4) + y(y + 4)$

66. $x^3(x - 2) + 6(x - 2)$

67. $(a + b)(a - b) + a(a + b)$

68. $y^2(x - y) - 4(y - x)$

Solutions:

61. $(x - 3)(x + 5)$ 63. $(q - 5)(y - 10)$ 65. $(y + 4)(x^3 + y)$ 67. $(a + b)(2a - b)$

Factoring by Grouping

- This is a strategy that we can use to factor polynomials that have FOUR terms.
- This strategy has three steps:
 - (1) **Draw parentheses** to make TWO groups
 - (2) **Factor out the GCF from each** of the two groups
 - (3) **Factor out the common binomial** factor (if there is one).

Use factoring by grouping to factor these polynomials.

EX $ax + 2x - 2a - 4$

$$(ax + 2x) - (2a + 4)$$

$$x(a + 2) - 2(a + 2)$$

$$(a + 2)(x - 2)$$

EXERCISES:

Factor by grouping.

- | | |
|-----------------------------|-----------------------------|
| 69. $t^3 - 3t^2 + 2t - 6$ | 70. $x^3 + 6x^2 + 2x + 12$ |
| 71. $16x^3 + 8x^2 + 2x + 1$ | 72. $4u^3 - 2u^2 - 6u + 3$ |
| 73. $x^3 - 3x - x^2 + 3$ | 74. $x^3 + 7x - 3x^2 - 21$ |
| 75. $3x^2 + x^3 - 18 - 6x$ | 76. $5x^2 + 10x^3 + 4 + 8x$ |
| 77. $ky^2 - 4ky + 2y - 8$ | 78. $ay^2 + 3ay + 3y + 9$ |
| 79. $3a + ab + 3c + bc$ | 80. $x^2 - 2x + xy - 2y$ |
| 81. $h^2 - hk + hr - kr$ | 82. $p^3 + 2p^2 + 4p + 8$ |
| 83. $p^2 - 2pq + pr - 2qr$ | 84. $3hk - 2k - 12h + 8$ |

Solutions:

69. $(t - 3)(t^2 + 2)$ 71. $(2x + 1)(8x^2 + 1)$ 73. $(x - 1)(x^2 - 3)$ 75. $(x + 3)(x^2 - 6)$ 77. $(y - 4)(ky + 2)$
 79. $(a + c)(3 + b)$ 81. $(h - k)(h + r)$ 83. $(p - 2q)(p + r)$