# CHAPTER 9 REVIEW

## Important Concepts



## Important Facts and Formulas

Addition and Subtraction Identities

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$
$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Cofunction Identities

$$\sin x = \cos\left(\frac{\pi}{2} - x\right) \qquad \cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\tan x = \cot\left(\frac{\pi}{2} - x\right) \qquad \cot x = \tan\left(\frac{\pi}{2} - x\right)$$

$$\sec x = \csc\left(\frac{\pi}{2} - x\right) \qquad \csc x = \sec\left(\frac{\pi}{2} - x\right)$$

Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half-Angle Identities

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

## Review Exercises

Section **9.1**

**In Exercises 1–4, simplify the given expression.**

**1.** $\dfrac{\sin^2 t + (\tan^2 t + 2\tan t - 4) + \cos^2 t}{3\tan^2 t - 3\tan t}$

**2.** $\dfrac{\sec^2 t \csc t}{\csc^2 t \sec t}$

**3.** $\dfrac{\tan^2 x - \sin^2 x}{\sec^2 x}$

**4.** $\dfrac{(\sin x + \cos x)(\sin x - \cos x) + 1}{\sin^2 x}$

In Exercises 5–11, determine graphically whether the equation could not possibly be an identity, or write a proof showing that it is.

**5.** $\sin^4 t - \cos^4 t = 2\sin^2 t - 1$

**6.** $1 + 2\cos^2 t + \cos^4 t = \sin^4 t$

**7.** $\dfrac{\sin t}{1 - \cos t} = \dfrac{1 + \cos t}{\sin t}$

**8.** $\dfrac{\sin^2 t}{\cos^2 t} + 1 = \dfrac{1}{\cos^2 t}$

**9.** $\dfrac{\cos^2(\pi + t)}{\sin^2(\pi + t)} - 1 = \dfrac{1}{\sin^2 t}$

**10.** $\tan x + \cot x = \sec x \csc x$

**11.** $(\sin x + \cos x)^2 - \sin 2x = 1$

In Exercises 12–16, prove the given identity.

12. $\dfrac{\sec x + 1}{\tan x} = \dfrac{\tan x}{\sec x - 1}$

13. $\dfrac{\cos^4 x - \sin^4 x}{1 - \tan^4 x} = \cos^4 x$

14. $\dfrac{1 + \tan^2 x}{\tan^2 x} = \csc^2 x$

15. $\sec x - \cos x = \sin x \tan x$

16. $\tan^2 x - \sec^2 x = \cot^2 x - \csc^2 x$

17. $\sqrt{\dfrac{1 - \cos^2 x}{1 - \sin^2 x}} = ?$

a. $|\tan x|$

b. $|\cot x|$

c. $\sqrt{\dfrac{1 - \sin^2 x}{1 - \cos^2 x}}$

d. $\sec x$

e. undefined

18. $\dfrac{1}{(\csc x)(\sec^2 x)} = ?$

a. $\dfrac{1}{(\sin x)(\cos^2 x)}$

b. $\sin x - \sin^3 x$

c. $\dfrac{1}{(\sin x)(1 + \tan^2 x)}$

d. $\sin x - \dfrac{1}{1 + \tan^2 x}$

e. $1 + \tan^3 x$

Section 9.2

In Exercises 19–20, prove the given identity.

19. $\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y$

20. $\dfrac{\cos(x - y)}{\cos x \cos y} = 1 + \tan x \tan y$

21. Evaluate the following in exact form, where the angles $\alpha$ and $\beta$ satisfy the conditions:

$$\sin \alpha = \frac{4}{5} \text{ for } \frac{\pi}{2} < \alpha < \pi \qquad \tan \beta = \frac{7}{24} \text{ for } \pi < \beta < \frac{3\pi}{2},$$

a. $\sin(\beta + \alpha)$ b. $\tan(\beta - \alpha)$ c. $\cos(\alpha - \beta)$

22. If $\tan x = \frac{4}{3}$ and $\pi < x < \frac{3\pi}{2}$, and $\cot y = -\frac{5}{12}$ with $\frac{3\pi}{2} < y < 2\pi$, find $\sin(x - y)$.

23. If $\sin x = -\frac{12}{13}$ with $\pi < x < \frac{3\pi}{2}$, and $\sec y = \frac{13}{12}$ with $\frac{3\pi}{2} < y < 2\pi$, find $\cos(x + y)$.

24. If $\sin x = \frac{1}{4}$ and $0 < x < \frac{\pi}{2}$, then $\sin\left(\frac{\pi}{3} + x\right) = ?$

25. If $\sin x = -\frac{2}{5}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\cos\left(\frac{\pi}{4} + x\right) = ?$

26. Find the exact value of $\sin \frac{5\pi}{12}$.

27. Express $\sec(x - \pi)$ in terms of $\sin x$ and $\cos x$.

**Section 9.2.A**

28. Find the angle of inclination of the straight line through the points $(2, 6)$ and $(-2, 2)$.

29. Find one of the angles between the line $L$ through the points $(-3, 2)$ and $(5, 1)$ and the line $M$, which has slope 2.

**Section 9.3**

30. Evaluate the following in exact form, where the angles $\alpha$ and $\beta$ satisfy the conditions:

$$\sin\alpha = \frac{44}{125} \text{ for } \frac{\pi}{2} < \alpha < \pi \qquad \tan\beta = -\frac{15}{112} \text{ for } \frac{3\pi}{2} < \beta < 2\pi$$

**a.** $\sin\frac{\beta}{2}$ **b.** $\cos 2\alpha$

**c.** $\tan 2\alpha$ **d.** $\cos(\alpha - \beta) + \cos(\alpha + \beta)$

**In Exercises 31–34, prove the given identity.**

31. $\dfrac{1 - \cos 2x}{\tan x} = \sin 2x$

32. $\dfrac{\tan x - \sin x}{2\tan x} = \sin^2\dfrac{x}{2}$

33. $2\cos x - 2\cos^3 x = \sin x \sin 2x$

34. $\sin 2x = \dfrac{1}{\tan x + \cot 2x}$

35. If $\tan x = \frac{5}{12}$ and $\sin x > 0$, find $\sin 2x$.

36. If $\cos x = \frac{15}{17}$ and $0 < x < \frac{\pi}{2}$, find $\sin\frac{x}{2}$.

37. If $\sin x = 0$, is it true that $\sin 2x = 0$? Justify your answer.

38. If $\cos x = 0$, is it true that $\cos 2x = 0$? Justify your answer.

39. Show $\sqrt{2 + \sqrt{3}} = \dfrac{\sqrt{2} + \sqrt{6}}{2}$ by computing $\cos\frac{\pi}{12}$ in two ways, using the half-angle identity and the subtraction identity for cosine.

40. True or false: $2\sin x = \sin 2x$. Justify your answer.

41. If $\sin x = 0.6$ and $0 < x < \frac{\pi}{2}$, find $\sin 2x$.

42. If $\sin x = 0.6$ and $0 < x < \frac{\pi}{2}$, find $\sin\frac{x}{2}$.

**Section 9.4**

**Solve the equation. Find exact solutions when possible and approximate ones otherwise.**

43. $5\tan x = 2\sin 2x$

44. $\cos 2x = \cos x$

45. $2\cos x + \sin x = 0$

46. $\sin 2x + \cos x = 0$

**75.** $2\left(\sin^2\frac{1}{2}\alpha - \sin^2\frac{1}{2}\theta\right)$

$= 2\left(\frac{1-\cos 2\left(\frac{1}{2}\alpha\right)}{2} - \frac{1-\cos 2\left(\frac{1}{2}\theta\right)}{2}\right)$

$= 2\left(\frac{1-\cos\alpha - 1 + \cos\theta}{2}\right)$

$= \cos\theta - \cos\alpha$

### Section 9.4, page 608

**1.** no solution

**3.** $x = 0, \pi, 2\pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

**5.** $x = \frac{\pi}{2}, \frac{3\pi}{2}$

**7.** $x = 0, 2\pi$

**9.** $x = \frac{\pi}{2}, \frac{3\pi}{2}$

**11.** $x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

**13.** $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

**15.** $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

**17.** $x = \frac{\pi}{3}, \frac{5\pi}{3}$

**19.** $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

**21.** $x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$

**23.** $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

**25.** $x = \frac{\pi}{3}, \frac{5\pi}{3}$

**27.** $x = \frac{5\pi}{12}, \frac{13\pi}{12}$

**29.** $x = -\pi, 0, \pi$

**31.** $x = \frac{\pi}{3}, -\pi, \pi$

**33.** $x = \frac{2\pi}{5}, \frac{6\pi}{5}, 2\pi, 0, \frac{4\pi}{3}$

**35.** $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, 0, \pi, 2\pi$

**37. a.** $f(x) = 2\sin\left(x + \frac{\pi}{3}\right)$

**b.** 2

**c.** $x = \frac{\pi}{6}$

**39. a.** $f(x) = \sqrt{2}\sin\left(x + \frac{7\pi}{4}\right)$

**b.** $f(x) = \sqrt{2}$

**c.** $x = \frac{3\pi}{4}$

### Chapter 9 Review, page 611

**1.** $\frac{1}{3} + \cot t$

**3.** $\sin^4 x$

**5.** $\sin^4 t - \cos^4 t = (\sin^2 t - \cos^2 t)(\sin^2 t + \cos^2 t) = [\sin^2 t - (1 - \sin^2 t)](1) = 2\sin^2 t - 1$

**7.** $\frac{\sin t}{1-\cos t} = \frac{\sin t}{1-\cos t}\cdot\frac{(1+\cos t)}{(1+\cos t)}$

$= \frac{\sin t(1+\cos t)}{1-\cos^2 t} = \frac{\sin t(1+\cos t)}{\sin^2 t} = \frac{1+\cos t}{\sin t}$

**9.** Not an identity

**11.** $(\sin x + \cos x)^2 - \sin 2x$

$= \sin^2 x + 2\sin x\cos x + \cos^2 x - 2\sin x\cos x$

$= \sin^2 x + \cos^2 x = 1$

**13.** $\frac{\cos^4 x - \sin^4 x}{1 - \tan^4 x} = \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{(1-\tan^2 x)(1+\tan^2 x)}$

$= \frac{\cos^2 x - \sin^2 x}{(1-\tan^2 x)\sec^2 x} = \frac{(\cos^2 x - \sin^2 x)}{\left(1 - \frac{\sin^2 x}{\cos^2 x}\right)\frac{1}{\cos^2 x}}$

$= \frac{(\cos^2 x - \sin^2 x)\cos^4 x}{\cos^2 x - \sin^2 x} = \cos^4 x$

**15.** $\sec x - \cos x = \frac{1}{\cos x} - \cos x$

$= \frac{1-\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$

$= \sin x\frac{\sin x}{\cos x} = \sin x\tan x$

**17.** (a)

**19.** $\cos(x+y)\cos(x-y)$

$= [\cos x\cos y - \sin x\sin y][\cos x\cos y + \sin x\sin y]$

$= \cos^2 x\cos^2 y - \sin^2 x\sin^2 y$

$= \cos^2 x(1 - \sin^2 y) - (1 - \cos^2 x)\sin^2 y$

$= \cos^2 x - \cos^2 x\sin^2 y - \sin^2 y + \cos^2 x\sin^2 y$

$= \cos^2 x - \sin^2 y$

**21. a.** $-\frac{3}{5}$ **b.** $\frac{117}{44}$ **c.** $\frac{44}{125}$

**23.** $-\frac{120}{169}$

**25.** $\frac{\sqrt{42} + 2\sqrt{2}}{10}$

**27.** $-\frac{1}{\cos x}$

**29.** $\approx$ 1.23 radians

**31.** $\frac{1-\cos 2x}{\tan x} = \frac{1-(1-2\sin^2 x)}{\tan x} = \frac{2\sin^2 x}{\tan x}$

$= \frac{2\sin^2 x}{\frac{\sin x}{\cos x}} = \frac{2\sin^2 x\cos x}{\sin x}$

$= 2\sin x\cos x = \sin 2x$

**33.** $2\cos x - 2\cos^3 x = 2\cos x(1 - \cos^2 x)$

$= 2\cos x(\sin^2 x) = \sin x(2\sin x\cos x)$

**35.** $\frac{120}{169}$

**37.** Yes. $\sin 2x = 2\sin x\cos x = 2(0)\cos x = 0$

**39.** $\cos\left(\frac{\pi}{12}\right) = \cos\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right]$

$= \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}; \cos\left(\frac{\pi}{12}\right) =$

$$\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}$$
$$=\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)+\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)=\frac{\sqrt{6}+\sqrt{2}}{4}. \text{ So,}$$
$$\frac{\sqrt{2+\sqrt{3}}}{2}=\frac{\sqrt{6}+\sqrt{2}}{4} \text{ or } \sqrt{2+\sqrt{3}}=\frac{\sqrt{2}+\sqrt{6}}{2}$$

**41.** 0.96 **43.** $x = k\pi$ **45.** $x \approx 2.0344 + k\pi$

### Chapter 9 **Can Do Calculus**, page 615

**1. a.** 1; sin $x$ is changing at a rate of 1 unit per increase in $x$ when $x = 0$.

**b.** $\frac{\sqrt{2}}{2}$; sin $x$ is increasing approximately 0.7071 per unit increase in $x$ when $x = \frac{\pi}{4}$.

**c.** 0; sin $x$ is not changing per unit increase in $x$ when $x = \frac{\pi}{2}$

**d.** $\frac{\sqrt{3}}{2}$; sin $x$ is increasing approximately 0.8660 per unit increase in $x$ when $x = \frac{\pi}{6}$.

**2.** $-\sin x$

**3. a.** 0; cos $x$ is not changing per unit increase in $x$ when $x = 0$.

**b.** $-\frac{\sqrt{2}}{2}$; cos $x$ is decreasing approximately 0.7071 units per unit increase in $x$ when $x = \frac{\pi}{4}$

**c.** $-1$; cos $x$ is decreasing 1 unit per unit increase in $x$ when $x = \frac{\pi}{2}$.

**d.** $-\frac{1}{2}$; cos $x$ is decreasing $\frac{1}{2}$ unit per unit increase in $x$ when $x = \frac{\pi}{6}$.

## Chapter 10

### Section 10.1, page 622

**1.** $a = 4.2$, $\angle B = 125.0°$, $\angle C = 35.0°$

**3.** $c = 13.9$, $\angle A = 22.5°$, $\angle B = 39.5°$

**5.** $a = 24.4$, $\angle B = 18.4°$, $\angle C = 21.6°$

**7.** $c = 21.5$, $\angle A = 33.5°$, $\angle B = 67.9°$

**9.** $\angle A = 120°$, $\angle B = 21.8°$, $\angle C = 38.2°$

**11.** $\angle A = 24.1°$, $\angle B = 30.8°$, $\angle C = 125.1°$

**13.** $\angle A = 38.8°$, $\angle B = 34.5°$, $\angle C = 106.7°$

**15.** $\angle A = 34.1°$, $\angle B = 50.5°$, $\angle C = 95.4°$

**17.** 54.2° at vertex (0, 0); 48.4° at vertex (5, −2); 77.4° at vertex (1, −4)

**19.** 334.9 km **21.** 63.7 ft

**23.** 84.9° **25.** 8.4 km

**27.** 231.9 ft **29.** 154.5 ft

**31.** 4.7 cm and 9.0 cm **33.** 33.44°

**35.** 978.7 mi

**37.** $AB = 24.27$, $AC = 21.23$, $BC = 19.5$, $\angle A = 50.2°$, $\angle B = 56.8°$, $\angle C = 73.0°$

**39.** 16.99 m

### Section 10.2, page 634

**1.** $\angle C = 110°$, $b = 2.5$, $c = 6.3$

**3.** $\angle B = 14°$, $b = 2.2$, $c = 6.8$

**5.** $\angle A = 88°$, $a = 17.3$, $c = 12.8$

**7.** $\angle C = 41.5°$, $b = 9.7$, $c = 10.9$

**9.** 7.3 **11.** 32.5 **13.** 82.3 **15.** 31.4

**17.** No solution

**19.** $\angle A_1 = 55.2°$, $\angle C_1 = 104.8°$, $c_1 = 14.1$; $\angle A_2 = 124.8°$, $\angle C_2 = 35.2°$, $c_2 = 8.4$

**21.** No solution

**23.** $\angle B_1 = 65.8°$, $\angle A_1 = 58.2°$, $a_1 = 10.3$; $\angle B_2 = 114.2°$, $\angle A_2 = 9.8°$, $a_2 = 2.1$

**25.** $\angle C = 72°$, $b = 14.7$, $c = 15.2$

**27.** $a = 9.8$, $\angle B = 23.3°$, $\angle C = 81.7°$

**29.** $\angle A = 18.6°$, $\angle B = 39.6°$, $\angle C = 121.9°$

**31.** $c = 13.9$, $\angle A = 60.1°$, $\angle B = 72.9°$

**33.** $\angle C = 39.8°$, $\angle A = 77.7°$, $a = 18.9$

**35.** No solution **37.** 6.5 **39.** About 7691

**41.** 135.5 m **43.** 5.4° **45.** 5 ft

**47.** 5.3° **49.** 30.1 km **51.** About 9642 ft

**53. a.** Use the Law of Cosines in triangle $ABD$ to find $\angle ABD$; then $\angle EBA$ is $180° - \angle ABD$. (Why?) Use the Law of Cosines in triangle $ABC$ to find $\angle CAB$; then $\angle EAB$ is $180° - \angle CAB$. You now have two of the angles in triangle $EAB$ and can easily find the third. Use these angles, side $AB$, and the Law of Sines to find $AE$.

**b.** 94.24 ft

**55.** 13.36 m **57.** 5.8 gal **59.** 11.18 sq units

**61.** No such triangle exists because the sum of the lengths of any two sides of a triangle must be greater than the length of the third side, which is not the case here.