PreCalculus – Trigonometry Assignment #2

PART A – Solving trig equations. For Questions 1,2,3 & 4, state

(i) the reference angle, if possible

(ii) the quadrants in which the angle lies, if possible

(iii) the angle in radians for one revolution: $0 \le \theta \le 2\pi$

(iv) all coterminal angles, using an expression involving $n \in I : \theta \in R$

1. Solve the following one-step trig equations:

   a) $\tan(\theta) = -\frac{\sqrt{3}}{3}$

   b) $\csc(\theta) = -1$

   c) $\cot(\theta) = \textit{undefined}$

   d) $\sec(\theta) = 0$

2. Solve the following trig equations by collecting like terms:

   a) $\sin(\theta) = -\sin(\theta) + \sqrt{3}$

   b) $2\cos\left(\frac{\theta}{2}\right) - 1 = \cos\left(\frac{\theta}{2}\right) - 2$

3. Solve for θ in the following equations (use substitution if needed ➔ i.e. let a = trigθ).

   a) $\sin^2(\theta) = 1$

   b) $2\cos^2(\theta) - 1 = 0$

   c) $\sqrt{3}\cot^2(\theta) = -\cot(\theta)$

   d) $2\sin^2(\theta) + 3\sin(\theta) + 1 = 0$

4. Factor the common term first and then solve for θ :

   a) $\sin(\theta)\cos(\theta) + \cos(\theta) = 0$

   b) $\cot(\theta)\csc(\theta) + 2\cot(\theta) = 0$

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5. Solve for θ both for one revolution and all coterminal angles to θ .

   a) $\sin\left(\theta - \frac{\pi}{4}\right) = -1$

   b) $2\cos\left(\theta + \frac{2\pi}{3}\right) + 1 = 0$

   c) $\tan\left(\frac{2\theta}{3}\right) - 1 = 0$

   d) $\cos^2(3\theta) = 1$

PART B – NO CALCULATOR – USE IDENTITIES to simplify/evaluate and leave answers in EXACT FORM

6. Given that $\tan(8\theta) = -\frac{2}{5}$ and $\frac{\pi}{2} < 8\theta < \pi$, evaluate the following exactly:

   a) $\cot(8\theta)$ c) $\sin(4\theta)$ e) $\tan(4\theta)$ g) $\sec(16\theta)$

   b) $\sec(8\theta)$ d) $\csc(4\theta)$ f) $\cot(-4\theta)$ h) $\tan(-16\theta)$

7. Evaluate the following exactly using sum/difference identities and special triangles:

   a) $\cos(75°)$ c) $\sin(75°)$ e) $\tan(\alpha+\beta)$ if $\tan(\alpha) = \frac{2}{3}$ and $\tan(\beta) = 2$

   b) $\sec\left(-\frac{\pi}{12}\right)$ d) $\csc\left(\frac{\pi}{12}\right)$ f) $\cot(\beta-\alpha)$ if $\cos(\alpha) = \frac{1}{3}$ and $\csc(\beta) = \frac{5}{3}$

8. Show that the value of $\cos(195) - \cos(105) = -\frac{\sqrt{2}}{2}$ using:

   a) a sum or difference identity and special triangles.

   b) a double-angle identity, a half-angle identity and special triangles.

9. Prove that $\sin(x-\pi) = -\sin(x)$ using: a) an algebraic approach b) a graphical approach

10. Prove that the solutions to $\cos^2(\theta) - \sin^2(\theta) = 0$ for $0 \le \theta \le 2\pi$ are equivalent to the zeroes of $y = \cos(2\theta)$ by solving the equation $\cos^2(\theta) - \sin^2(\theta) = 0$ algebraically and by graphing $y = \cos(2\theta)$. State an identity as a conclusion to your findings for $\theta \in R$.

11. Prove that the solutions to $\sin(x)\cos(x) = 0$ for $x \in R$ are equivalent to the zeroes of $y = \frac{1}{2}\sin(2x)$ by solving the equation algebraically and by graphing $y = \frac{1}{2}\sin(2x)$.

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12. CALCULATOR ACTIVE: Graph the function $y = 3\sin(x)\cos^2(x) - \sin^3(x)$ for $0 \le x \le 2\pi$; use xscl = $\frac{\pi}{6}$.

    a) What is the period of this function? [Hint: How many cycles are there in 2π rad?]

    b) Determine a simpler equation for this function in terms of sin(x) and state an identity for $3\sin(x)\cos^2(x) - \sin^3(x)$.

    c) Prove the resulting identity algebraically.