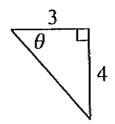
ii Volume $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$
$= \frac{1}{3}\pi \times 3^2 \times 4 + \frac{2}{3}\pi \times 3^3$
$\approx 37.7 + 56.5 \text{ m}^3$
$\approx 94.2 \text{ m}^3$

b **i**



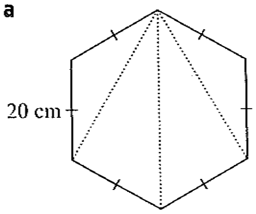
$\tan\theta = \frac{4}{3}$
$\therefore \quad \theta = \tan^{-1}\left(\frac{4}{3}\right)$
$\therefore \quad \theta \approx 53.1^\circ$

ii slant height $s = \sqrt{3^2 + 4^2}$ {Pythagoras}
$= \sqrt{25}$
$= 5$ m

iii Total surface area
= surface area of hemisphere + surface area of cone
$= \frac{1}{2} \times 4\pi r^2 + \pi r s$
$= \frac{1}{2} \times 4 \times \pi \times 3^2 + \pi \times 3 \times 5$
$\approx 104 \text{ m}^2$

c Weight of icecream $\approx 104 \times 1.23 \approx 128$ kg

8 **a**

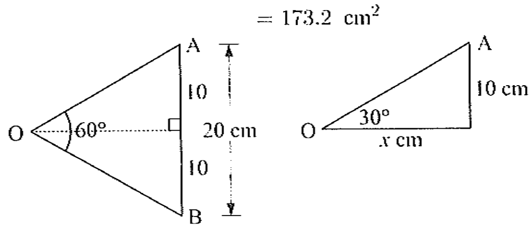


The shape can be divided into 4 triangles with each triangle having an angle sum of 180°.
$\therefore$ angle sum hexagon
$= 4 \times 180^\circ = 720^\circ$
$\therefore$ each angle
$= \frac{720^\circ}{6} = 120^\circ$

b **i** As OA = OB, triangle AOB is isosceles.
As each vertex is bisected then $\measuredangle OAB = \frac{120^\circ}{2} = 60^\circ$
$\therefore$ triangle AOB has all angles 60° (i.e., an equilateral triangle) $\therefore$ $\measuredangle AOB = 60^\circ$

ii ΔAOB is equilateral.

iii The sign is made up of 6 equilateral triangles.
The area of each triangle $= \frac{1}{2} \times 20 \times 20 \times \sin 60^\circ$.
$= 173.2 \text{ cm}^2$



$\therefore$ total area of figure $= 6 \times 173.2$
$= 1039.2$
$\approx 1040 \text{ cm}^2$

c **i** Height (y) of sign = length OA $\times 2 = 40$ cm

ii Now $\tan 30^\circ = \frac{10}{x}$ and so $x = \frac{10}{\tan 30^\circ} \approx 17.32$
Width (x) of sign $\approx 2 \times 17.32 \approx 34.6$ cm

iii Area = height $\times$ width
$= 40 \times 34.6$
$= 1386 \text{ cm}^2$
$= 1390 \text{ cm}^2$

d Wasted area = area of rectangle − area of hexagon
$= 1386 - 1039$
$= 347 \text{ cm}^2$
$\therefore$ cost of wasted material $= 0.35 \times 347$ EUD
$= 121.45$ EUD

SOLUTIONS TO TOPIC 6 — STATISTICS

Short questions

1 **a** Discrete data. 14, 16, 18, 23, 24, 25, 26, 26, 34

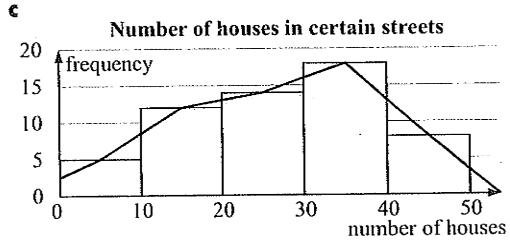
b Mean $= \frac{14 + 16 + \ldots\ldots + 34}{9} = 22.9$, i.e., 23 customers/h

Median is $\frac{n+1}{2} = \frac{9+1}{2} = 5$ i.e., 5th value
$\therefore$ the median = 24 customers
Mode = 26
Range = 34 − 14 = 20

c Total income $= 206 \times \$14.20 = \$2925.20$

2 **a** Discrete data.

b and **c**



c Modal class is 30 - 39

3 **a** $\frac{n+1}{2} = \frac{22+1}{2} = 11.5$th value
But, 11th value = 12th value = 22,
$\therefore$ median = 22 birds/day.

Q_1 lies between the 5th and 6th values
i.e., $Q_1 = \frac{14+17}{2} = 15.5$ birds/day

Q_3 lies between the 17th and 18th values
i.e., $Q_3 = \frac{35+36}{2} = 35.5$ birds/day

b IQR $= 35.5 - 15.5 = 20$ birds/day
$Q_1 - 1.5\times$ IQR $= 15.5 - 30 < 0$
$Q_3 + 1.5\times$ IQR $= 35.5 + 30 = 65.5$ birds/day
As the maximum number of birds present was 49, there are no outliers.

c On 5 days there are more than 35 birds in the park.
Let X be the number of birds in the park,
then $P(X > 35) = \frac{5}{22}$.

4 **a** Continuous data.

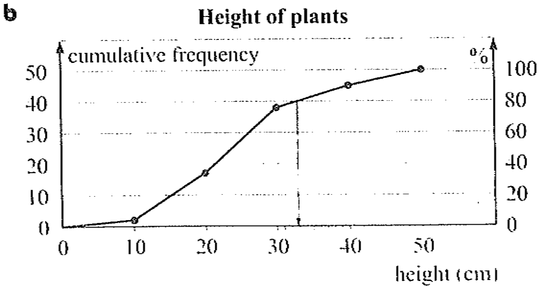
b Mid-interval values are: 4.5, 14.5, 24.5, 34.5, 44.5

c Lower boundary 9.5, upper boundary 19.5.

d Mean $= \frac{(4.5 \times 2) + (14.5 \times 15) + \ldots\ldots + (44.5 \times 5)}{50}$
$= 24.1$

5 **a** Cumulative frequencies: 2, 17, 38, 45, 50.

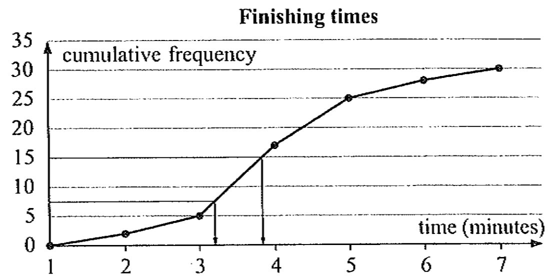
b



c 80th percentile is approximately 32 cm.

d 80th percentile is 40th plant. There are 10 plants taller than the 80th percentile.

6



a median $\approx 15^{\text{th}}$ score ≈ 3.8 (or 3.9) min

b $Q_1 \approx 7\frac{1}{2}^{\text{th}}$ score ≈ 3.2 min

c 5 finish within 3 minutes and

25 finish within 5 min

$\therefore$ 20 finish between 3 and 5 minutes.

7 **a** 9, 10, a, 13, b, 16, 21

Sum $= 7 \times 14 = 98$

$(a + b) = 98 - (9 + 10 + 13 + 16 + 21) = 29$

In order, $a = 13$, $b = 16$.

b The median lies between 9 and 11 and so the median $= 10$.

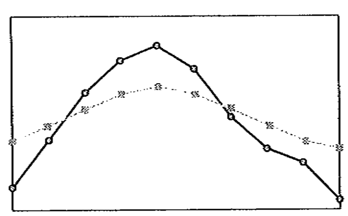
Since median $=$ mean, mean $= 10$.

$$\text{mean} = \frac{1 + 5 + 9 + 11 + 16 + p}{6}$$

$$\therefore \quad 10 = \frac{42 + p}{6}$$

$\therefore \quad 42 + p = 60$ and so $p = 18$

8 **a**



b Set 2 has the higher standard deviation, i.e., greater dispersion of data.

c Set 1 is likely to have the smaller IQR.

d Set 2 is likely to have more people on higher wages. Data is more spread and the median wage is slightly higher.

9 **a** $165 < \text{height} \leqslant 175$

b The frequency table is

Height (rounded, cm)	*Frequency*
140	2
150	6
160	14
170	17
180	9
190	2

Total number of students is 50.

Using calculator, mean height $= 166$ cm.

Standard deviation is 11.5 cm.

c $\overline{x} + 2s = 166 + 23 = 189$

There are at least 2 students taller than 2 standard deviations above the mean height.

10 **a** Midpoints are 89.5, 109.5, 129.5, 149.5, 169.5, 189.5

Using a calculator, mean rent $\approx \$138.49$

Standard deviation $\approx \$21.60$

b $30 + 14 + 1 = 45$ houses have rent greater than \$140.

There are 89 houses.

$P(\text{rent} > \$140) = \frac{45}{89}$

c $\overline{x} + 1s = \$138.49 + \$21.60 = \$160.09 \approx \$160$

Percentage of rent above \$160 is $\dfrac{14+1}{89} \times 100\%$

$= 16.9\%$

11 **a** Site 3 has the greatest range.

b Site 2 has the smallest spread.

c Site 1 has the highest median weight.

d The heaviest fungi were found at Site 3.

e Site 1 has the highest proportion of weights above 40 g.

f All sites have the same proportion (25%) above Q_3.

12 **a** $Q_1 \approx 175$, $Q_2 \approx 190$, $Q_3 \approx 200$.

b Range $\approx 220 - 130 = 90$

c IQR $\approx 200 - 175 = 25$

d $Q_1 - 1.5\times$ IQR $= 167 - 1.5(25) = 129.5$.

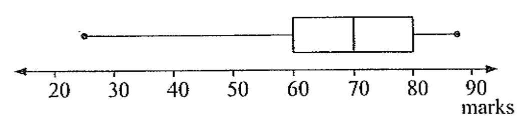
The minimum of 130 is only just not an outlier.

13 We start by displaying the data on a stem and leaf plot.

2	5
3	
4	8 9
5	7 9
6	1 2 7 8
7	0 2 5 5 6 8 8
8	1 2 5 7

a min $= 25$, $Q_1 = 60$, $Q_2 = 71$, $Q_3 = 78$, max $= 87$

b

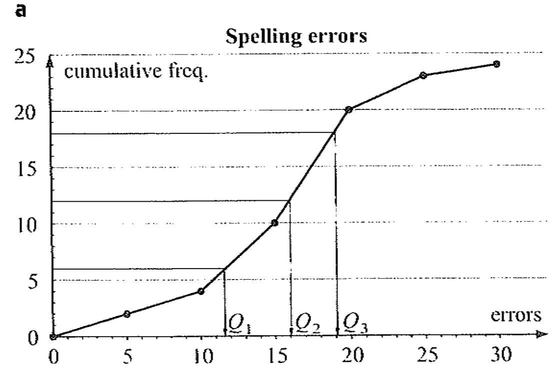


c IQR $= 78 - 60 = 18$

$Q_1 - 1.5\times$ IQR $= 60 - 1.5(18) = 33$

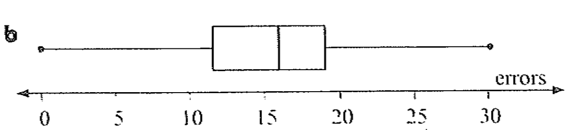
The minimum mark is an outlier. Student did not study; poor attendance or the student had a very bad day.

14 **a**



min $= 0$. $Q_1 = 12$. $Q_2 = 16$. $Q_3 = 19$. max $= 30$

b



15 **a** Median is 5th value = \$310 000

b Q_1 is between 2nd and 3rd value = \$261 000,
Q_3 is between 7th and 8th value = \$335 000
IQR = 335 000 − 261 000 = 74 000

c $Q_3 + 1.5(74\,000) = 446\,000$
The highest priced house is an outlier.
Median (with outlier omitted) = \$295 000

d Percentage change is

$$\frac{310\,000 - 295\,000}{310\,000} \times 100\% = 4.84\%$$

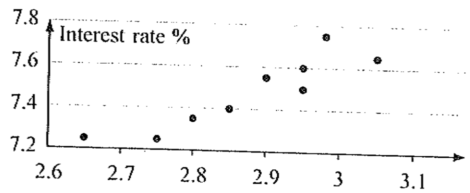
16 **a** $r = \frac{s_{xy}}{s_x s_y} = \frac{405}{17.4 \times 25.6} = 0.909$

b strong, positive, relationship

c
$$(y - \overline{y}) = \frac{s_{xy}}{(s_x)^2}(x - \overline{x})$$
$$\therefore\ (y - 110) = \frac{405}{17.4^2}(x - 63)$$
$$\therefore\ y - 110 = 1.34(x - 63)$$
$$\therefore\ y - 110 = 1.34x - 84.4$$
$$\therefore\ y = 1.34x + 25.6$$

d $y = 1.34 \times 70 + 25.6 \approx 119$

17 **a**

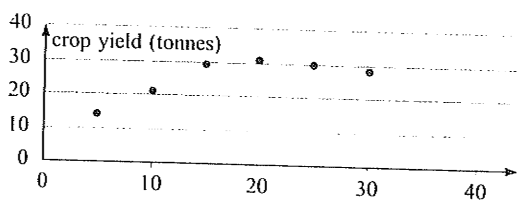


b $r = 0.915$

c There is a strong, positive, linear relationship between exchange rate and interest rate.

18 **a** $r = 0.795$ A moderate, positive relationship may exist between crop yield and rainfall.

b



c The relationship between rainfall and crop yield does not appear to be linear and so r may not be appropriate for this data.

19 **a** The 2×2 contingency table is:

	Y_1	Y_2	*sum*
X_1	32	14	46
X_2	25	19	44
sum	57	33	90

The expected frequency table is:

	Y_1	Y_2
X_1	$\frac{46 \times 57}{90} = 29$	$\frac{46 \times 33}{90} = 17$
X_2	$\frac{44 \times 57}{90} = 28$	$\frac{44 \times 33}{90} = 16$

b The χ^2 calculation is:

f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$	$\frac{(f_o - f_e)^2}{f_e}$
32	29	3	9	0.310
14	17	−3	9	0.529
25	28	3	9	0.321
19	16	3	9	0.563
			Total	1.723

So $\chi^2_{calc} = 1.72$.

20 **a** H_0: Plant height is independent of light conditions.
H_A: Plant height is dependent on light conditions.

b The 3×2 contingency table is:

	$Ht < 60$ cm	$Ht \geqslant 60$ cm	*sum*
Sunlight	37	43	80
Shade	22	18	40
Dark	25	19	44
sum	84	80	164

The expected frequency table is:

	$Ht < 60$ cm	$Ht \geqslant 60$ cm
Sunlight	$\frac{80 \times 84}{164} = 41.0$	$\frac{80 \times 80}{164} = 39.0$
Shade	$\frac{40 \times 84}{164} = 20.5$	$\frac{40 \times 80}{164} = 19.5$
Dark	$\frac{44 \times 84}{164} = 22.5$	$\frac{44 \times 80}{164} = 21.5$

The χ^2 calculation is:

f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$	$\frac{(f_o - f_e)^2}{f_e}$
37	41.0	−4.0	16	0.390
43	39.0	4.0	16	0.410
22	20.5	1.5	2.25	0.110
18	19.5	−1.5	2.25	0.115
25	22.5	−2.5	6.25	0.278
19	21.5	2.5	6.25	0.291
			Total	1.594

$\chi^2_{calc} \approx 1.59$ (the calculated value may vary slightly depending on the rounding)

c $\chi^2_{2,\,05} = 5.991$

d $\chi^2_{calc} < \chi^2_{crit}$ do not reject H_0
The nursery's claim is justified according to this data. There is no significant difference in the height of the plants and the conditions they are growing under.

21 **a** Using a gdc, $\chi^2_{calc} = 6.88$
There are $(3-1)(3-1) = 4$ degrees of freedom
and $\chi^2_{4,\,0.05} = 9.488$
Since $\chi^2_{calc} < \chi^2_{4,\,0.05}$, we do not reject H_0, and accept that the factors are independent.
Note also that the p-value is $p = 0.142$
Since $p > 0.05$ we do not reject H_0.

b Expected value in Row 1/Column 1 is less than 5.

c Combined table

	Factor Y_a	Factor Y_b	Factor Y_c
Factor X_{a+b}	27	38	35
Factor X_c	16	11	22

$\chi^2_{calc} = 3.6246$ $\chi^2_{2,\,0.05} = 5.991$ **or** $p = 0.163\,27$

$\chi^2_{calc} < \chi^2_{2,\,0.05}$ or $p > 0.05$ do not reject H_0
Factors are independent.

d Adding rows led to slightly greater p-value, but the difference was not significant.

22 a $y = -0.185x + 44.1$
Number of potatoes $= -0.185$(median weight) $+ 44.1$

b **i** Number of potatoes $= -0.185(100) + 44.1 = 26$
ii Number of potatoes $= -0.185(200) + 44.1 = 7.1$

c The first calculation is likely to be more reliable - it is an interpolated value. 200 grams is outside the range and the second calculation is an extrapolation.

23 a $y = -0.555x + 71.3$ $r = -0.647$

b The outlier is (50, 12).
With the outlier removed, $y = -0.637x + 80.5$ and $r = -0.986$.

c The slope of the line is steeper (gradient has become more negative). The relationship has changed from weak/moderate to very strong.

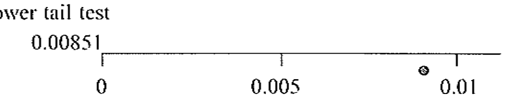
24 a $y = 2.43x + 32.0$

b $70 = 2.43$ (hours) $+ 32.0$
Tony studied for $\dfrac{70 - 32}{2.43} = 15.6$ hours.

c The y-intercept (32%) is the estimate of the result for a student who did not do any study.
The gradient of the line indicates that the result will increase by 2.43% for each additional hour studied.

25 a Using a gdc, $\chi^2_{calc} = 0.0171$
With 2 degrees of freedom and assuming H_0
$P(\chi^2_{calc} \geqslant 0.0171) = 0.991\,49$
Probability value $p = 0.991\,49$
lower tail $1 - p = 0.008\,51$

b lower tail test

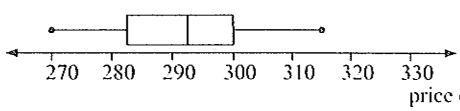


c 1% level: reject H_0
0.5% level: do not reject H_0
At the 1% level, the deviation of the data from expected values may not necessarily be due to random chance. At the 0.5% level, the deviations from expected values are reasonable.

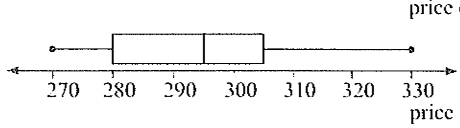
Long questions

1 a i $\dfrac{n+1}{2} = \dfrac{25+1}{2} = 13$
so, the median is the 13th value. $\therefore$ median = \$293
Range $= 316 - 271 = 45$
Q_1 is $\left(\frac{n+1}{4}\right)$th value $= \left(\frac{25+1}{4}\right)$th value = 6.5th value,
i.e., between 6th and 7th values, so $Q_1 = \$283$.
Q_3 is 3×6.5th = 19.5th value,
i.e., between 19th and 20th values, so $Q_3 = \$301$.

ii



b



c **i** New prices are more spread out.
ii Mean price has increased slightly. Some printers have increased in price; others have decreased.

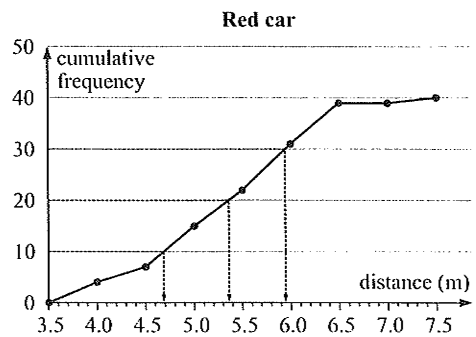
d Mean price has increased by \$2 from \$293 to \$295
Percentage increase is $\frac{2}{293} \times 100\% = 0.683\%$.

2 a Mean $= \dfrac{3.6 + 5.4 + 4.5 + \ldots\ldots + 4.5 + 6.4}{40} = 5.265$
Using a gdc, standard deviation is 0.856.

b

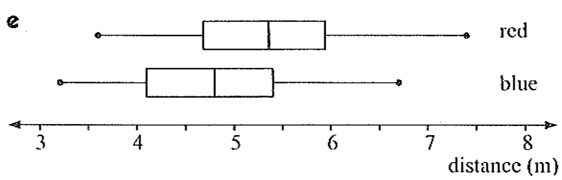
Distance (m)	*Cumulative frequency*
3.5 - < 4.0	4
4.0 - < 4.5	7
4.5 - < 5.0	15
5.0 - < 5.5	22
5.5 - < 6.0	31
6.0 - < 6.5	39
6.5 - < 7.0	39
7.0 - < 7.5	40

c



d Median ≈ 4.8, $Q_1 \approx 5.4$, $Q_3 \approx 5.9$

e



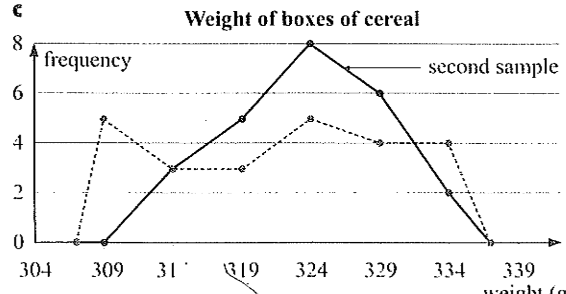
f All values of the five-number summary, min, Q_1, median, Q_3 and max, for the red car are higher than those for the blue car. There is no evidence to support the view that the cars were made by the same machine.

3 a Mean $= \dfrac{312 + 312 + 308 + \ldots\ldots + 330 + 309 + 329}{24}$
$= 319$ g
Range $= 334 - 306 = 28$ g

b

Weight (g)	*Frequency*	*Weight (g)*	*Frequency*
305 - 309	5	320 - 324	5
310 - 314	3	325 - 329	4
315 - 319	3	330 - 334	4

c



d The stated weight is higher than the average (mean) contents.

e Mean

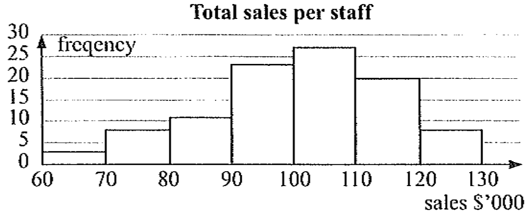
$= \dfrac{312 \times 3 + 317 \times 5 + 322 \times 8 + 327 \times 6 + 332 \times 2}{24}$

$= 322$ g

f on graph

g The mean weight is now higher. The spread of weights is less. The evidence suggests that an improvement in the production process has occurred.

4 a **Total sales per staff**



b minimum is $120 000, maximum is $129 999.99

c Mean $= \dfrac{65 \times 3 + 75 \times 8 + 85 \times 11 + \ldots\ldots + 125 \times 8}{100}$

$= \$100\,500$

Using technology, standard deviation is $14 800.

d **i** $\bar{x} + 2s = 100\,500 + 2 \times 14\,800 = 130\,100$

ii No bonuses were paid.

e The top 8 sales staff sold more than $120 000.

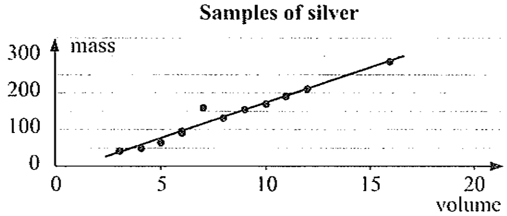
This is $120 000 − $100 500 = $19 500 above the mean

and $\dfrac{19\,500}{14\,800} = 1.32$ standard deviations above the mean.

f $\bar{x} - 1.385s = 100\,500 - 1.385 \times 14\,800 = \$80\,002$

Approximately 11 staff may be retrenched.

5 a **Samples of silver**



b **i** $\bar{x} = \dfrac{3 + 6 + 4 + \ldots\ldots 10 + 11}{12} = 8.08 \text{ cm}^3$

$\bar{y} = \dfrac{40 + 95 + 50 + \ldots\ldots + 170 + 190}{12} = 137$ g

ii on graph

c An imperfect, positive relationship appears to exist between volume and mass of the samples of silver.

d **i** $r = 0.980$

e **i** $M = 19.5V - 24.7$

ii $M = 19.5 \times 7 - 24.7 = 112$ g

iii Percentage error between the given and expected mass is $\dfrac{160 - 112}{112} \times 100\% = 42.9\%$.

6 a **i** $r = \dfrac{s_{xy}}{s_x s_y} = \dfrac{33.9}{7.25 \times 4.78} = 0.978$

ii A strong, positive relationship appears to exist between age and annual income.

b $(y - \bar{y}) = \dfrac{s_{xy}}{s_x^2}(x - \bar{x})$

$\therefore \quad y - 20 = \dfrac{33.9}{7.25^2}(x - 26.3)$

$\therefore \quad y - 20 = 0.645(x - 26.3)$

$\therefore \quad y = 0.645x - 17 + 20$

$\therefore \quad y = 0.645x + 3$

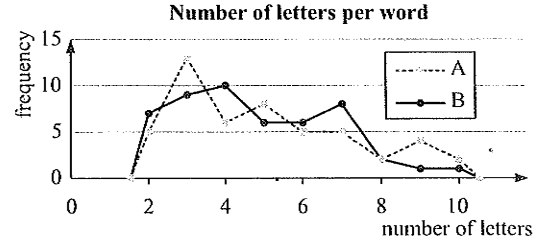
i.e., income $= 0.645\,(\text{age}) + 3$

c **i** At age 30, income $= 0.645(30) + 3 = 22\,350$

ii At age 60, income $= 0.645(60) + 3 = 41\,700$

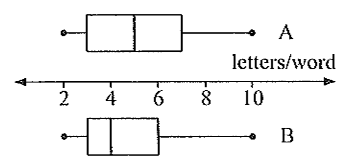
d The age of 30 is within the given data range but 60 is outside the range. Predicting the income at 30 years is an interpolation and is more reliable than the extrapolation required for 60.

7 a **Number of letters per word**



b

	A	B
min	2	3
Q_1	3	3
med	5	4
Q_3	7	6
max	10	10



c A: mean = 5, standard deviation = 2.28

B: mean = 4.76, standard deviation = 2.02

d

	< 5	$\geqslant 5$	*sum*
Story A	24	26	50
Story B	26	24	50
sum	50	50	100

e H_0: There is no difference in the stories.

Expected values:

	< 5	$\geqslant 5$
Story A	$\frac{50 \times 50}{100} = 25$	$\frac{50 \times 50}{100} = 25$
Story B	$\frac{50 \times 50}{100} = 25$	$\frac{50 \times 50}{100} = 25$

$\chi^2_{calc} = 0.16 \quad \chi^2_{1,\,0.1} = 2.706 \quad or \quad p = 0.689$

$\chi^2_{calc} < \chi^2_{crit} \quad or \quad p > 0.10$ hence, do not reject H_0

f There is no significant difference between the stories.

g The graphical evidence suggests that the stories may have been written by different authors.

The mean of B is lower and the standard deviation is smaller again, suggesting that the stories are by different authors.

However, the chi-square test suggests that the difference is not significant.

Overall, the statistical evidence is not strong enough to support the claim that the two stories were written by different authors.

8 a H_0: Choice of breakfast cereal is independent of gender.

b Contingency table:

	Muesli	Rolled Oats	Corn Flakes	Weetbix	*sum*
Female	2	7	24	12	45
Male	4	15	13	13	45
sum	6	22	37	25	90

Expected values:

	Muesli	Rolled Oats	Corn Flakes	Weetbix
Female	$\frac{45\times6}{90}$ $=3$	$\frac{45\times22}{90}$ $=11$	$\frac{45\times37}{90}$ $=18.5$	$\frac{45\times25}{90}$ $=12.5$
Male	$\frac{45\times6}{90}$ $=3$	$\frac{45\times22}{90}$ $=11$	$\frac{45\times37}{90}$ $=18.5$	$\frac{45\times25}{90}$ $=12.5$

c Combined contingency table:

	Muesli or Rolled Oats	Corn Flakes	Weetbix	*sum*
Female	9	24	12	45
Male	19	13	13	45
sum	28	37	25	90

Expected values:

	Muesli or Rolled Oats	Corn Flakes	Weetbix
Female	$\frac{28\times45}{90}=14$	18.5	12.5
Male	$\frac{28\times45}{90}=14$	18.5	12.5

d $\chi^2_{calc} = \frac{(9-14)^2}{14} + \frac{(24-18.5)^2}{18.5} + \frac{(12-12.5)^2}{12.5} + ...$

$+ \frac{(19-14)^2}{14} + \frac{(13-18.5)^2}{18.5} + \frac{(13-12.5)^2}{12.5} = 6.88$

e $df = (2-1)(3-1) = 2$; $\chi^2_{2,\,0.05} = 5.991$

$\chi^2_{calc} > \chi^2_{crit}$ hence reject H_0

f The choice of breakfast cereal is dependent on gender.

g $\chi^2_{calc} = 6.89$ $\chi^2_{3,\,0.05} = 7.815$ *or* $p = 0.075\,62$

$\chi^2_{calc} < \chi^2_{crit}$ *or* $p = 0.075\,62 > 0.05$ hence do not reject H_0

He would have concluded that the choice of breakfast cereal and gender are not related.

SOLUTIONS TO TOPIC 7 — CALCULUS

Short questions

1 a $y = x^3 - 4.5x^2 - 6x + 13$

$\therefore \frac{dy}{dx} = 3x^2 - 2\times4.5x - 6$

$\therefore \frac{dy}{dx} = 3x^2 - 9x - 6$

b When the gradient of the tangent is 6.

$\frac{dy}{dx} = 6$

$\therefore 3x^2 - 9x - 6 = 6$

$\therefore 3x^2 - 9x - 12 = 0$

$\therefore 3(x^2 - 3x - 4) = 0$

$\therefore 3(x-4)(x+1) = 0$

$\therefore x = 4$ or $x = -1$

So the x-coordinates of the points are 4 and -1.

2 a $y = ax^2 + bx + c$ $\therefore \frac{dy}{dx} = 2ax + b$

b When $x = k$, $\frac{dy}{dx} = 0$ $\therefore 2ax + b = 0$

$\therefore 2ax = -b$

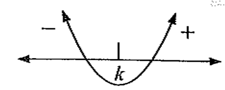
$\therefore x = -\frac{b}{2a}$

i.e., $k = -\frac{b}{2a}$

c If $\frac{dy}{dx} < 0$, y is decreasing.

d When $x < k$, $\frac{dy}{dx} < 0$, and when $x > k$, $\frac{dy}{dx} > 0$.

$\therefore \frac{dy}{dx}$ has sign diagram:



$\therefore x = k$ is a local minimum.

3 a $y = \frac{7}{x^3}$ $\therefore y = 7x^{-3}$

b $\frac{dy}{dx} = 7(-3)x^{-4} = -21x^{-4}$

$\therefore \frac{dy}{dx} = \frac{-21}{x^4}$

4 a $y = \frac{2x^4 - 4x^2 - 3}{x}$

$\therefore y = \frac{2x^4}{x} - \frac{4x^2}{x} - \frac{3}{x}$

$\therefore y = 2x^3 - 4x - 3x^{-1}$

b $\frac{dy}{dx} = 2(3)x^2 - 4 - 3(-1)x^{-2}$

$\therefore \frac{dy}{dx} = 6x^2 - 4 + \frac{3}{x^2}$

c At $x = -1$, $\frac{dy}{dx} = 6(-1)^2 - 4 + \frac{3}{(-1)^2}$

$= 6 - 4 + 3$

$= 5$

$\therefore$ at $x = -1$, the gradient is 5.

5 a $f(x) = 3x^2 - 4x^{-1} + 7$

$\therefore f'(x) = 3(2)x^1 - 4(-1)x^{-2}$

$= 6x + \frac{4}{x^2}$

b $f'(x) = 6x + 4x^{-2}$

$\therefore f''(x) = 6 + 4(-2)x^{-3}$

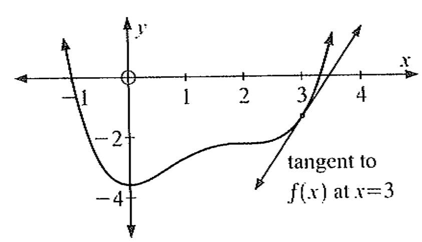
$= 6 - \frac{8}{x^3}$

6 a i $f(x)$ is increasing on $x > 0$.

ii $f(x)$ is decreasing on $x < 0$.

b $f'(x) = 0$ at $x = 0$ and $x = 2$.

c



7 a $f(x) = x^4 - 6x^2 - x + 3$

$\therefore f'(x) = 4x^3 - 12x - 1$

b $f''(x) = 12x^2 - 12$

c Max/min values of $f'(x)$ occur when

$f''(x) = 0$

$\therefore 12x^2 - 12 = 0$

$\therefore 12(x-1)(x+1) = 0$

$\therefore x = \pm1$