2 Chapter Summary

What did you learn?

	Review Exercises
Section 2.1	
☐ Analyze graphs of quadratic functions.	1, 2
☐ Write quadratic functions in standard form and use the results to sketch graphs of functions.	3–10
☐ Find minimum and maximum values of functions in real-life applications.	11, 12
Section 2.2	
☐ Use transformations to sketch graphs of polynomial functions.	13–16
☐ Use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions.	17–22
☐ Find and use zeros of polynomial functions as sketching aids.	23–28
☐ Use the Intermediate Value Theorem to help locate zeros of polynomial functions.	29–32
Section 2.3	
☐ Use long division to divide polynomials by other polynomials.	35–42
☐ Use synthetic division to divide polynomials by binomials of the form $(x - k)$.	43–48
☐ Use the Remainder and Factor Theorems.	49–54
☐ Use the Rational Zero Test to determine possible rational zeros of polynomial functions.	55–60
☐ Use Descartes's Rule of Signs and the Upper and Lower Bound Rules to find zeros of polynomials.	61–64
Section 2.4	
☐ Use the imaginary unit i to write complex numbers.	65–68
☐ Add, subtract, and multiply complex numbers.	69–76
☐ Use complex conjugates to write the quotient of two complex numbers in standard form.	77–80
☐ Plot complex numbers in the complex plane.	81–86
Section 2.5	
☐ Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function.	87–90
☐ Find all zeros of polynomial functions, including complex zeros.	91–100
☐ Find conjugate pairs of complex zeros.	101–104
☐ Find zeros of polynomials by factoring.	105–108
Section 2.6	
☐ Find the domains of rational functions.	109–120
☐ Find horizontal and vertical asymptotes of graphs of rational functions.	109–120
☐ Use rational functions to model and solve real-life problems.	121, 122
Section 2.7	
☐ Analyze and sketch graphs of rational functions.	123–130
☐ Sketch graphs of rational functions that have slant asymptotes.	131–134
☐ Use rational functions to model and solve real-life problems.	135, 136
Section 2.8	
☐ Classify scatter plots.	137–140
☐ Use scatter plots and a graphing utility to find quadratic models for data.	141
☐ Choose a model that best fits a set of data.	141

2 Review Exercises

2.1 In Exercises 1 and 2, use a graphing utility to graph each function in the same viewing window. Describe how the graph of each function is related to the graph of $y = x^2$.

1. (a) $y = 2x^2$ (b) $y = -2x^2$
   (c) $y = x^2 + 2$ (d) $y = (x + 5)^2$
2. (a) $y = x^2 - 4$ (b) $y = 4 - x^2$
   (c) $y = (x - 1)^2$ (d) $y = \frac{1}{2}x^2 - 1$

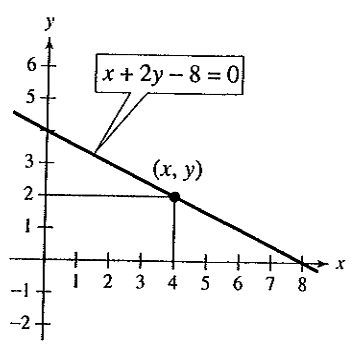
In Exercises 3–6, sketch the graph of the quadratic function. Identify the vertex and the intercept(s).

3. $f(x) = \left(x + \frac{3}{2}\right)^2 + 1$
4. $f(x) = (x - 4)^2 - 4$
5. $f(x) = \frac{1}{3}(x^2 + 5x - 4)$
6. $f(x) = 3x^2 - 12x + 11$

In Exercises 7–10, write the standard form of the quadratic function that has the indicated vertex and whose graph passes through the given point. Verify your result with a graphing utility.

7. Vertex: $(1, -4)$; Point: $(2, -3)$
8. Vertex: $(2, 3)$; Point: $(0, 2)$
9. Vertex: $(-2, -2)$; Point: $(-1, 0)$
10. Vertex: $\left(-\frac{1}{4}, \frac{3}{2}\right)$; Point: $(-2, 0)$

11. ***Numerical, Graphical, and Analytical Analysis*** A rectangle is inscribed in the region bounded by the x-axis, the y-axis, and the graph of $x + 2y - 8 = 0$, as shown in the figure.



(a) Write the area A as a function of x. Determine the domain of the function in the context of the problem.

(b) Use the *table* feature of a graphing utility to create a table showing possible values of x and the corresponding areas of the rectangle. Use the table to estimate the dimensions that will produce a maximum area.

(c) Use a graphing utility to graph the area function. Use the graph to approximate the dimensions that will produce a maximum area.

(d) Write the area function in standard form to find algebraically the dimensions that will produce a maximum area.

(e) Compare your results in parts (b), (c), and (d).

12. ***Cost*** A textile manufacturer has daily production costs of $C = 10{,}000 - 110x + 0.45x^2$, where C is the total cost (in dollars) and x is the number of units produced. Use the *table* feature of a graphing utility to determine how many units should be produced each day to yield a minimum cost.

2.2 In Exercises 13–16, sketch the graph of $y = x^n$ and each specified transformation.

13. $y = x^4$
    (a) $f(x) = (x + 5)^4$ (b) $f(x) = x^4 - 4$
    (c) $f(x) = 3 + x^4$ (d) $f(x) = \frac{1}{4}(x - 2)^4$
14. $y = x^5$
    (a) $f(x) = (x + 4)^5$ (b) $f(x) = 6 + x^5$
    (c) $f(x) = 3 - \frac{1}{2}x^5$ (d) $f(x) = 2(x + 3)^5$
15. $y = x^6$
    (a) $f(x) = x^6 - 2$ (b) $f(x) = -\frac{1}{4}x^6$
    (c) $f(x) = -\frac{1}{2}x^6 - 5$ (d) $f(x) = -(x + 7)^6 - 5$
16. $y = x^3$
    (a) $f(x) = -x^3 + 4$ (b) $f(x) = (x + 2)^3 - 1$
    (c) $f(x) = -\frac{1}{3}x^3 + 1$ (d) $f(x) = -(x + 8)^3$

Graphical Analysis **In Exercises 17 and 18, use a graphing utility to graph the functions f and g in the same viewing window. Zoom out far enough so that the right-hand and left-hand behaviors of f and g appear identical.**

17. $f(x) = \frac{1}{2}x^3 - 2x + 1,\quad g(x) = \frac{1}{2}x^3$
18. $f(x) = -x^4 + 2x^3,\quad g(x) = -x^4$

In Exercises 19–22, use the Leading Coefficient Test to determine the right-hand and left-hand behavior of the graph of the polynomial function.

19. $f(x) = -x^2 + 6x + 9$
20. $f(x) = \frac{1}{2}x^3 + 2x$
21. $g(x) = \frac{3}{4}(x^4 + 3x^2 + 2)$
22. $h(x) = -x^5 - 7x^2 + 10x$

In Exercises 23–28, (a) use a graphing utility to graph the function, (b) use the graph to approximate any zeros, and (c) find the zeros algebraically.

23. $g(x) = x^4 - x^3 - 2x^2$
24. $h(x) = -2x^3 - x^2 + x$
25. $f(t) = t^3 - 3t$
26. $f(x) = -(x + 6)^3 - 8$
27. $f(x) = x(x + 3)^2$
28. $f(t) = t^4 - 4t^2$

In Exercises 29–32, (a) use the Intermediate Value Theorem and a graphing utility to find intervals of length 1 in which the polynomial function is guaranteed to have a zero, (b) use the *zero* or *root* feature of a graphing utility to approximate the zeros of the function, and (c) verify your results in part (a) by using the *table* feature of a graphing utility.

29. $f(x) = x^3 + 2x^2 - x - 1$
30. $f(x) = 0.24x^3 - 2.6x - 1.4$
31. $f(x) = x^4 - 6x^2 - 4$
32. $f(x) = 2x^4 + \frac{7}{2}x^3 - 2$

2.3 ***Graphical Analysis*** **In Exercises 33 and 34, use a graphing utility to graph the two equations in the same viewing window. Use the graphs to verify that the expressions are equivalent. Verify the results algebraically.**

33. $y_1 = \dfrac{x^2}{x - 2}, \quad y_2 = x + 2 + \dfrac{4}{x - 2}$
34. $y_1 = \dfrac{x^4 + 1}{x^2 + 2}, \quad y_2 = x^2 - 2 + \dfrac{5}{x^2 + 2}$

In Exercises 35–42, use long division to divide.

35. $\dfrac{24x^2 - x - 8}{3x - 2}$
36. $\dfrac{4x^2 + 7}{3x - 2}$
37. $\dfrac{x^4 - 3x^2 + 2}{x^2 - 1}$
38. $\dfrac{3x^4}{x^2 - 1}$
39. $(5x^3 - 13x^2 - x + 2) \div (x^2 - 3x + 1)$
40. $(x^4 + x^3 - x^2 + 2x) \div (x^2 + 2x)$
41. $\dfrac{6x^4 + 10x^3 + 13x^2 - 5x + 2}{2x^2 - 1}$
42. $\dfrac{x^4 - 3x^3 + 4x^2 - 6x + 3}{x^2 + 2}$

In Exercises 43–48, use synthetic division to divide.

43. $(0.25x^4 - 4x^3) \div (x + 2)$
44. $(0.1x^3 + 0.3x^2 - 0.5) \div (x - 5)$
45. $(6x^4 - 4x^3 - 27x^2 + 18x) \div \left(x - \frac{2}{3}\right)$
46. $(2x^3 + 2x^2 - x + 2) \div \left(x - \frac{1}{2}\right)$
47. $(3x^3 - 10x^2 + 12x - 22) \div (x - 4)$
48. $(2x^3 + 6x^2 - 14x + 9) \div (x - 1)$

In Exercises 49 and 50, use synthetic division to find each function value. Use a graphing utility to verify your results.

49. $f(x) = x^4 + 10x^3 - 24x^2 + 20x + 44$
(a) $f(-3)$ (b) $f(-1)$
50. $g(t) = 2t^5 - 5t^4 - 8t + 20$
(a) $g(-4)$ (b) $g\left(\sqrt{2}\right)$

In Exercises 51–54, (a) verify the given factor(s) of the function *f*, (b) find the remaining factors of *f*, (c) use your results to write the complete factorization of *f*, (d) list all real zeros of *f*, and (e) confirm your results by using a graphing utility to graph the function.

	Function	*Factor(s)*
51.	$f(x) = x^3 + 4x^2 - 25x - 28$	$(x - 4)$
52.	$f(x) = 2x^3 + 11x^2 - 21x - 90$	$(x + 6)$
53.	$f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$	$(x + 2)$, $(x - 3)$
54.	$f(x) = x^4 - 11x^3 + 41x^2 - 61x + 30$	$(x - 2)$, $(x - 5)$

In Exercises 55 and 56, use the Rational Zero Test to list all possible rational zeros of *f*. Use a graphing utility to verify that the zeros of *f* are contained in the list.

55. $f(x) = 4x^3 - 11x^2 + 10x - 3$
56. $f(x) = 10x^3 + 21x^2 - x - 6$

In Exercises 57–60, find all the zeros of the function.

57. $f(x) = 6x^3 - 5x^2 + 24x - 20$

58. $f(x) = x^3 - 1.3x^2 - 1.7x + 0.6$

59. $f(x) = 6x^4 - 25x^3 + 14x^2 + 27x - 18$

60. $f(x) = 5x^4 + 126x^2 + 25$

In Exercises 61 and 62, use Descartes's Rule of Signs to determine the possible numbers of positive and negative real zeros of the function.

61. $g(x) = 5x^3 + 3x^2 - 6x + 9$

62. $h(x) = -2x^5 + 4x^3 - 2x^2 + 5$

In Exercises 63 and 64, use synthetic division to verify the upper and lower bounds of the real zeros of *f*.

63. $f(x) = 4x^3 - 3x^2 + 4x - 3$
    Upper bound: $x = 1$; Lower bound: $x = -\frac{1}{4}$

64. $f(x) = 2x^3 - 5x^2 - 14x + 8$
    Upper bound: $x = 8$; Lower bound: $x = -4$

2.4 **In Exercises 65–68, write the complex number in standard form.**

65. $6 + \sqrt{-25}$
66. $-\sqrt{-12} + 3$
67. $-2i^2 + 7i$
68. $-i^2 - 4i$

In Exercises 69–76, perform the operations and write the result in standard form.

69. $(7 + 5i) + (-4 + 2i)$

70. $\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

71. $5i(13 - 8i)$
72. $(1 + 6i)(5 - 2i)$
73. $(10 - 8i)(2 - 3i)$
74. $i(6 + i)(3 - 2i)$
75. $(3 + 7i)^2 + (3 - 7i)^2$
76. $(4 - i)^2 - (4 + i)^2$

In Exercises 77–80, write the quotient in standard form.

77. $\frac{6 + i}{i}$
78. $\frac{4}{-3i}$
79. $\frac{3 + 2i}{5 + i}$
80. $\frac{1 - 7i}{2 + 3i}$

In Exercises 81–86, plot the complex number in the complex plane.

81. $2 - 5i$
82. $-1 + 4i$
83. $-6i$
84. $7i$
85. 3
86. -2

2.5 **In Exercises 87–90, find all the zeros of the function.**

87. $f(x) = 3x(x - 2)^2$

88. $f(x) = (x - 4)(x + 9)^2$

89. $f(x) = (x + 4)(x - 6)(x - 2i)(x + 2i)$

90. $g(t) = (t - 8)(t - 5)^2(t - 3 + i)(t - 3 - i)$

In Exercises 91–94, find all the zeros of the function and write the polynomial as a product of linear factors. Use a graphing utility to graph the function to verify your results graphically.

91. $f(x) = 2x^4 - 5x^3 + 10x - 12$

92. $g(x) = 3x^4 - 4x^3 + 7x^2 + 10x - 4$

93. $h(x) = x^3 - 7x^2 + 18x - 24$

94. $f(x) = 2x^3 - 5x^2 - 9x + 40$

In Exercises 95–100, (a) find all the zeros of the function, (b) write the polynomial as a product of linear factors, (c) use your factorization to determine the *x*-intercepts of the graph of the function, and (d) use a graphing utility to verify that the real zeros are the only *x*-intercepts.

95. $f(x) = x^3 - 4x^2 + 6x - 4$

96. $f(x) = x^3 - 5x^2 - 7x + 51$

97. $f(x) = x^3 + 6x^2 + 11x + 12$

98. $f(x) = 2x^3 - 9x^2 + 22x - 30$

99. $f(x) = x^4 + 34x^2 + 225$

100. $f(x) = x^4 + 10x^3 + 26x^2 + 10x + 25$

In Exercises 101–104, find a polynomial function with real coefficients that has the given zeros. (There are many correct answers.)

101. $-2, -2, -5i$
102. $4, 4, 2i$
103. $1, -4, -3 + 5i$
104. $-4, -4, 1 + \sqrt{3}i$

In Exercises 105–108, write the polynomial (a) as the product of factors that are irreducible over the *rationals*, (b) as the product of linear and quadratic factors that are irreducible over the *reals*, and (c) in completely factored form.

105. $f(x) = x^4 + 2x^2 - 8$

106. $f(x) = x^4 - x^3 - x^2 + 5x - 20$
    (*Hint:* One factor is $x^2 - 5$.)

107. $f(x) = x^4 - 2x^3 + 8x^2 - 18x - 9$

(*Hint:* One factor is $x^2 + 9$.)

108. $f(x) = x^4 - 4x^3 + 3x^2 + 8x - 16$

(*Hint:* One factor is $x^2 - x - 4$.)

2.6 In Exercises 109–120, (a) find the domain of the function and (b) identify any horizontal and vertical asymptotes.

109. $f(x) = \dfrac{x - 8}{1 - x}$ **110.** $f(x) = \dfrac{5x}{x + 12}$

111. $f(x) = \dfrac{2}{x^2 - 3x - 18}$

112. $f(x) = \dfrac{2x^2 + 3}{x^2 + x + 3}$

113. $f(x) = \dfrac{7 + x}{7 - x}$ **114.** $f(x) = \dfrac{6x}{x^2 - 1}$

115. $f(x) = \dfrac{4x^2}{2x^2 - 3}$ **116.** $f(x) = \dfrac{3x^2 - 11x - 4}{x^2 + 2}$

117. $f(x) = \dfrac{2x - 10}{x^2 - 2x - 15}$

118. $f(x) = \dfrac{x^3 - 4x^2}{x^2 + 3x + 2}$

119. $f(x) = \dfrac{x - 2}{|x| + 2}$ **120.** $f(x) = \dfrac{2x}{|2x - 1|}$

121. ***Seizure of Illegal Drugs*** The cost C, in millions of dollars, for the U.S. government to seize $p\%$ of an illegal drug as it enters the country is given by

$$C = \frac{528p}{100 - p}, \quad 0 \le p < 100.$$

(a) Find the cost of seizing 25%, 50%, and 75% of the illegal drug.

(b) Use a graphing utility to graph the function. Be sure to choose an appropriate viewing window. Explain why you chose the values you used in your viewing window.

(c) According to this model, would it be possible to seize 100% of the drug? Explain.

122. ***Wildlife*** A biology class performs an experiment comparing the quantity of food consumed by a certain kind of moth with the quantity supplied. The model for the experimental data is given by

$$y = \frac{1.568x - 0.001}{6.360x + 1}, \quad x > 0$$

where x is the quantity (in milligrams) of food supplied and y is the quantity (in milligrams) eaten (see figure). At what level of consumption will the moth become satiated?

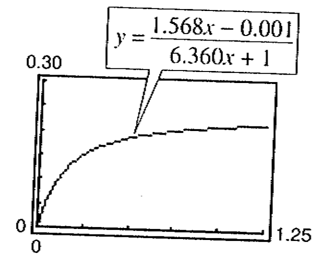


Figure for 122

2.7 In Exercises 123–130, sketch the graph of the rational function by hand. As sketching aids, check for intercepts, vertical asymptotes, and horizontal asymptotes. Use a graphing utility to verify your graph.

123. $f(x) = \dfrac{2x - 1}{x - 5}$ **124.** $f(x) = \dfrac{x - 3}{x - 2}$

125. $f(x) = \dfrac{2x}{x^2 + 4}$ **126.** $f(x) = \dfrac{2x^2}{x^2 - 4}$

127. $f(x) = \dfrac{x^2}{x^2 + 1}$ **128.** $f(x) = \dfrac{5x}{x^2 + 1}$

129. $f(x) = \dfrac{2(x^2 - 16)}{x^2 + 2x - 8}$ **130.** $f(x) = \dfrac{3x^2 - 6x}{x^2 - 4}$

In Exercises 131–134, sketch the graph of the rational function by hand. As sketching aids, check for intercepts, vertical asymptotes, horizontal asymptotes, and slant asymptotes.

131. $f(x) = \dfrac{2x^3}{x^2 + 1}$ **132.** $f(x) = \dfrac{x^3}{3x^2 - 6}$

133. $f(x) = \dfrac{x^2 - x + 1}{x - 3}$ **134.** $f(x) = \dfrac{2x^2 + 7x + 3}{x + 1}$

135. ***Wildlife*** The Parks and Wildlife Commission introduces 80,000 fish into a large human-made lake. The population N of the fish, in thousands, is given by

$$N = \frac{20(4 + 3t)}{1 + 0.05t}, \quad t \ge 0$$

where t is time in years.

(a) Use a graphing utility to graph the function.

(b) Use the graph in part (a) to find the populations when $t = 5$, $t = 10$, and $t = 25$.

(c) What is the maximum number of fish in the lake as time increases? Explain your reasoning.

136. ***Page Design*** A page that is x inches wide and y inches high contains 30 square inches of print. The top and bottom margins are 2 inches deep and the margins on each side are 2 inches wide.

(a) Draw a diagram that gives a visual representation of the problem.

(b) Show that the total area A of the page is given by

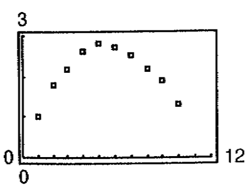
$$A = \frac{2x(2x + 7)}{x - 4}.$$

(c) Determine the domain of the function based on the physical constraints of the problem.

(d) Use a graphing utility to graph the area function and approximate the page size such that the minimum amount of paper will be used. Verify your answer numerically using the *table* feature of a graphing utility.

2.8 In Exercises 137–140, determine whether the scatter plot could best be modeled by a linear model, a quadratic model, or neither.

137.



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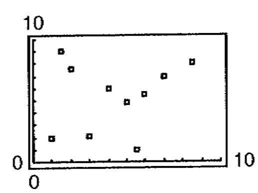

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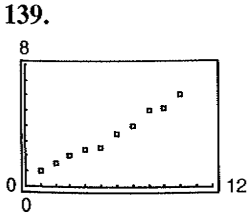

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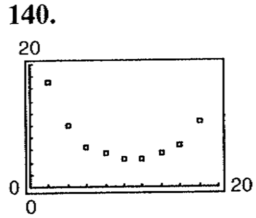

141. ***Consumer Awareness*** The table shows the average price P (in dollars) for a personal computer from 1997 to 2002. (Source: Consumer Electronics Association)

Year	Average price, P
1997	1450
1998	1300
1999	1100
2000	1000
2001	900
2002	855

Table for 141

(a) Use a graphing utility to create a scatter plot of the data. Let t represent the year, with $t = 7$ corresponding to 1997.

(b) Use the *regression* feature of a graphing utility to find a linear model for the data.

(c) Use a graphing utility to graph the linear model with the scatter plot from part (a).

(d) Use the *regression* feature of a graphing utility to find a quadratic model for the data.

(e) Use a graphing utility to graph the quadratic model with the scatter plot from part (a).

(f) Determine which model best fits the data and use the model you chose to predict the average price for a personal computer in 2008. Does your answer seem reasonable? Explain.

Synthesis

142. ***Error Analysis*** Describe the error.

$$-i(\sqrt{-4} - 1) = -i(4i - 1)$$
$$= -4i^2 - i$$
$$= 4 - i$$

True or False? **In Exercises 143 and 144, determine whether the statement is true or false. Justify your answer.**

143. The graph of $f(x) = \dfrac{2x^3}{x + 1}$ has a slant asymptote.

144. A fourth-degree polynomial with real coefficients can have -5, $-8i$, $4i$, and 5 as its zeros.

145. ***Think About It*** What does it mean for a divisor to divide evenly into a dividend?

146. ***Writing*** Write a paragraph discussing whether every rational function has a vertical asymptote.

2 Chapter Test

Take this test as you would take a test in class. After you are finished, check your work against the answers given in the back of the book.

1. Describe how the graph of g differs from the graph of $f(x) = x^2$.
(a) $g(x) = 6 - x^2$ (b) $g(x) = \left(x - \frac{3}{2}\right)^2$

2. Identify the vertex and intercepts of the graph of $y = x^2 + 4x + 3$.

3. Write an equation of the parabola shown at the right.

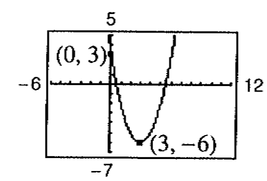

Figure for 3

4. Divide using long division: $(3x^3 + 4x - 1) \div (x^2 + 1)$.

5. Divide using synthetic division: $(2x^4 - 5x^2 - 3) \div (x - 2)$.

In Exercises 6 and 7, list all the possible rational zeros of the function. Use a graphing utility to graph the function and find all the rational zeros.

6. $g(t) = 2t^4 - 3t^3 + 16t - 24$ **7.** $h(x) = 3x^5 + 2x^4 - 3x - 2$

In Exercises 8–10, perform the operation and write the result in standard form.

8. $(-8 - 3i) + (-1 - 15i)$ **9.** $(2 + i)(6 - i)$

10. $(4 + 3i) - (5 + i)$

11. Write the quotient in standard form: $\dfrac{3i}{7 + i}$.

In Exercises 12 and 13, use the *zero* or *root* feature of a graphing utility to approximate (accurate to three decimal places) the real zeros of the function.

12. $f(x) = x^4 - x^3 - 1$ **13.** $f(x) = 3x^5 + 2x^4 - 12x - 8$

In Exercises 14–16, sketch the graph of the rational function. As sketching aids, check for intercepts, vertical asymptotes, horizontal asymptotes, and slant asymptotes.

14. $h(x) = \dfrac{4}{x^2} - 1$ **15.** $g(x) = \dfrac{x^2 + 2}{x - 1}$ **16.** $f(x) = \dfrac{2x^2 + 9}{5x^2 + 2}$

17. The table shows the number C of U.S. Supreme Court cases waiting to be tried for the years 1995 to 2000. (Source: Office of the Clerk, Supreme Court of the United States)

(a) Use a graphing utility to create a scatter plot of the data. Let t represent the year, with $t = 5$ corresponding to 1995.

(b) Use the *regression* feature of a graphing utility to find a quadratic model for the data.

(c) Use a graphing utility to graph the model with the scatter plot from part (a). Is the quadratic model a good fit for the data?

(d) Use the model to predict the year in which there will be 15,000 U.S. Supreme Court cases waiting to be tried.

Year	Cases, C
1995	7565
1996	7602
1997	7692
1998	8083
1999	8445
2000	8965

Table for 17