Worksheet 1.10 Further Algebra

Section 1 Distributive Laws

When collecting like terms, we need to know things like

$$\begin{aligned} 2(a+b) &= 2a+2b \\ \text{and } 3(a-b) &= 3a-3b \end{aligned}$$

This is called the distributive law, which is a rule that holds true in the arithmetic of numbers. In general

$$a(b+c) = ab+ac$$

Example 1 :

$$\begin{aligned} 5(x+2y) &= 5x+10y \\ 3x(x+y) &= 3x^2+3xy \\ -2(a+b) &= -2a-2b \\ -3(a-b) &= -3a+3b \end{aligned}$$

These are just extensions of the rules for removing brackets when an addition or subtraction sign is in front of the brackets. Here are some examples that involve collecting like terms:

$$\begin{aligned} x(y+2)-3(y+2x) &= xy+2x-3y-6x \\ &= xy-3y+2x-6x \\ &= xy-3y-4x \\ a(b+c)+b(a-c) &= ab+ac+ba-bc \\ &= ab+ac+ab-bc \\ &= 2ab+ac-bc \end{aligned}$$

Notice that, in the second example, we also used the commutative law: $ab = ba$.

Exercises:

1. Expand the following by removing the brackets:

(a) $6(x+3)$
(b) $2(3x-4)$
(c) $-3(x+y)$
(d) $5(m-4)$
(e) $x(x+y)$
(f) $y(y^2-2)$
(g) $t(3t+4)$
(h) $4(y-5)$
(i) $-x(x+2)$
(j) $-3(m-n)$

2. Remove the brackets and collect like terms in each of the following:

(a) $2(m+4)+3(m+6)$
(b) $4(t-2)-3(t+1)$
(c) $7(m-3)-2(m-4)$
(d) $-4(x+1)+3(x+2)$
(e) $5(2t+1)+3(t+2)$
(f) $4(x-3)+2(5-x)$
(g) $x(x+4)+x(x-3)$
(h) $t(t+1)-4(t+2)$
(i) $m(m-4)+2(m+1)$
(j) $3(x+2)+4x$

Section 2 Further Distributive Laws

Example 1 : Sometimes we need to expand and simplify algebraic expressions like

$$(x+2)(x+3)$$

If we think of $(x+3)$ as a single term for the time being, and call it A, so that $A=x+3$, then

$$\begin{aligned}
(x+2)(x+3) &= (x+2)A \\
&= xA+2A && \text{by the distributive law} \\
&= x(x+3)+2(x+3) && \text{substituting for } A \\
&= x^2+3x+2x+6 \\
&= x^2+5x+6 && \text{collecting like terms}
\end{aligned}$$

Example 2 :

$$\begin{aligned}(x-5)(y+2) &= (x-5)B \qquad \text{if} \quad B = y+2 \\ &= xB - 5B \\ &= x(y+2) - 5(y+2) \\ &= xy + 2x - 5y - 10\end{aligned}$$

When we are doing these calculations we normally multiply each term in the first bracket by each term in the second and leave out the middle steps. However, putting in all the steps at first is a good idea.

Example 3 :

$$\begin{aligned}(y+2)(y-1) &= y(y-1) + 2(y-1) \\ &= y^2 - y + 2y - 2 \\ &= y^2 + y - 2\end{aligned}$$

To expand expressions which have one set of brackets inside another, just follow the rules already given, although it is best to do the inner-most brackets first then the outer ones.

Example 4 :

$$\begin{aligned}(x + (5-y)2) + 5(x - 3(y+1)) &= (x + 10 - 2y) + 5(x - 3y - 3) \\ &= x + 10 - 2y + 5x - 15y - 15 \\ &= x + 5x - 2y - 15y + 10 - 15 \\ &= 6x - 17y - 5\end{aligned}$$

Note that it is the convention for the constant term, in this case -5, to be put last in the expression when writing an answer, and for the other terms to be written in alphabetical order.

Exercises:

1. Expand the following and collect like terms:

   (a) $(x+2)(x+5)$

   (b) $(x+4)(x+1)$

(c) $(y+2)(y-3)$
(d) $(m+7)(m-5)$
(e) $(x+8)(x-3)$
(f) $(2x+1)(x+2)$
(g) $(3m+2)(m-4)$
(h) $(x+3)(x-3)$
(i) $(y+5)(y-5)$
(j) $(m+5)^2$

2. Expand the following and collect like terms:

   (a) $(2x+3(x+1))+4(x+2)$
   (b) $(8x-2(x+3))+3(x+4)$
   (c) $(2x+4(x+1))+2(x+3(x+4))$
   (d) $(2m+3(m+4))-2(m+1)$
   (e) $2(4m-6)-5(2m+3(m+1))$

Section 3 Introduction to Substitution

Algebraic expressions are useful to us as a way of representing quantities without assigning explicit numerical values. For example, the perimeter of a rectangle is twice the length plus twice the width. Let the length be represented by l, the width by w, and the perimeter by P. Then we have the formula

$$P = 2l + 2w$$

We can use this formula to find the perimeter of any rectangle, once the width and length are given. If a rectangle is 5cm long and 2 cm wide, then the perimeter is

$$P = 2 \times 5 + 2 \times 2 = 10 + 4 = 14$$

so the perimeter is 14 cm.

Example 1 :
If $x = 4$ and $y = 3$, what does $xy + 5x$ equal?
Well, $xy + 5x = 4 \times 3 + 5 \times 4 = 12 + 20 = 32$.