5 Chapter Summary

What did you learn?

Section 5.1	Review Exercises
☐ Recognize and write the fundamental trigonometric identities.	1–10
☐ Use the fundamental trigonometric identities to evaluate trigonometric functions, simplify trigonometric expressions, and rewrite trigonometric expressions.	11–24
Section 5.2	
☐ Verify trigonometric identities.	25–36
Section 5.3	
☐ Use standard algebraic techniques to solve trigonometric equations.	37–48
☐ Solve trigonometric equations of quadratic type.	49–52
☐ Solve trigonometric equations involving multiple angles.	53–58
☐ Use inverse trigonometric functions to solve trigonometric equations.	59–62
Section 5.4	
☐ Use sum and difference formulas to evaluate trigonometric functions, verify identities, and solve trigonometric equations.	63–84
Section 5.5	
☐ Use multiple-angle formulas to rewrite and evaluate trigonometric functions.	85–94
☐ Use power-reducing formulas to rewrite and evaluate trigonometric functions.	95–98
☐ Use half-angle formulas to rewrite and evaluate trigonometric functions.	99–110
☐ Use product-to-sum and sum-to-product formulas to rewrite and evaluate trigonometric functions.	111–122

5 Review Exercises

5.1 **In Exercises 1–10, name the trigonometric function equivalent to the expression.**

1. $\dfrac{1}{\cos x}$
2. $\dfrac{1}{\sin x}$
3. $\dfrac{1}{\sec x}$
4. $\dfrac{1}{\tan x}$
5. $\sqrt{1-\cos^2 x}$
6. $\sqrt{1+\tan^2 x}$
7. $\csc\left(\dfrac{\pi}{2}-x\right)$
8. $\cot\left(\dfrac{\pi}{2}-x\right)$
9. $\sec(-x)$
10. $\tan(-x)$

In Exercises 11–14, use the given values to evaluate (if possible) all six trigonometric functions of the angle.

11. $\sin x = \dfrac{4}{5}, \quad \cos x = \dfrac{3}{5}$
12. $\tan \theta = \dfrac{2}{3}, \quad \sec \theta = \dfrac{\sqrt{13}}{3}$
13. $\sin\left(\dfrac{\pi}{2}-x\right) = \dfrac{1}{\sqrt{2}}, \quad \sin x = -\dfrac{1}{\sqrt{2}}$
14. $\csc\left(\dfrac{\pi}{2}-\theta\right) = 3, \quad \sin \theta = \dfrac{2\sqrt{2}}{3}$

In Exercises 15–22, use the fundamental identities to simplify the expression. Use the *table* feature of a graphing utility to check your result numerically.

15. $\dfrac{1}{\cot^2 x + 1}$
16. $\dfrac{\sec^2 x - 1}{\sec x - 1}$
17. $\dfrac{\sin^2 \alpha - \cos^2 \alpha}{\sin^2 \alpha - \sin \alpha \cos \alpha}$
18. $\dfrac{\sin^3 \beta + \cos^3 \beta}{\sin \beta + \cos \beta}$
19. $\tan^2 \theta(\csc^2 \theta - 1)$
20. $\csc^2 x(1-\cos^2 x)$
21. $\tan\left(\dfrac{\pi}{2}-x\right)\sec x$
22. $\dfrac{\sin(-x)\cot x}{\sin\left(\dfrac{\pi}{2}-x\right)}$

23. ***Rate of Change*** The rate of change of the function $f(x) = 2\sqrt{\sin x}$ is given by the expression $\sin^{-1/2} x \cos x$. Show that this expression can also be written as $\cot x\sqrt{\sin x}$.

24. ***Rate of Change*** The rate of change of the function $f(x) = \csc x - \cot x$ is the expression $\csc^2 x - \csc x \cot x$. Show that this expression can also be written as $(1-\cos x)/\sin^2 x$.

5.2 **In Exercises 25–36, verify the identity.**

25. $\cos x(\tan^2 x + 1) = \sec x$
26. $\sec^2 x \cot x - \cot x = \tan x$
27. $\sin^3 \theta + \sin \theta \cos^2 \theta = \sin \theta$
28. $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$
29. $\sin^5 x \cos^2 x = (\cos^2 x - 2\cos^4 x + \cos^6 x)\sin x$
30. $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x)\cos x$
31. $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \dfrac{1-\sin\theta}{|\cos\theta|}$
32. $\sqrt{1-\cos x} = \dfrac{|\sin x|}{\sqrt{1+\cos x}}$
33. $\dfrac{\csc(-x)}{\sec(-x)} = -\cot x$
34. $\dfrac{1+\sec(-x)}{\sin(-x)+\tan(-x)} = -\csc x$
35. $\sin^2 x + \sin^2\left(\dfrac{\pi}{2}-x\right) = 1$
36. $\csc x \sin\left(\dfrac{\pi}{2}-x\right) = \cot x$

5.3 **In Exercises 37–48, solve the equation.**

37. $2\sin x - 1 = 0$
38. $\tan x + 1 = 0$
39. $\sin x = \sqrt{3} - \sin x$
40. $4\cos x = 1 + 2\cos x$
41. $3\sqrt{3}\tan x = 3$
42. $\frac{1}{2}\sec x - 1 = 0$
43. $3\csc^2 x = 4$
44. $4\tan^2 x - 1 = \tan^2 x$
45. $4\cos^2 x - 3 = 0$
46. $\sin x(\sin x + 1) = 0$
47. $\sin x - \tan x = 0$
48. $\csc x - 2\cot x = 0$

In Exercises 49–58, find all solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to check your answers.

49. $2\cos^2 x - \cos x = 1$
50. $2\sin^2 x - 3\sin x = -1$
51. $\cos^2 x + \sin x = 1$
52. $\sin^2 x + 2\cos x = 2$
53. $2\sin 2x - \sqrt{2} = 0$
54. $\sqrt{3}\tan 3x = 0$
55. $\cos 4x(\cos x - 1) = 0$
56. $3\csc^2 5x = -4$

57. $\cos 4x - 7\cos 2x = 8$

58. $\sin 4x - \sin 2x = 0$

In Exercises 59–62, use the inverse functions where necessary to find all solutions of the equation in the interval $[0, 2\pi)$.

59. $\sin^2 x - 2\sin x = 0$

60. $2\cos^2 x + 3\cos x = 0$

61. $\tan^2\theta + \tan\theta - 12 = 0$

62. $\sec^2 x + 6\tan x + 4 = 0$

5.4 **In Exercises 63–66, find the exact values of the sine, cosine, and tangent of the angle.**

63. $285° = 315° - 30°$

64. $345° = 300° + 45°$

65. $\frac{25\pi}{12} = \frac{11\pi}{6} + \frac{\pi}{4}$

66. $\frac{19\pi}{12} = \frac{11\pi}{6} - \frac{\pi}{4}$

In Exercises 67–70, write the expression as the sine, cosine, or tangent of an angle.

67. $\sin 140° \cos 50° + \cos 140° \sin 50°$

68. $\cos 45° \cos 120° - \sin 45° \sin 120°$

69. $\frac{\tan 25° + \tan 10°}{1 - \tan 25° \tan 10°}$

70. $\frac{\tan 68° - \tan 115°}{1 + \tan 68° \tan 115°}$

In Exercises 71–76, find the exact value of the trigonometric function given that $\sin u = \frac{3}{4}$ and $\cos v = -\frac{5}{13}$. (Both u and v are in Quadrant II.)

71. $\sin(u + v)$

72. $\tan(u + v)$

73. $\tan(u - v)$

74. $\sin(u - v)$

75. $\cos(u + v)$

76. $\cos(u - v)$

In Exercises 77–82, verify the identity.

77. $\cos\left(x + \frac{\pi}{2}\right) = -\sin x$

78. $\sin\left(x - \frac{3\pi}{2}\right) = \cos x$

79. $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

80. $\sin(\pi - x) = \sin x$

81. $\cos 3x = 4\cos^3 x - 3\cos x$

82. $\frac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

In Exercises 83 and 84, find the solutions of the equation in the interval $[0, 2\pi)$.

83. $\sin\left(x + \frac{\pi}{2}\right) - \sin\left(x - \frac{\pi}{2}\right) = \sqrt{3}$

84. $\cos\left(x + \frac{3\pi}{4}\right) - \cos\left(x - \frac{3\pi}{4}\right) = 0$

5.5 **In Exercises 85–88, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.**

85. $\sin u = -\frac{5}{7}, \quad \pi < u < \frac{3\pi}{2}$

86. $\cos u = \frac{4}{5}, \quad \frac{3\pi}{2} < u < 2\pi$

87. $\tan u = -\frac{2}{9}, \quad \frac{\pi}{2} < u < \pi$

88. $\cos u = -\frac{2}{\sqrt{5}}, \quad \frac{\pi}{2} < u < \pi$

In Exercises 89–92, use double-angle formulas to verify the identity algebraically. Use a graphing utility to check your result graphically.

89. $6\sin x\cos x = 3\sin 2x$

90. $4\sin x\cos x + 2 = 2\sin 2x + 2$

91. $1 - 4\sin^2 x\cos^2 x = \cos^2 2x$

92. $\sin 4x = 8\cos^3 x\sin x - 4\cos x\sin x$

93. ***Projectile Motion*** A baseball leaves the hand of the first baseman at an angle of θ with the horizontal and with an initial velocity of $v_0 = 80$ feet per second. The ball is caught by the second baseman 100 feet away. Find θ if the range r of a projectile is given by $r = \frac{1}{32}v_0^2 \sin 2\theta$.

94. ***Projectile Motion*** Use the equation in Exercise 93 to find θ when a golf ball is hit with an initial velocity of $v_0 = 50$ feet per second and lands 77 feet away.

In Exercises 95–98, use the power-reducing formulas to rewrite the expression in terms of the first power of the cosine.

95. $\sin^6 x$

96. $\cos^4 x \sin^4 x$

97. $\cos^4 2x$

98. $\sin^4 2x$

In Exercises 99–102, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

99. $105°$

100. $67° 30'$

101. $\frac{7\pi}{8}$

102. $\frac{11\pi}{12}$

In Exercises 103–106, find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.

103. $\sin u = \frac{3}{5}, \quad 0 < u < \frac{\pi}{2}$

104. $\tan u = \frac{5}{8}, \quad \pi < u < \frac{3\pi}{2}$

105. $\cos u = -\frac{2}{7}, \quad \frac{\pi}{2} < u < \pi$

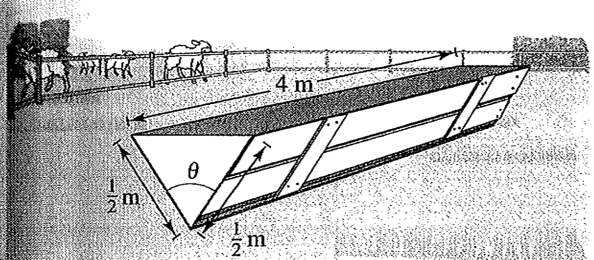
106. $\sec u = -6, \quad \frac{\pi}{2} < u < \pi$

In Exercises 107 and 108, use the half-angle formulas to simplify the expression.

107. $-\sqrt{\frac{1 + \cos 10x}{2}}$

108. $\frac{\sin 6x}{1 + \cos 6x}$

Geometry **In Exercises 109 and 110, a trough for feeding cattle is 4 meters long and its cross sections are isosceles triangles with two equal sides of $\frac{1}{2}$ meter (see figure). The angle between the equal sides is θ.**



109. Write the trough's volume as a function of $\theta/2$.

110. Write the volume of the trough as a function of θ and determine the value of θ such that the volume is maximum.

In Exercises 111–114, use the product-to-sum formulas to write the product as a sum or difference.

111. $6 \sin \frac{\pi}{4} \cos \frac{\pi}{4}$

112. $4 \sin 15° \sin 45°$

113. $\sin 3\alpha \sin 2\alpha$

114. $\cos 4\theta \sin 6\theta$

In Exercises 115–118, use the sum-to-product formulas to write the sum or difference as a product.

115. $\cos 3\theta + \cos 2\theta$

116. $\sin 5\theta + \sin 3\theta$

117. $\sin\left(x + \frac{\pi}{4}\right) - \sin\left(x - \frac{\pi}{4}\right)$

118. $\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right)$

Harmonic Motion **In Exercises 119–122, a weight is attached to a spring suspended vertically from a ceiling. When a driving force is applied to the system, the weight moves vertically from its equilibrium position. This motion is described by the model**

$$y = 1.5 \sin 8t - 0.5 \cos 8t$$

where y is the distance from equilibrium in feet and t is the time in seconds.

119. Write the model in the form
$y = \sqrt{a^2 + b^2} \sin(Bt + C)$.

120. Use a graphing utility to graph the model.

121. Find the amplitude of the oscillations of the weight.

122. Find the frequency of the oscillations of the weight.

Synthesis

True or False? **In Exercises 123–126, determine whether the statement is true or false. Justify your answer.**

123. If $\frac{\pi}{2} < \theta < \pi$, then $\cos \frac{\theta}{2} < 0$.

124. $\sin(x + y) = \sin x + \sin y$

125. $4 \sin(-x) \cos(-x) = -2 \sin 2x$

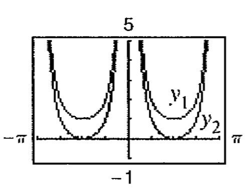
126. $4 \sin 45° \cos 15° = 1 + \sqrt{3}$

127. List the reciprocal identities, quotient identities, and Pythagorean identities from memory.

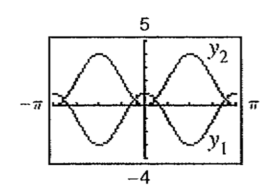
128. Is $\cos \theta = \sqrt{1 - \sin^2 \theta}$ an identity? Explain.

In Exercises 129 and 130, use the graphs of y_1 and y_2 to determine how to change y_2 to a new function y_3 such that $y_1 = y_3$.

129. $y_1 = \sec^2\left(\frac{\pi}{2} - x\right)$
$y_2 = \cot^2 x$



130. $y_1 = \frac{\cos 3x}{\cos x}$
$y_2 = (2 \sin x)^2$



5 Chapter Test

Take this test as you would take a test in class. After you are finished, check your work against the answers given in the back of the book.

1. If $\tan\theta = \frac{6}{5}$ and $\cos\theta < 0$, use the fundamental identities to evaluate the other five trigonometric functions of θ.
2. Use the fundamental identities to simplify $\csc^2\beta(1 - \cos^2\beta)$.
3. Factor and simplify $\dfrac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x}$.
4. Add and simplify $\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$.
5. Determine the values of θ, $0 \le \theta < 2\pi$, for which $\tan\theta = -\sqrt{\sec^2\theta - 1}$ is true.
6. Use a graphing utility to graph the functions $y_1 = \cos x + \sin x \tan x$ and $y_2 = \sec x$. Make a conjecture about y_1 and y_2. Verify your result algebraically.

In Exercises 7–12, verify the identity.

7. $\sin\theta\sec\theta = \tan\theta$
8. $\sec^2 x\tan^2 x + \sec^2 x = \sec^4 x$
9. $\dfrac{\csc\alpha + \sec\alpha}{\sin\alpha + \cos\alpha} = \cot\alpha + \tan\alpha$
10. $\cos\left(x + \dfrac{\pi}{2}\right) = -\sin x$
11. $\cos(\pi - \theta) + \sin\left(\dfrac{\pi}{2} + \theta\right) = 0$
12. $(\sin x + \cos x)^2 = 1 + \sin 2x$
13. Find the exact value of $\tan 105°$.
14. Rewrite $\sin^4 x\tan^2 x$ in terms of the first power of the cosine.
15. Use a half-angle formula to simplify the expression $\dfrac{\sin 4\theta}{1 + \cos 4\theta}$.
16. Write $6\sin 4\theta\sin 6\theta$ as a sum or difference.
17. Write $\cos 5\theta + \cos 3\theta$ as a product.

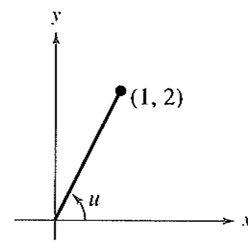


Figure for 23

In Exercises 18–21, find all solutions of the equation in the interval $[0, 2\pi)$.

18. $\tan^2 x + \tan x = 0$
19. $\sin 2\alpha - \cos\alpha = 0$
20. $4\cos^2 x - 3 = 0$
21. $\csc^2 x - \csc x - 2 = 0$
22. Use a graphing utility to approximate the solutions of the equation $5\cos x - x = 0$ in the interval $[0, 2\pi)$ accurate to three decimal places.
23. Use the figure at the right to find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$.
24. The *index of refraction* n of a transparent material is the ratio of the speed of light in a vacuum to the speed of light in the material. For the glass triangular prism in the figure at the right, $n = 1.5$ and $\alpha = 60°$. Find the angle θ for the glass prism if

$$n = \frac{\sin(\theta/2 + \alpha/2)}{\sin(\theta/2)}.$$

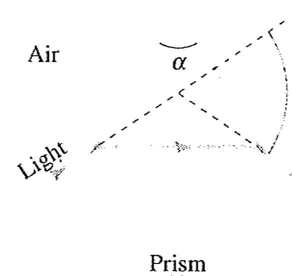


Figure for 24