CHAPTER 5 REVIEW

Important Facts and Formulas

- *Rational Exponents:*

$$c^{\frac{1}{n}} = \sqrt[n]{c}$$

$$(c^t)^{\frac{1}{k}} = (c^{\frac{1}{k}})^t = c^{\frac{t}{k}}$$

- *Laws of Exponents:*

$$c^r c^s = c^{r+s} \qquad (cd)^r = c^r d^r$$

$$\frac{c^r}{c^s} = c^{r-s} \qquad \left(\frac{c}{d}\right)^r = \frac{c^r}{d^r}$$

$$(c^r)^s = c^{rs} \qquad c^{-r} = \frac{1}{c^r}$$

- $g(x) = \log x$ is the inverse function of $f(x) = 10^x$:

$$10^{\log v} = v \text{ for all } v > 0 \quad \text{and} \quad \log 10^u = u \text{ for all } u$$

- $g(x) = \ln x$ is the inverse function of $f(x) = e^x$:

$$e^{\ln v} = v \text{ for all } v > 0 \quad \text{and} \quad \ln e^u = u \text{ for all } u$$

- $h(x) = \log_b x$ is the inverse function of $k(x) = b^x$:

$$b^{\log_b v} = v \text{ for all } v > 0 \quad \text{and} \quad \log_b (b^u) = u \text{ for all } u$$

- *Logarithm Laws:* For all $v, w > 0$ and any k:

$$\ln(vw) = \ln v + \ln w \qquad \log_b (vw) = \log_b v + \log_b w$$

$$\ln\left(\frac{v}{w}\right) = \ln v - \ln w \qquad \log_b \left(\frac{v}{w}\right) = \log_b v - \log_b w$$

$$\ln(v^k) = k \ln v \qquad \log_b (v^k) = k \log_b v$$

- *Exponential Growth Functions:*

$$f(x) = P(1 + r)^x \quad (0 < r < 1)$$

$$f(x) = Pa^x \quad (a > 1)$$

$$f(x) = Pe^{kx} \quad (k > 0)$$

- *Exponential Decay Functions:*

$$f(x) = P(1 - r)^x \quad (0 < r < 1)$$

$$f(x) = Pa^x \quad (0 < a < 1)$$

$$f(x) = Pe^{kx} \quad (k < 0)$$

- *Logistic Function:* $f(x) = \dfrac{a}{1 + be^{-kx}}$

- *Compound Interest Formula:* $A = P(1 + r)^t$

- *Continuous Compounding:* $A = Pe^{rt}$

- *Radioactive Decay Function:* $f(x) = P(0.5)^{\frac{x}{h}}$

- *Change of Base Formula:* $\log_b v = \dfrac{\ln v}{\ln b}$

Review Exercises

Section 5.1

In Exercises 1–6, simplify the expression.

1. $\sqrt{\sqrt[3]{c^{12}}}$
2. $\left(\sqrt[3]{4c^3d^2}\right)^3\left(c\sqrt{d}\right)^2$
3. $\left(a^{-\frac{2}{3}}b^{\frac{2}{5}}\right)\left(a^3b^6\right)^{\frac{4}{3}}$
4. $\dfrac{(3c)^{\frac{3}{5}}(2d)^{-2}(4c)^{\frac{1}{2}}}{(4c)^{\frac{1}{5}}(2d)^4(2c)^{-\frac{3}{2}}}$
5. $\left(u^{\frac{1}{4}} - v^{\frac{1}{4}}\right)\left(u^{\frac{1}{4}} + v^{\frac{1}{4}}\right)$
6. $c^{\frac{3}{2}}\left(2c^{\frac{1}{2}} + 3c^{-\frac{3}{2}}\right)$

In Exercises 7 and 8, simplify and write the expression without radicals or negative exponents.

7. $\dfrac{\sqrt[3]{6c^4d^{14}}}{\sqrt[3]{48c^{-2}d^2}}$
8. $\dfrac{(8u^5)^{\frac{1}{4}}\,2^{-1}u^{-3}}{2u^8}$

9. Rationalize the numerator and simplify: $\dfrac{\sqrt{2x + 2h + 1} - \sqrt{2x + 1}}{h}$

10. Rationalize the denominator: $\dfrac{5}{\sqrt{x} - 3}$

Section 5.2

In Exercises 11–16, list the transformations needed to transform the graph of $f(x) = 5^x$ into the graph of the given function.

11. $g(x) = -2 \cdot 5^x$
12. $h(x) = 5^{3x}$
13. $k(x) = 5^{-\frac{1}{2}x}$
14. $g(x) = 5^{2-x}$
15. $h(x) = 5^x + 4$
16. $h(x) = -5^{x+2}$

In Exercises 17 and 18, find a viewing window (or windows) that shows a complete graph of the function.

17. $f(x) = 2^{x^2-x-2}$
18. $g(x) = \dfrac{850}{1 + 5e^{-0.4x}}$

19. Compunote offers a starting salary of $60,000 with $1000 yearly raises. Calcuplay offers a starting salary of $30,000 with a 6% raise each year.
 a. Complete the following table for each company.

Year	Compunote	Year	Calcuplay
1	$60,000	1	$30,000
2	$61,000	2	$31,800
3		3	
4		4	
5		5	

 b. For each company write a function that gives your salary in terms of years employed.
 c. If you plan on staying with the company for only five years, which job should you take to earn the most money?
 d. If you plan on staying with the company for 20 years, which is your best choice?

20. A computer software company claims that the following function models the "learning curve" for their software.

$$P(t) = \frac{100}{1 + 48.2e^{-0.52t}}$$

where t is measured in months and $P(t)$ is the average percent of the software program's capabilities mastered after t months.

a. Initially what percent of the program is mastered?
b. After 6 months what percent of the program is mastered?
c. Roughly, when can a person expect to "learn the most in the least amount of time"?
d. If the company's claim is true, how many months will it take to have completely mastered the program?

Section 5.3

21. Phil borrows \$800 at 9% annual interest, compounded annually.
    a. How much does he owe after 6 years?
    b. If he pays off the loan at the end of 6 years, how much interest will he owe?

22. If you invest \$5000 for 5 years at 9% annual interest, how much more will you make if interest is compounded continuously than if it is compounded quarterly?

23. Mary Karen invests \$2000 at 5.5% annual interest, compounded monthly.
    a. How much is her investment worth in 3 years?
    b. When will her investment be worth \$12,000?

24. If a \$2000 investment grows to \$5000 in 14 years, with interest compounded annually, what is the interest rate?

25. Company sales are increasing at 6.5% per year. If sales this year are \$56,000, write the rule of a function that gives the sales in year x (where $x = 0$ corresponds to the present year).

26. The population of Potterville is decreasing at an annual rate of 1.5%. If the population is 38,500 now, what will be the population x years from now?

27. The half-life of carbon-14 is 5730 years. How much carbon-14 remains from an original 16 grams after 12,000 years?

28. How long will it take for 4 grams of carbon-14 to decay to 1 gram?

Section 5.4

In Exercises 29–34, translate the given exponential statement into an equivalent logarithmic one.

29. $e^{6.628} = 756$
30. $e^{5.8972} = 364$
31. $e^{r^2-1} = u + v$
32. $e^{a-b} = c$
33. $10^{2.8785} = 756$
34. $10^{c-d} = t$

In Exercises 35–38, translate the given logarithmic statement into an equivalent exponential one.

35. $\ln 1234 = 7.118$
36. $\ln(ax + b) = y$
37. $\ln(rs) = t$
38. $\log 1234 = 3.0913$

39. Find $\log(-0.01)$.

In Exercises 40–43, describe the transformation from $f(x) = \log x$ or $g(x) = \ln x$ to the given function. Give the domain and range of the given function.

40. $h(x) = -\frac{1}{2}\log(x + 3)$

41. $k(x) = \log(4 - x)$

42. $h(x) = \ln(3x)$

43. $k(x) = 3 \ln x - 5$

44. You are conducting an experiment about memory. The people who participate agree to take a test at the end of your course and every month thereafter for a period of two years. The average score for the group is given by the model $M(t) = 91 - 14 \ln(t + 1)$, $0 \le t \le 24$, where t is time in months after the first test.

- **a.** What is the average score on the initial exam?
- **b.** What is the average score after three months?
- **c.** When will the average drop below 50%?
- **d.** Is the magnitude of the rate of memory loss greater in the first month after the course (from $t = 0$ to $t = 1$) or after the first year (from $t = 12$ to $t = 13$)?
- **e.** Hypothetically, if the model could be extended past $t = 24$ months, would it be possible for the average score to be 0%?

Section 5.5

In Exercises 45–48, evaluate the given expression without using a calculator.

45. $\ln e^3$

46. $\ln e$

47. $e^{\ln \frac{3}{4}}$

48. $e^{\ln (x+2y)}$

49. Simplify: $3 \ln \sqrt{x} + \frac{1}{2} \ln x$

50. Simplify: $\ln(e^{4e})^{-1} + 4e$

In Exercises 51 and 52, write the given expression as a single logarithm.

51. $\ln 3x - 3 \ln x + \ln 3y$

52. $4 \ln x - 2(\ln x^3 + 4 \ln x)$

53. Which of the following statements is *true*?

- **a.** $\ln 10 = (\ln 2)(\ln 5)$
- **b.** $\ln\left(\frac{e}{6}\right) = \ln e + \ln 6$
- **c.** $\ln\left(\frac{1}{7}\right) + \ln 7 = 0$
- **d.** $\ln(-e) = -1$
- **e.** None of the above is true.

54. Which of the following statements is *false*?

- **a.** $10 (\log 5) = \log 50$
- **b.** $\log 100 + 3 = \log 10^5$
- **c.** $\log 1 = \ln 1$
- **d.** $\frac{\log 6}{\log 3} = \log 2$
- **e.** All of the above are false.

55. What is the domain of the function $f(x) = \ln\left(\frac{x}{x - 1}\right)$?

Section 5.5.A

In Exercises 56 and 57, translate the given logarithmic statement into an equivalent exponential one.

56. $\log_5 (cd - k) = u$

57. $\log_d (uv) = w$

58. Write $\log_7 7x + \log_7 y - 1$ as a single logarithm.

59. $\log_{20} 400 = ?$

60. If $\log_3 9^{x^2} = 4$, what is x?

Use the following six graphs for Exercises 61 and 62.

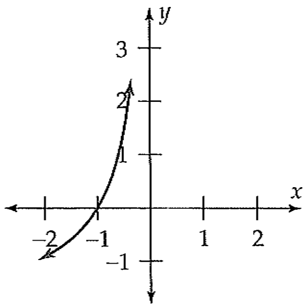

Figure I

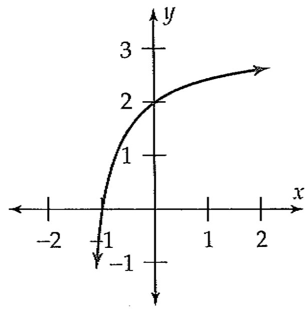

Figure II

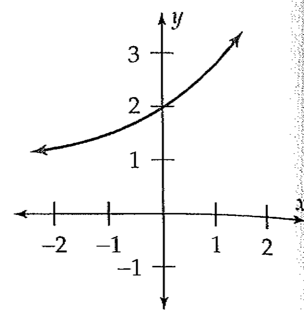

Figure III

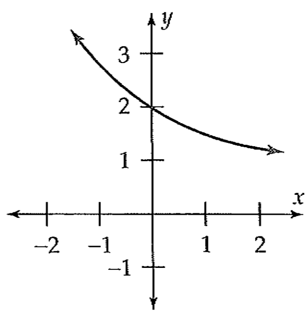

Figure IV

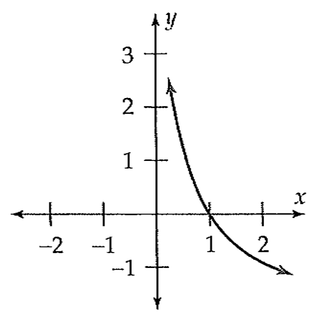

Figure V

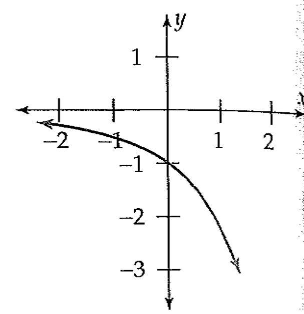

Figure VI

61. If $b > 1$, then the graph of $f(x) = -\log_b x$ could possibly be
a. I **c.** V **e.** none of these
b. IV **d.** VI

62. If $0 < b < 1$ then the graph of $g(x) = b^x + 1$ could possibly be
a. II **c.** IV **e.** none of these
b. III **d.** VI

Section **5.6**

In Exercises 63–71, solve the equation for x.

63. $8^x = 4^{x^2-3}$ **64.** $e^{3x} = 4$ **65.** $2 \cdot 4^x - 5 = -4$

66. $725e^{-4x} = 1500$ **67.** $u = c + d \ln x$ **68.** $2^x = 3^{x+3}$

69. $\ln x + \ln(3x - 5) = \ln 2$ **70.** $\ln(x + 8) - \ln x = 1$

71. $\log(x^2 - 1) = 2 + \log(x + 1)$

72. At a small community college the spread of a rumor through the population of 500 faculty and students can be modeled by

$$\ln n - \ln(1000 - 2n) = 0.65t - \ln 998,$$

where n is the number of people who have heard the rumor after t days.
a. How many people know the rumor initially (at $t = 0$)?
b. How many people have heard the rumor after four days?
c. Roughly, in how many weeks will the entire population have heard the rumor?
d. Use the properties of logarithms to write n as a function of t; in other words solve the model above for n in terms of t.

e. Enter the function you found in part **d** into your calculator and use the table feature to check your answers to parts **a**, **b**, and **c**. Do they agree?
f. Graph the function. Over what time interval does the rumor seem to "spread" the fastest?

73. The half-life of polonium (^{210}Po) is 140 days. If you start with 10 milligrams, how much will be left at the end of a year?

74. An insect colony grows exponentially from 200 to 2000 in 3 months. How long will it take for the insect population to reach 50,000?

75. Hydrogen-3 decays at a rate of 5.59% per year. Find its half-life.

76. The half-life of radium-88 is 1590 years. How long will it take for 10 grams to decay to 1 gram?

77. How much money should be invested at 8% per year, compounded quarterly, in order to have $1000 in 10 years?

78. At what annual interest rate should you invest your money if you want to double it in 6 years?

79. One earthquake measures 4.6 on the Richter scale. A second earthquake is 1000 times more intense than the first. What does it measure on the Richter scale?

Section 5.7

80. The table below gives the population of Austin, Texas.

Year	1950	1970	1980	1990	2000
Population	132,459	253,539	345,890	465,622	656,562

a. Sketch a scatter plot of the data, with $x = 0$ corresponding to 1950.
b. Find an exponential model for the data.
c. Use the model to estimate the population of Austin in 1960 and 2005.

81. The wind-chill factor is the temperature that would produce the same cooling effect on a person's skin if there were no wind. The table shows the wind-chill factors for various wind speeds when the temperature is 25 °F.

Wind speed (mph)	0	5	10	15	20	25	30	35	40	45
Wind chill temperature (°F)	25	19	15	13	11	9	8	7	6	5

[*Source: National Weather Service*]

a. What does a 20-mph wind make 25 °F feel like?
b. Sketch a scatter plot of the data, with $x = 0$ corresponding to 0 mph.
c. Explain why an exponential model would be appropriate.
d. Find an exponential model for the data.
e. According to the model, what is the wind-chill factor for a 23-mph wind?