Precalculus: Unit 1 Test Name: _____

PART A – CALCULATOR INACTIVE

Questions # 1-12 are MULTIPLE CHOICE. Write the CAPITAL LETTER of the correct response on the answer sheet. Each correct answer is worth 2 points. (24)

1. The symmetry of $f(x) = \sqrt{x}$ is

   A) even B) odd C) neither odd nor even

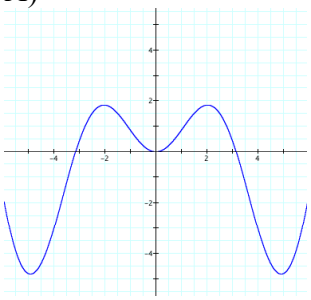
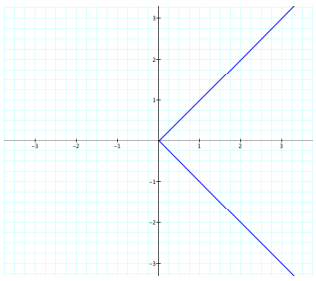
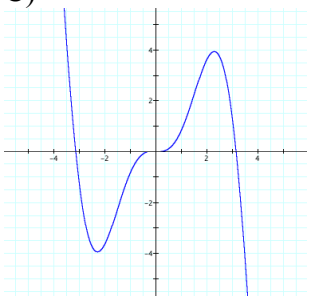
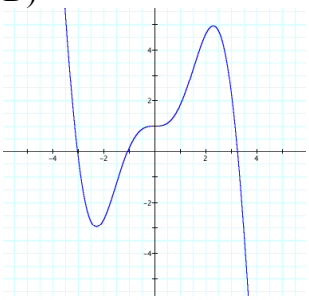
2. The symmetry of $f(x) = |x| + x^2$ is

   A) even B) odd C) neither odd nor even

3. The horizontal asymptotes of $f(x) = \dfrac{x}{x^2+1}$ is/are:

   A) x = 0 B) y = 1 C) y = -1 D) y = 0 E) there are none

4. The function with odd symmetry is

   A)  B)  C)  D) 

5. The inverse of the function $f(x) = \dfrac{2x^3 - 1}{4}$ is $f^{-1}(x) =$

   A) $\dfrac{\sqrt[3]{4x+1}}{2}$ B) $\sqrt[3]{\dfrac{4x+1}{2}}$ C) $4\left(\dfrac{\sqrt[3]{x}}{2}+1\right)$ D) $\sqrt[3]{2x+2}$

6. Which of the following is an unbounded function?

   A) $y = e^x$ B) $y = \text{int}(x)$ C) $y = \sin x$ D) $y = \dfrac{1}{1+e^{-x}}$

7. The "end behaviour" of $f(x) = 2 + e^{-x}$ as x→ +∞ is:

   A) x → +2 B) y → 0 C) y → +2 D) y → +∞ E) cannot be determined

8. The tables of values below represent values for the function $y = f(x)$.

x	y1
-1.00	-.333
0.000	-.500
1.000	-1.00
2.000	undef
3.000	1.000

x	y1
-.020	-.495
-.010	-.498
0.000	-.500
.010	-.503
.020	-.505

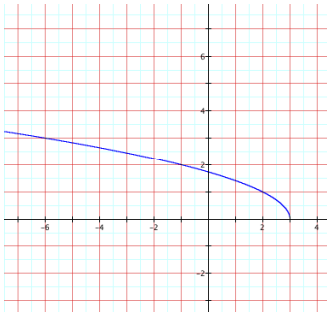
x	y1
.980	-.980
.990	-.990
1.000	-1.00
1.010	-1.01
1.020	-1.02

x	y1
1.980	-50.0
1.990	-100.
2.000	undef
2.010	100.0
2.020	50.00

According to the tables, what is the equation of the vertical asymptote?

A) $x = 0$ B) $x = 1$ C) $x = 2$ D) $x = 3$

9. The domain of the inverse of the graph given at the right is



A) x ε R, x ≤ 3
B) y ε R, y ≥ 0
C) x ε R, x ≥ 0
D) y ε R, y ≤ 3

10. What are the transformations on $y = f(x)$ given by $y = -f(x+3)$?

A) reflection through the x axis and horizontal translation of right 3
B) reflection through the y axis and horizontal translation of right 3
C) reflection through the x axis and horizontal translation of left 3
D) reflection through the y axis and horizontal translation of left 3

11. The point $A(3,2)$ lies on the graph of $y = g(x)$. To what point A' does A get mapped to by the transformed function $y = 5g(3x)$?

A) $A'(9,10)$ B) $A'(1,10)$ C) $A'\left(9,\frac{2}{5}\right)$ D) $A'\left(1,\frac{2}{5}\right)$

12. The point $A'(3,2)$ lies on the graph of the transformed function $y = 5g(3x)$. What point A was on the original (basic) function $y = g(x)$?

A) $A(9,10)$ B) $A(1,10)$ C) $A\left(9,\frac{2}{5}\right)$ D) $A\left(1,\frac{2}{5}\right)$

PART B – CALCULATOR ACTIVE

Show all work for the following questions on a separate sheet of paper. Provide complete algebraic solutions. Be clear and concise in your communication and neat in your presentation. You may verify your answer using your calculator, but full credit will only be awarded for the correct answer and proper format

13. Determine the equation of the composition $g \circ f(x)$ (i.e. $g\left[f\left(x\right)\right]$) given $f\left(x\right) = \sqrt{4x-1}$ and $g\left(x\right) = 2x^2 - 3$. State the domain of the composite function. Explain this domain. (4)

14. Given the functions from Q1 for f(x) and g(x), determine the equation and then the domain for:

a) f(x) + g(x) b) $f(x) \times g(x)$ c) $\frac{f(x)}{g(x)}$ (4)

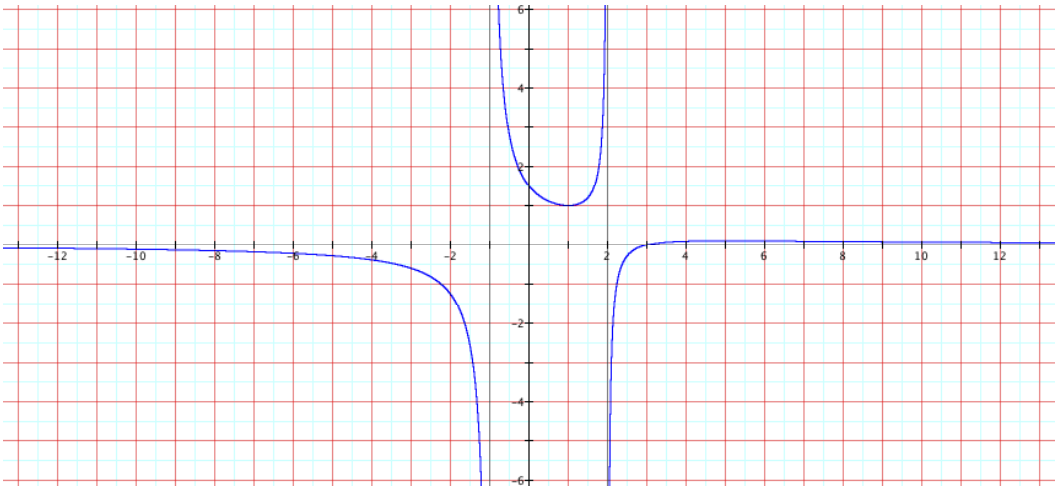
15. Determine the symmetry (odd, even, neither) of $f\left(x\right) = \frac{\left|x\right| + 3x^4}{x + \sin x}$ using proper algebraic format. You may verify your answer using your calculator, but full credit will only be awarded for the correct answer and proper format. (4)

16. Find the domain, range for the inverse of $g(x) = \sqrt{4-x} - 2$. Show supporting algebraic. (4)

17. Given the rational function $f(x) = \frac{x+1}{3x-2}$, determine the equation of the inverse of the function and its domain and range. (4)

Show all work for the following questions on this sheet of paper. Be concise in your communication and neat in your presentation.

18. The graph of the function $f(x) = \dfrac{x-3}{x^2-x-2}$ is given below.



b) State the type (VA, HA) and equations for all asymptotes of this function. Sketch these asymptotes on the graph as dotted lines (use a ruler). (2)

c) Is the function bounded, bounded above, bounded below or unbounded? (1)

d) Is $f(x)$ continuous for x ε R? Explain. (1)

e) State the coordinates of any local maxima or local minima. (1)

f) State the interval(s) on which this function is increasing. (1)

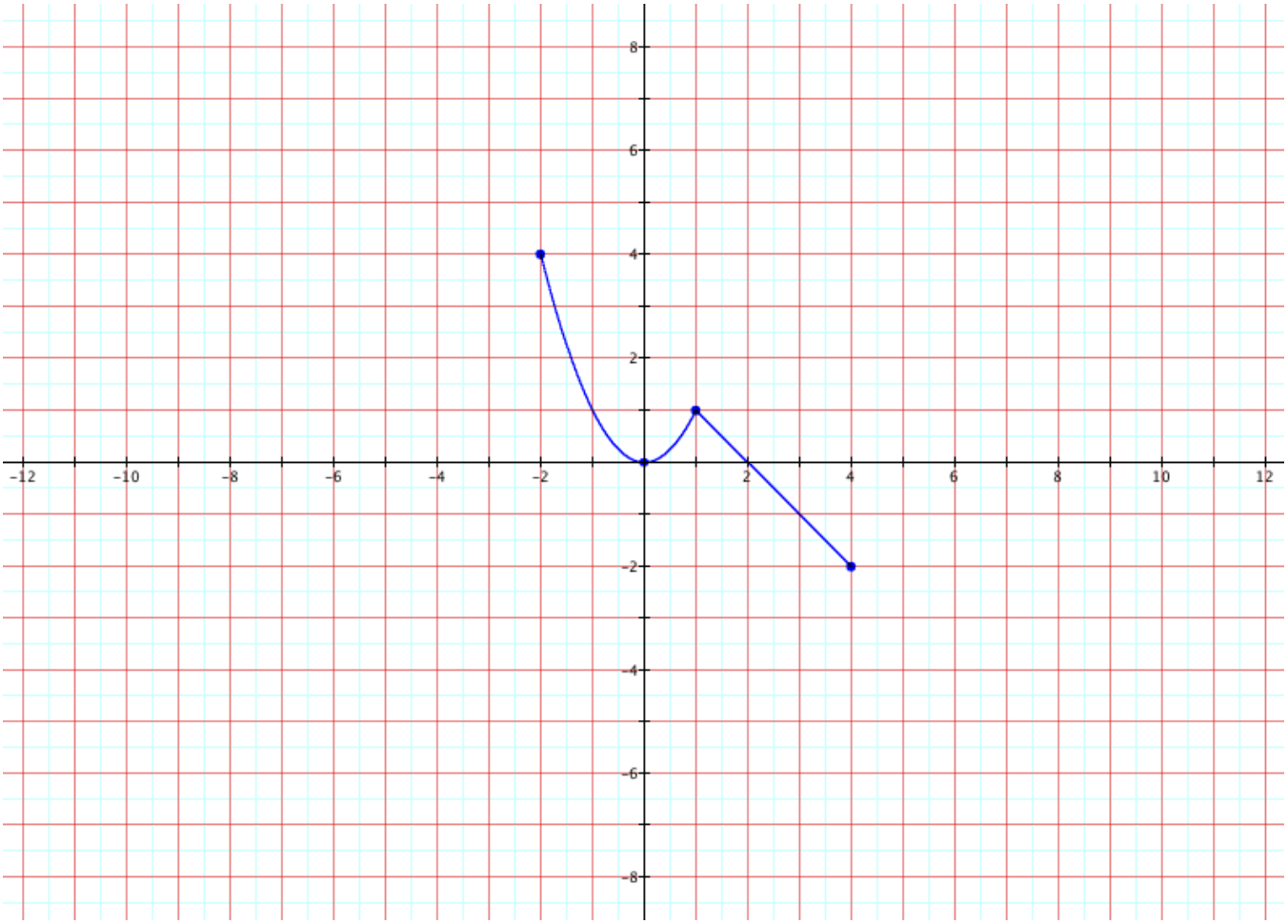
g) State the range of the graph of $f^{-1}(x)$. (1)

h) Is $f(x)$ one-to-one? Why or why not? (1)

Show all work for the following questions on this sheet of paper. Be clear and concise in your communication and neat in your presentation.

19. Given the graph of the function $y = f(x)$. List the transformations indicated by each transformed equation and sketch the transformed graph of $y = -2f\left(1 - \frac{1}{2}x\right) + 1$ on the grid provided.

(10)



PART C – CALCULATOR INACTIVE

Show all work for the following questions on a separate sheet of paper. Provide complete algebraic solutions. Be clear and concise in your communication and neat in your presentation.

20. For the parabola $f(x) = -2x^2 - 12x - 19$, (8)

a) Complete the square and express the equation in vertex form.
b) State the coordinates of the vertex.
c) Find the equation of $f^{-1}(x)$ (the inverse)
d) Restrict the domain of f(x) such that $f^{-1}(x)$ is a function. Explain why you restrict the domain.

21. Determine the domain of $y = \sqrt[4]{x^2 - 3x - 10}$ and state it in correct notation. (5)

22. In this question, you will work with a piecewise function made from our basic functions and some simple transformations of our basic functions. So let $f(x) = \begin{cases} x & -5 < x < -3 \\ 6 - x^2 & ;-3 \le x < 2 \\ \dfrac{1}{x-2} & 2 \le x \le 4 \end{cases}$. (14)

a. Graph f(x). You may find a table of values helpful (although more time consuming than simply knowing your basic functions!)
b. On the domain of xε[-5,4], find and classify any discontinuities on f(x).
c. State the range of f(x).
d. Is the function one to one? Explain your reasoning.
e. Evaluate $f^{-1}(5)$.

PART D – Extra Credit Work – Applications of Functions

1. The following data comes from an experiment performed by Mr. S's class last year, where we measured the temperature of a hot cup of coffee as it cooled over the course of a class. The room temperature when we carried out our experiment was 26.8^{o}.

Time	0	10	20	30	40	50
Temp	82.5	65.5	57.7	51.1	47	43.1

2. Determine a cubic regression equation for the data.
   a. Write the regression equation and the r^2 value.
   b. Give one valid reason for selecting a cubic regression to "fit" the data.
   c. Give one valid reason for NOT selecting a cubic regression to "fit" the data.

3. Determine the domain of the function in the context of the data. Explain your answer.

4. Determine the range of the function in the context of the data. Explain your answer.

5. Here are some function terms that you should address in deciding on a reasonable function to model the data: bounded, increase & decrease, maximum and minimum points, end behaviour. Using these 4 function concepts, explain how each these function concepts would (or would not) apply to the data, specifically in helping you decide what regression would seem most reasonable given the context of the data (changing temperature of my coffee) (For example: Would you expect the function to be bounded? If so why? If not, why not? Would you expect the function to be increasing or decreasing? Over what domain? Would you expect the function to have maximum or minimum points? What would be the predicted end behaviour for the function that best models the data?)

6. Now that you have considered some functional analysis features, decide which of our 12 basic functions would you expect to be best suited for modeling this data? Explain why (refer to some of your ideas from Q5).

7. If the data were best modeled by an exponential model ($y = e^x$), determine reasonable transformations of e^x ($y = ae^{bx} + d$) such that you can predict the equation of the exponential function that best fits the data points. Explain the process by which you arrived at your final answer (in the form of $y = ae^{bx} + d$.