Review of Exponential Relations

Integrated Math 2

Concepts to Know – From Video Notes/ HW & Lesson Notes

- Zero and Integer Exponents
- Exponent Laws
- Scientific Notation
- Analyzing Data Sets (M&M Lab & HW/video HW)
- Word Problems – Modeling with Exponential Relations
- Graphing Exponential Graphs with & without Calculator
- Solving Exponential Equations

Key formulas

Basic Formula with RATIOS: $y = C(a)^x$

Basic Formula with RATES: $y = C(1 + r)^x$

Compound Interest Formula: $FV = PV\left(1 + \frac{r}{n}\right)^{nt}$

Doubling Formula: $y = C(2)^{\frac{t}{D}}$

Half Life Formula: $y = C\left(\frac{1}{2}\right)^{\frac{t}{H}}$

Equations from Data Sets

Determine an equation to model these data sets

X	0	1	2	3	4	5	6
y	5	10	20	40	80	160	320

X	4	5	6	7	8	9	10
y	5	10	20	40	80	160	320

Modeling Example #1

- The following data table shows the relationship between the time (in hours after a rain storm in Manila) and the number of bacteria (#/mL of water) in water samples from the Pasig River:

Time (hours)	# of Bacteria
0	100
1	196
2	395
3	806
4	1570
5	3154
6	6215
7	12600
8	25300

Modeling Example #1

- (a) Graph the data on a scatter plot
- (b) How do you know the data is exponential rather than quadratic?
- (c) How can you analyze the numeric data (no graphs) to conclude that the data is exponential?
- (d) Write an equation to model the data. Define your variables carefully.

Modeling Example #2

- The value of Mr S car is depreciating over time. I bought the car new in 2002 and the value of my car (in thousands) over the years has been tabulated below:

Year	200 2	200 3	200 4	200 5	200 6	200 7	200 8	200 9	201 0
Value	40	36	32.4	29.2	26.2	23.6	21.3	19.1	17.2

- (a) Graph the data on a scatter plot
- (b) How do you know the data is exponential rather than quadratic?
- (c) How can you analyze the numeric data (no graphs) to conclude that the data is exponential?
- (d) Write an equation to model the data. Define your variables carefully.

Modeling Example #3

- The following data table shows the historic world population since 1950:

Year	1950	1960	197 0	1980	1990	1995	2000	2005	2010
Pop (in millions)	2.56	3.04	3.71	4.45	5.29	5.780	6.09	6.47	6.85

- (a) Graph the data on a scatter plot
- (b) How do you know the data is exponential rather than quadratic?
- (c) How can you analyze the numeric data (no graphs) to conclude that the data is exponential?
- (d) Write an equation to model the data. Define your variables carefully.

Review of Exponent Laws

- product of powers: $3^4 \times 3^6$
- $3^4 \times 3^6 = 3^{4+6}$ ➔ add exponents if bases are **equal**

- quotient of powers: $3^9 \div 3^2$
- $3^9 \div 3^2 = 3^{9-2}$ ➔ subtract exponents if bases are **equal**

- power of a power: $(3^2)^4$
- $(3^2)^4 = 3^{2 \times 4}$ ➔ multiply powers

- power of a product: $(3 \times a)^5$
- $(3 \times a)^5 = 3^5 \times a^5 = 243a^5$ ➔ distribute the exponent

- power of a quotient: $(a/3)^5$
- $(a/3)^5 = a^5 \div 3^5 = a^5/243$ ➔ distribute the exponent

Examples

- ex 1. Simplify the following expressions:
- (i) $(3a^2b)(-2a^3b^2)$
- (ii) $(2m^3)^4$
- (iii) $(-4p^3q^2)^3$

- ex 2. Simplify $(6x^5y^3/8y^4)^2$

- ex 3. Simplify $(-6x^{-2}y)(-9x^{-5}y^{-2})$ / $(3x^2y^{-4})$ and express answer with positive exponents

- ex 4. Evaluate the following
- (i) $(3/4)^{-2}$
- (ii) $(-6)^0$ / (2^{-3})
- (iii) $(2^{-4} + 2^{-6})$ / (2^{-3})

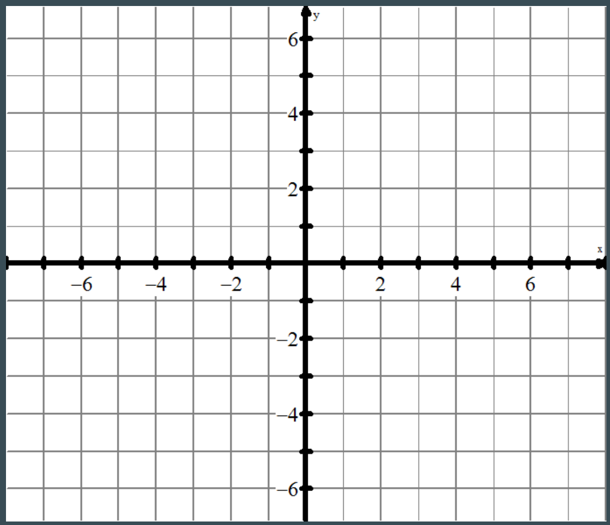
Scientific Notation

- http://www.kutasoftware.com/FreeWorksheets/Alg1Worksheets/Writing%20Scientific%20Notation.pdf

Graphs of Exponential Relations

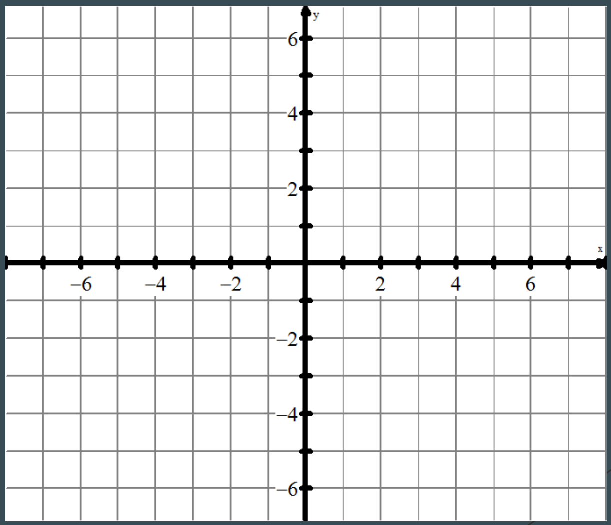
Graph $y = -(2)^x + 3$

Label three points & the asymptote & state range



Graph $y = 5(0.5)^x - 2$

Label three points & the asymptote & state range



(E) Solving Strategies – Algebraic Solution #1

- This prior observation sets up our general equation solving strategy => get both sides of an equation expressed in the same base

- ex. Solve and verify the following:

- (a) $(\frac{1}{2})^x = 4^{2-x}$ (b) $3^{y+2} = 1/27$
- (c) $(1/16)^{2a-3} = (1/4)^{a+3}$ (d) $3^{2x} = 81$
- (e) $5^{2x-1} = 1/125$ (f) $36^{2x+4} = \sqrt{(1296^x)}$

Compound Interest

(a) $4000 borrowed for 4 years at 3%/a, compounded annually	**(b) $7500 invested for 6 years at 6%/a, compounded monthly**
(c) $15 000 borrowed for 5 years at 2.4%/a, compounded quarterly	(d) $28 200 invested for 10 years at 5.5%/a, compounded semiannually
(e) $850 financed for 1 year at 3.65%/a, compounded daily	(f) $2225 invested for 47 weeks at 5.2%/a, compounded weekly

Working with Exponential Models

- Populations can also grow exponentially according to the formula $P = P_o(1.0125)^n$. If a population of 4,000,000 people grows according to this formula, determine:
- 1. the population after 5 years
- 2. the population after 12.25 years
- 3. when will the population be 6,500,000
- 4. what is the average annual rate of increase of the population

Working with Exponential Models

- The value of a car depreciates according to the exponential equation $V(t) = 25{,}000(0.8)^t$, where t is time measured in years since the car's purchase. Determine:

- 1. the car's value after 5 years
- 2. the car's value after 7.5 years
- 3. when will the car's value be $8,000
- 4. what is the average annual rate of decrease of the car's value?

Examples with Applications

- Example 1 → Radioactive materials decay according to the formula $N(t) = N_0(1/2)^{t/h}$ where N_0 is the initial amount, t is the time, and h is the half-life of the chemical, and the (1/2) represents the decay factor. If Radon has a half life of 25 days, how long does it take a 200 mg sample to decay to 12.5 mg?

Examples with Applications

- Example 2 → A bacterial culture doubles in size every 25 minutes. If a population starts with 100 bacteria, then how long will it take the population to reach 2,000,000?

Examples with Applications

- ex 3. The half-life of radium-226 is 1620 a. Starting with a sample of 120 mg, after how many years is only 40 mg left?

- ex 4. Find the length of time required for an investment of $1000 to grow to $4,500 at a rate of 9% p.a. compounded quarterly.