SUMMARY

SETS

- A set is a collection of numbers or objects.
  For example, $A = \{\text{factors of } 16\}$
- The Universal set is the set of all things being considered.
  The symbol U is used to represent the universal set.
- $a \in A$ is read as a is a member (element) of the set A.

SUBSETS, INTERSECTION, UNION, COMPLEMENT

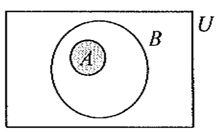
Given two sets, A and B:

- $A \subset B$ reads A is a **subset** of B and every element of A is also an element of B.

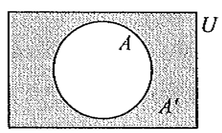
  $A \cap B$ represents the **intersection** and is the set made up of elements which are **in both** A and B.
- $A \cup B$ represents the **union** and is the set made up of elements which are **in** A **or** B **or both.**
- A' represents the **complement** of A and is the set made up of all elements which are in the Universal set and **not in** A.

Subsets, intersection, union and complement can be represented by shaded regions on a Venn diagram.

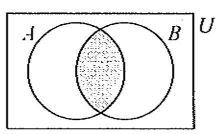
$A \subset B$



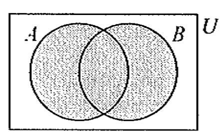
A'



$A \cap B$



$A \cup B$



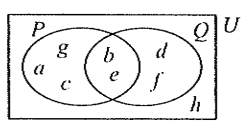
COUNTING IN A SET

$n(A)$ is used to represent the number of elements in a set.

For example, if $A = \{a, b, c, d, e\}$, then $n(A) = 5$.

Example:

For this Venn diagram



find **a** $n(P \cap Q)$ **b** $n(P \cup Q)$ **c** $n(Q')$

Solution:

a $P \cap Q = \{b, e\}$ $\therefore$ $n(P \cap Q) = 2$

b $P \cup Q = \{a, b, c, d, e, f, g\}$ $\therefore$ $n(P \cup Q) = 7$

c $Q' = \{a, c, g, h\}$ $\therefore$ $n(Q') = 4$

PROBLEM SOLVING WITH VENN DIAGRAMS

Venn diagrams can be used to solve some problems.

Example:

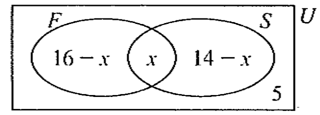
A class of 30 pupils contains students who study French, students who study Spanish, students who study both French and Spanish and students who study neither. 16 students study French, 14 students study Spanish and 5 study neither.

How many students study both French and Spanish.

Solution:

Let x be the number of students who study both subjects.

Then $16 - x$ must study French only and $14 - x$ study Spanish only.



Now, $16 - x + x + 14 - x + 5 = 30$

$\therefore \quad 35 - x = 30$

$\therefore \quad x = 5$

So, 5 students study both French and Spanish.

LOGIC

You must understand and be able to use:

- Propositions, symbolic notation, p, q, and r for propositions, compound propositions.
- **Conjunction:** $p \wedge q$, p and q relates to the region $P \cap Q$ on a Venn diagram.
- **Disjunction:** $p \vee q$, p or q relates to the region $P \cup Q$ on a Venn diagram.
- **Exclusive disjunction:** $p \underline{\vee} q$ relates to the region $P \cup Q$ but not $P \cap Q$ on a Venn diagram.
- **Negation:** $\neg p$ "not p" relates to the region P' on a Venn diagram.
- **Implication:** $p \Rightarrow q$ "If p then q."
- **Equivalence:** $p \Leftrightarrow q$ "p if and only if q."
- **Converse:** $q \Rightarrow p$ "If q then p."
- **Inverse:** $\neg p \Rightarrow \neg q$ "If not p then not q."
- **Contrapositive:** $\neg q \Rightarrow \neg p$ "If not q then not p."

TRUTH TABLES

p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \vee q$	$p \underline{\vee} q$	$p \Rightarrow q$
T	T	F	F	T	T	F	T
T	F	F	T	F	T	T	F
F	T	T	F	F	T	T	T
F	F	T	T	F	F	F	T

Terms

- **Logical equivalence:** propositions having the same truth set.
- **Tautology:** propositions whose truth set is all true.
- **Contradiction:** propositions whose truth set is all false.

Example:

Let p be the proposition $x = 2$
and q be the proposition $3x^2 - 2x - 2 = 6$.

a Write down the proposition $p \Rightarrow q$.

b Is this proposition true of false?

c Write down the converse of this compound proposition in words.

d Is this proposition true or false? Give a reason for your answer.

Solution:

a If $x = 2$, then $3x^2 - 2x - 2 = 6$.

b True.

c If $3x^2 - 2x - 2 = 6$, then $x = 2$.

d False. $3x^2 - 2x - 2 = 6 \Rightarrow (3x + 4)(x - 2) = 0$
$\Rightarrow x = 2$ or $x = -\frac{4}{3}$.

Example:

a Draw the truth tables for the compound propositions $(\neg p \vee q) \Rightarrow q$ and $p \wedge \neg q$.

b Are the two compound propositions logically equivalent? Give a reason for your answer.

Solution:

a

p	q	$\neg p$	$\neg p \vee q$	$(\neg p \vee q) \Rightarrow q$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	F

p	q	$\neg q$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

b No, the final truth table columns are not the same.

PROBABILITY

- The probability of an event A occurring is given by $\mathrm{P}(A) = \dfrac{n(A)}{n(U)}$.
- The probability of the complement of an event A occurring is $\mathrm{P}(A') = 1 - \mathrm{P}(A)$.

LAWS OF PROBABILITY

- $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$.
- **Mutually exclusive events** are events that share no common outcomes. This means, $A \cap B = \varnothing$ and so $\mathrm{P}(A \cap B) = 0$ and $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$.
- **Conditional probability**. The probability of event A occurring given that event B has occurred is

$$\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}.$$

- **Independent events** are events where the occurrence of one of the events does not affect the occurrence of the other event. Consequently, $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$.

USING DIAGRAMS TO FIND PROBABILITIES

Tree diagrams, Venn diagrams and lattices can be used to find probabilities.

Example:

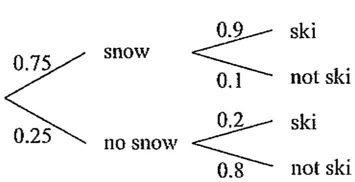
The probability that it will snow tomorrow is 0.75. If it snows the probability that I will go skiing is 0.9. If it does not snow the probability that I will go skiing is 0.2.

a Draw a tree diagram to illustrate this information.

b Find the probability that I will go skiing tomorrow.

Solution:

a



b $\mathrm{P}(\text{skiing}) = 0.75 \times 0.9 + 0.25 \times 0.2 = 0.725$

Example:

Two events are such that $\mathrm{P}(A) = 0.6$, $\mathrm{P}(B) = 0.5$ and $\mathrm{P}(A \cup B) = 0.85$.

a Find $\mathrm{P}(A \cap B)$.

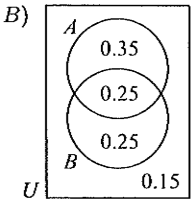
b Are A and B mutually exclusive? Give a reason for your answer.

c Are A and B independent? Give a reason for your answer.

Solution:

a

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$
$$\therefore \quad 0.85 = 0.6 + 0.5 - \mathrm{P}(A \cap B)$$
$$\therefore \quad \mathrm{P}(A \cap B) = 1.1 - 0.85 = 0.25$$



b $\mathrm{P}(A) + \mathrm{P}(B) = 1.1$ and $\mathrm{P}(A \cap B) = 0.85$ and as they are not equal, A and B are not mutually exclusive.

c $\mathrm{P}(A) \times \mathrm{P}(B) = 0.3$ and $\mathrm{P}(A \cap B) = 0.25$ and as they are not equal, A and B are not independent.

Example:

Two bags contain cards with numbers printed on them.
Bag 1 contains 5 cards with 0, 0, 1, 3 and 4 printed on them.
Bag 2 contains 4 cards with 0, 3, 4 and 5 on them.

a Draw a lattice diagram to illustrate this information.

b Find the probability that when one card is picked from each bag at random the total is an even number.

c Find the probability that given the total number is even it is less than 5.

Solution:

a

		Bag 1				
	Sum	0	0	1	3	4
Bag 2	0	0	0	1	3	4
	3	3	3	4	6	7
	4	4	4	5	7	8
	5	5	5	6	8	9

b $\mathrm{P}(\text{even number}) = \frac{8}{20}$

c $\mathrm{P}(\text{less than } 5 \mid \text{total even}) = \frac{4}{8}$

TOPIC 3 – SETS, LOGIC AND PROBABILITY (SHORT QUESTIONS)

1 Set $A = \{$multiples of 4 greater than 0 and less than 20$\}$
Set $B = \{$even numbers greater than 0 and less than 10$\}$

a Find $A \cup B$.

b Find $A \cap B$.

c Set C contains 3 elements and is a subset of A. Write down one possible set C.

2 $\mathbb{N}$ is the set of natural numbers, $\mathbb{Z}$ is the set of integers and $\mathbb{Q}$ is the set of rational numbers.

a Write down an element that is in:
i $\mathbb{Z}$ **ii** $\mathbb{Z}'$ **iii** $\mathbb{N} \cap \mathbb{Q}$ **iv** $\mathbb{Q}' \cap \mathbb{Z}$

b Draw a Venn diagram to represent the sets $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$.

3 Let $U = \{$positive integers greater than 7 and less than 19$\}$
$A = \{$multiples of 3$\}$
$B = \{$factors of 36$\}$

a List the elements of $A \cap B$.

b List the elements of A'.

c Represent the relationship between sets A and B on a Venn diagram.

d Are the following statements true or false?
i $B \subset A$ **ii** $n(A' \cap B') = 7$ **iii** $A \cup B = A$

4 If $U = \{$positive integers less than 20$\}$
$P = \{$prime numbers less than 20$\}$
$F = \{$factors of 24$\}$.

Find:

a $n(F)$ **b** $P \cap F$ **c** $P \cup F$ **d** $P' \cap F$

5 Let $U = \{x \in \mathbb{Z} \mid 1 < x < 50\}$.

A, B, and C are subsets of U such that,

$A = \{\text{factors of } 48\}$ $B = \{\text{multiples of } 6\}$

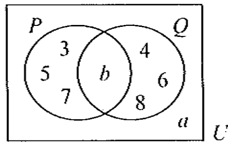
$C = \{\text{multiples of } 8\}$.

a List the elements in $A \cap B \cap C$.

b Find $n(B)$.

c List the elements in $A' \cap B$.

6 Given the information in the following Venn diagram:



$U = \{x \in \mathbb{N} \mid 1 < x < 10\}$

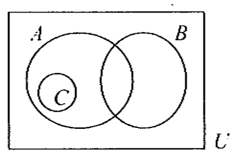
$P = \{\text{Prime Numbers}\}$

a Find the value of element b.

b Describe in words, the set Q.

c Find the value of element a.

7 The following Venn diagram shows the sets U, A, B, and C



a State whether the following propositions are true or false for the information shown in the Venn diagram.

i $A \cap B = \varnothing$ **ii** $A \cap C' = C$

iii $B \subset A'$ **iv** $C \subset (A \cap B)$

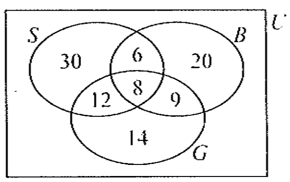
b Shade the region $A' \cap B$ on the Venn diagram above.

8 The Venn diagram below shows the number of members at a sports club who take part in various sporting activities where:

$S = \{\text{members who play soccer}\}$

$B = \{\text{members who play basketball}\}$

$G = \{\text{members who play golf}\}$.



Find the number of members who:

a play all 3 sports

b play soccer or basketball but not both

c play only golf

d do not play basketball

e play more than one sport

f play soccer but do not play golf.

9 Let p and q be the propositions:

p: The sun is shining. q: I take the dog for a walk.

Write the following propositions using words only.

a $p \Rightarrow q$

b $\neg p \vee q$

c Write in words the converse of $p \Rightarrow q$

d Write the following proposition in symbolic form.

"If I do not take the dog for a walk then the sun is not shining."

e Is the proposition in **d** the inverse, converse, or contrapositive of part **a**?

10 Consider each of the following propositions:

p: Peter has black hair. q: Peter plays baseball.

r: Peter is a student.

a Write each of the following arguments in symbols.

i If Peter does not play baseball then he is not a student.

ii If Peter does not have black hair then he is neither a student nor does he play baseball.

b Write the following argument in words: $\neg r \Rightarrow \neg(p \vee q)$.

11 Two propositions are given:

p: Wilson is an active dog. q: Wilson digs holes.

a Write the following propositions using words only.

i $\neg p \wedge \neg q$ **ii** $p \veebar q$ **iii** $\neg p \Rightarrow q$

b By completing this truth table state whether the argument in **a iii** is a tautology, a contradiction or neither.

p	q	$\neg p$	$\neg p \Rightarrow q$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

12 Given the two propositions:

p: I like the beach.

q: I live near the sea.

a Write the following in symbols and words:

i the inverse **ii** the contrapositive.

b Write in symbols the converse of the above propositions p and q.

13 Consider the two propositions p and q.

Complete the truth table below for the compound proposition $(p \Rightarrow \neg q) \vee (\neg p \Rightarrow q)$.

p	q	$\neg q$	$p \Rightarrow \neg q$	$\neg p$	$\neg p \Rightarrow q$	$(p \Rightarrow \neg q) \vee (\neg p \Rightarrow q)$
T	T	F		F		
T	F	T		F		
F	T	F		T		
F	F	T		T		

State whether the result above is a contradiction, a tautology or neither.

14 Three propositions are defined as follows:

p: The weather is hot. q: I have money.

r: I buy an ice cream.

a Write a sentence, in words only, for each of the following propositions:

i $(\neg r \wedge \neg q) \vee p$ **ii** $(p \wedge q) \Rightarrow r$ **iii** $(\neg p \vee \neg q) \Rightarrow \neg r$

b Write in words the contrapositive for the proposition $p \Rightarrow r$

15 p: Molly has a DVD player.

q: Molly likes watching movies.

a Write the following proposition using words only: $\neg p \Rightarrow \neg q$

b Complete the following truth table for $p \Rightarrow q$

p	q	$p \Rightarrow q$
T	T	
T	F	
F	T	
F	F	

c Complete the following truth table for $\neg p \Rightarrow \neg q$.

p	q	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	T	F	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	

d Are the two propositions $q \Rightarrow p$ and $p \Rightarrow q$ logically equivalent? Give a reason for your answer.

16 Consider the two propositions p and q.

a Complete the truth table for the compound proposition $(p \wedge q) \Rightarrow p$.

p	q	$p \wedge q$	$(p \wedge q) \Rightarrow p$
T	T	T	
T	F		
F	T		
F	F	F	

b Is the compound proposition $(p \wedge q) \Rightarrow p$ a contradiction, tautology, or neither?

17 Consider the two propositions p and q :

p: I eat an apple every day. q: I visit the doctor.

a State the negation of q.

b In words only, write the following propositions:

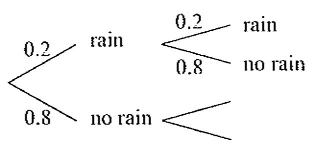
i $\neg p \Rightarrow q$ **ii** $(\neg p \vee q) \Rightarrow q$.

c For the compound proposition $(\neg p \vee q) \Rightarrow q$, state whether it is a tautology, contradiction or neither.

p	q	$\neg p$	$\neg p \vee q$	$(\neg p \vee q) \Rightarrow q$
T	T	F	T	T
T	F	F		T
F	T	T	T	
F	F	T		

18 The probability of rain on any day in December is 0.2.

The partially completed tree diagram below shows the possible outcomes when the weather for two consecutive days is considered.



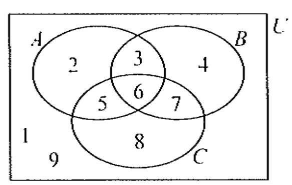
a Complete each of the boxes to finish the tree diagram.

b Use the tree diagram to determine the probability of:

i raining two days in a row

ii raining on one day only.

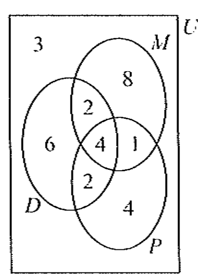
19 In the Venn diagram below sets A, B, and C are subsets of the Universal set $U = \{\text{Natural numbers less than } 10\}$.



a **i** Find $n(A)$. **ii** Find $n(A \cap B)$.

b List the elements in: **i** $A \cup B \cup C$ **ii** $(A' \cap B) \cup C$.

20 The Venn diagram below shows the number of students in a class who study various subjects.



M = {students who study Music}

P = {students who study Physics}

D = {students who study Drama}

Find the number of students who:

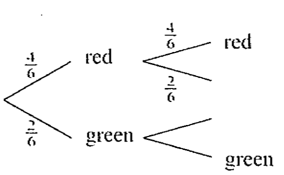
a study physics

b study Music or Drama but not both

c study Physics but not Music

d do not study Drama.

21 A die with 4 red faces and 2 green faces is rolled twice. Complete the following tree diagram to illustrate the possible outcomes.



Use the tree diagram to determine the probability of rolling:

a two reds

b two greens

c one of each colour.

22 Events A and B have the following probabilities.

$P(A) = 0.35$ $P(B) = 0.7$ $P(A \cup B) = 0.8$

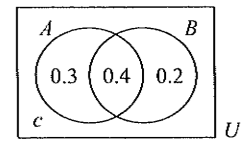
a Calculate $P(A \cap B)$.

b Represent this information on a Venn diagram.

c Find $P(A' \cap B')$.

d State, with a reason, whether events A and B are independent.

23 The following Venn Diagram shows the probabilities between two events A and B.



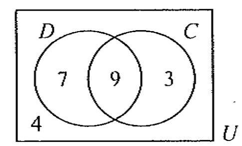
a Find the value of the probability c marked on the diagram.

b Use the Venn Diagram to determine:

i $P(A \cup B)$ **ii** $P(A \cap B)$ **iii** $P(A' \cap B)$

iv $P(A \mid B)$ **v** $P(B \mid A)$

24 The Venn Diagram below shows the number of students in a class that have dogs (D) and/or cats (C) for pets.



Determine the number of students:

a in the class

b who have both cats and dogs **c** who only have dogs

d who have at least one of the pets **e** who have no pets.

25 The table shows the number of different types of chocolates in a packet.

	soft	medium	hard
dark	10	24	15
light	18	14	9

a One chocolate is chosen at random. Find the probability that the chocolate is:

i dark or hard **ii** light, given that it is soft.

b If two chocolates had been chosen at random, find the probability that:

i both are medium **ii** at least one is soft and light.

TOPIC 3 – SETS, LOGIC AND PROBABILITY (LONG QUESTIONS)

1 Let: $U = \{x \in \mathbb{N} \mid x \leqslant 10\}$ $P = \{\text{multiples of } 2\}$

$Q = \{\text{multiples of } 3\}$ $R = \{\text{factors of } 12\}$.

a List the elements of:

i P **ii** Q **iii** R **iv** $P \cap Q \cap R$

b **i** Draw a Venn diagram to show the relationship between sets P, Q, and R.

ii Write the elements of U in the appropriate place on the Venn diagram.

c Describe in words the sets:

i $P \cup Q$ **ii** $P' \cap Q' \cap R'$

d Let p, q, and r be the statements: p: x is a multiple of 2

q: x is a multiple of 3

r: x is a factor of 12.

i On your Venn diagram in **b i** shade the region corresponding to $p \veebar q$

ii Use a truth table to find the values of $(p \wedge r) \Rightarrow (p \veebar q)$

Begin by writing the first three columns of your truth table in the following format:

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

iii Write down an element of U for which $(p \wedge r) \Rightarrow (p \veebar q)$ is true.

2 Consider the two propositions p and q and the compound proposition $\neg(p \wedge q) \Rightarrow \neg p \wedge \neg q$.

a **i** Copy the truth table below and complete the last four columns.

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$	$\neg(p \wedge q) \Rightarrow \neg p \wedge \neg q$
T	T	F	F				
T	F	F	T				
F	T	T	F				
F	F	T	T				

ii Is the above result a tautology, a contradiction, or neither?

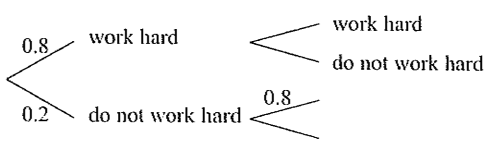
b Consider the two propositions: p: I work hard.
q: I get a promotion.

i Write in words the conjunction of propositions p and q.

ii Write in words the inverse of $p \Rightarrow q$.

c At the TULCO telecommunications company the probability of a worker working hard is 0.8.

Consider two randomly selected workers. A partially completed tree diagram is shown below.



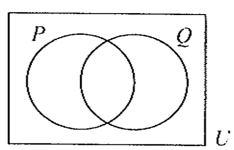
i Copy and complete the above tree diagram.

ii Use the tree diagram to find the probability that for two randomly selected workers:

a they both work hard

b only one works hard.

3 Consider two sets P and Q. The Venn diagram shows the relationship between them.



a On separate Venn Diagrams shade the regions corresponding to:

i $p \veebar q$ **ii** $\neg(p \wedge q)$ **iii** $\neg p \vee \neg q$.

b By referring to **a ii** and **a iii** explain whether $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

c Let: p: The bush has thorns.
q: The bush is a rose bush.

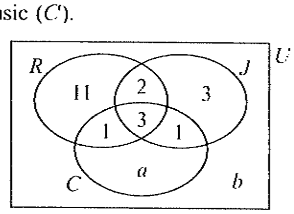
Write a sentence using only words for the following proposition: $\neg(p \wedge q) \Rightarrow \neg q$

d Construct a truth table for the compound proposition in **c**. Begin by writing the first two columns of your truth table in the following format. Comment on your result.

p	q
T	T
T	F
F	T
F	F

4 The Venn diagram below shows the number of students out of a class of 30 senior students who like Rock music (R), Jazz music (J) and Classical music (C).

17 like Rock music.
9 like Jazz, and
7 like Classical music.



a **i** Find the number of students in the region labelled a.

ii Describe in words the region containing 11 students.

iii Find the number of students in the region labelled b.

iv Describe region b in words.

b Draw a sketch of the Venn diagram and shade in the region $R' \cap J$.

c A student is chosen at random from this class. Determine the probability that this student:

i likes all three types of music

ii likes only Classical music

iii likes Rock music given that the student likes Jazz music.

d Two students are randomly selected from this class. Determine the probability that:

i both students like Rock music only

ii one student likes Rock only and the other likes all three types.

5 80 students were asked what type of television programme they had watched the previous evening.

35 watched Sport (S) 42 watched News (N)
50 watched Drama (D) 10 watched all three types
7 watched Sport and News only
12 watched News and Drama only
14 watched Sport and Drama only.

a **i** Draw a Venn diagram to illustrate the relationship between the three types of television programme watched.

ii On your Venn diagram indicate the number of students that belong to each region.

iii Determine the number of students who watch neither Sport nor Drama nor News.

b A student is selected at random. Determine the probability that the student:

i watched only Drama

ii watched Sport given that the student watched News.

c Let p and q be the propositions:

p: You watch sport on television.
q: You like sport.

i Write in words the following proposition: $p \Rightarrow q$.

ii The converse of the proposition $p \Rightarrow q$ is $q \Rightarrow p$. Write this proposition in words.

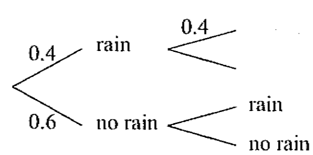
iii Consider the following proposition:

"If you do not like sport then you do not watch sport on television."

Express this statement using symbols only.

iv Is the proposition in **c iii** the inverse, the converse or the contrapositive of the proposition **c i**?

6 The probability of rain falling on any day in Dunedin is 0.4. The tree diagram below shows the possible outcomes when two consecutive days are considered.



a Copy and complete the tree diagram by filling in the boxes.

b Use the tree diagram to determine the probability of:

i rain on both days **ii** no rain on one day.

c Find the probability of the weather being fine for five consecutive days.

d Consider the propositions:

p: It is raining. q: I wear my raincoat.

Write the following propositions using words only.

i $p \Rightarrow q$ **ii** $\neg q \Rightarrow \neg p$

e Consider the compound proposition $\neg(p \wedge q) \Rightarrow \neg p$.

i Construct a truth table for this argument. Include headings of:

p q $p \wedge q$ $\neg(p \wedge q)$ $\neg p$ $\neg(p \wedge q) \Rightarrow \neg p$.

ii Give an example for when this argument is false.

7 **a** Two marbles are drawn from bag A containing 8 red and 5 blue marbles without replacement.

i Draw a tree diagram to represent the sample space.

ii Use the tree diagram to determine the probability of obtaining:

a two marbles that are the same colour

b at least one blue marble.

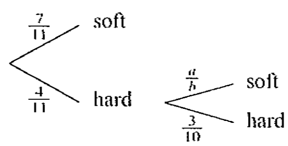
b A second bag B contains 7 red and 4 blue marbles. A five sided spinner with the numbers 1, 2, 3, 4, 5 on it is used to select from which bag the two marbles are taken. If an even number is spun then bag A is chosen and if an odd number is spun, bag B is selected. Draw a new tree diagram to represent all the possibilities and use the tree diagram to determine the probability of obtaining:

i an even number on the spinner

ii two blue marbles.

8 A box of chocolates contains 4 hard and 7 soft centres. One chocolate is selected randomly and eaten. A second chocolate is selected and eaten only if the first chocolate has a hard centre.

The tree diagram below represents the possible outcomes for this event.



a **i** Find the probability of choosing 2 chocolates.

ii Find the values of a and b.

iii Find the probability that both chocolates are hard centred.

iv Find the probability of selecting one of each type.

b A second box of chocolates contains all soft centres flavoured either with strawberry or cherry. There are 5 strawberry and 7 cherry flavoured chocolates. One chocolate is selected at random, eaten, then another is selected.

i Draw a tree diagram to represent all possible outcomes.

ii Use the tree diagram from **b i** to determine the probability that:

a both chocolates are strawberry

b the second chocolate is strawberry.

iii A child who does not like cherry flavoured chocolates randomly selects a chocolate from a new identical box, tries it and discards it if it is cherry flavoured. The child then selects another, repeating the process until a strawberry flavoured chocolate is found.

Determine the probability that the child selects a total of 4 chocolates.

TOPIC 4 **FUNCTIONS**

SUMMARY

DOMAIN AND RANGE

- The **domain** of a function is the set of all possible values that x may take.
- The **range** of a function is the set of all possible values that y takes.

FUNCTIONS AS MAPPINGS

A function f maps elements x of its domain to elements $y = f(x)$ of its range. This is written as $f : x \mapsto f(x)$.

Example:

$f : x \mapsto 3 - 4x$ maps the number x to the number $y = 3 - 4x$.

As a particular case, $-2 \mapsto 3 - 4(-2) = 11$

Here f maps the value -2 in its domain to 11 in its range.

Note: The function f is often written as $y = f(x)$, so the function in the example could be written as $y = 3 - 4x$ or $f(x) = 3 - 4x$.

MAPPING DIAGRAMS

A mapping diagram is a useful way of showing how elements are mapped from the domain to the range.

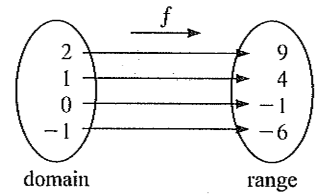
Example

Draw a mapping diagram for the function $f : x \mapsto 5x - 1$ with domain $\{x \in \mathbb{Z} \mid -1 \leqslant x \leqslant 2\}$.

Solution:

$f(-1) = 5(-1) - 1 = -6$ $\qquad f(0) = 5(0) - 1 = -1$

$f(1) = 5(1) - 1 = 4$ $\qquad f(2) = 5(2) - 1 = 9$



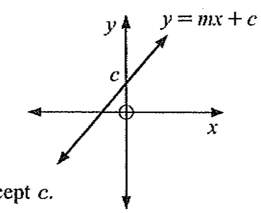
In this case, elements of the domain $\{-1, 0, 1, 2\}$ are mapped to elements of the range $\{-6, -1, 4, 9\}$.

FUNCTION TYPES

▶ **Linear**

A **linear function** is a function of the form $f : x \mapsto mx + c$ or $f(x) = mx + c$, where both m and c are constants.

The graph of a linear function $f(x) = mx + c$ is a straight line with gradient m and y-intercept c.



Example

Find the equation of the line passing through the points A(1, 5) and B(3, −3).

Solution:

$$\text{Gradient, } m = \frac{5 - -3}{1 - 3} = -4$$

Substituting (1, 5) in $y = mx + c$

gives $5 = -4 \times 1 + c$

$\therefore$ $5 = -4 + c$ and so $c = 9$

Equation is $y = -4x + 9$ {gradient-intercept form}

or, using $Ax + By = Ax_1 + By_1$

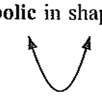
$4x + y = 4(1) + (5)$

$\therefore$ $4x + y = 9$

$\therefore$ $4x + y - 9 = 0$ {general form}

▶ **Quadratic**

- A **quadratic function** has form $f : x \mapsto ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$ where $a \neq 0$, and a, b and c are constants.
- The graph of a quadratic function is **parabolic** in shape.
- For $a > 0$, the graph opens upwards.



For $a < 0$, the graph opens downwards.

- Quadratic graphs have a vertical **axis of symmetry**, $x = \dfrac{-b}{2a}$.

Note: The axis of symmetry is midway between the x-intercepts.