## A. Lesson Context

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|---|---|---|---|
| BIG PICTURE of this UNIT: | • How can I analyze growth or decay patterns in data sets & contextual problems?<br>• How can I algebraically & graphically summarize growth or decay patterns?<br>• How can I compare & contrast linear and exponential models for growth and decay problems. | | |
| CONTEXT of this LESSON: | Where we've been<br><br>In Lessons 1,2,3, you generated & analyzed data from a variety of activities | Where we are<br><br>How do we work with equations that model growth & decay patterns | Where we are heading<br><br>How can I use equations that will help me make predictions about scenarios which feature exponential growth & decay? |

## (A) Lesson Objectives:

a. Write exponential equations to model real world applications that are specific to compound interest
b. Make predictions/extrapolations through numeric or algebraic analysis
c. Use multiple representations to solve the exponential equations that arise from real world applications

## (B) Compound vs Simple Interest

***Compound Interest***:

Ex. $1,000 earning 10% p.a. compounded annually

***Simple Interest***:

Ex. $1,000 earning 10% p.a simple interest

## *(C) COMPOUND INTEREST: Example to Investigate*

a. When my oldest son, Alexander, was born, my wife and I invested \$5,000 in an education fund for him. The education fund is earning 8% interest every year. You will develop an answer to my questions

   i. How much this investment is worth when Alexander starts university at the age of 19 years old?

   ii. When has the investment tripled its value?

Visualization: time line

Data table

| X (time in years) | Y (value in \$) |
|---|---|
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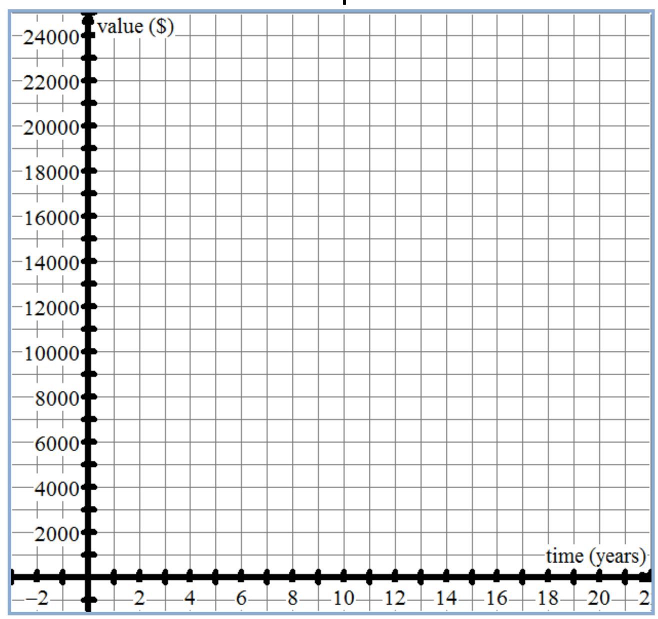
| | |
| | |

Eqn

Graph




(a) Value when Alex is 19 ➔ (write eqn)

(b) When has the investment tripled in value ➔ (write eqn)

(c) What assumption are you making ➔

## (D) Compound Interest Formula

b. When the interest is paid out at the END OF EVERY YEAR, we say that the compounding conditions are "compounded annually" at ***x***%/a.

c. In this case, our general formula for exponential equations is $Y = Y_0(b)^x$ will become "modified/rewritten/represented as $FV = PV(1 + r)^t$. The PARAMETERS (FV, PV, r, t) mean:

| FV ➔ | PV ➔ | r ➔ | t ➔ |
|---|---|---|---|
| | | | |

(E) Examples

d. Martina invests $5000 in a savings account that pays 5.25%/a, compounded annually. She does not make another deposit.
   i. Determine the amount in the account after 20 years.
   ii. Determine when the account has a balance of $12,500.

e. Natalie invests $18 000 at 8%/a, compounded annually.
   i. Determine the value of the investment after four years.
   ii. Find the interest earned at this time.

f. Determine the present value of an investment that will be worth $5000 in ten years. The interest rate is 4.8%/a, compounded annually.

## *(F) Examining Changes in the Compounding Conditions*

When interest is "paid" to the investor, it DOES NOT HAVE TO BE ANNUALLY!!!. What if an investor (like me) wants the interest paid MORE FREQUENTLY? How does this change the value of an investment?? How does it change the formula that I can use to predict future values?

Let's go back to my first example:

g. When my oldest son, Alexander, was born, my wife and I invested $5,000 in an education fund for him. The education fund is earning 8% interest every year. You will develop an answer to my questions
   i. How much this investment is worth when Alexander starts university at the age of 19 years old?
   ii. When has the investment tripled its value?

Now I will have 4 investment options that you will investigate:

OPTION A ➔ 8%/a compounded semi-annually

OPTION B ➔ 8%/a compounded quarterly

OPTION C ➔ 8%/a compounded monthly

OPTION D ➔ 8%/a compounded daily

Option you are investigating

Visualization: time line

Data table

| X (time in years) | Y (value in $) |
|---|---|
| | |
| | |
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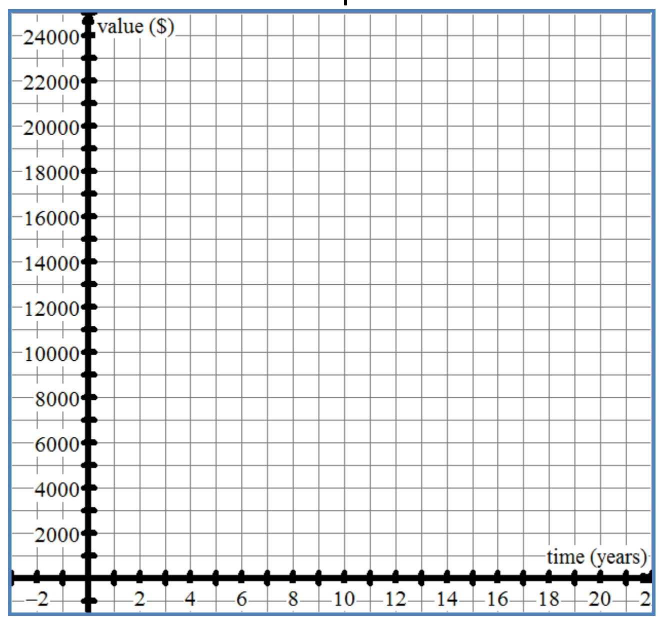
| | |

Eqn

Graph




(a) Value when Alex is 19 ➔ (write eqn)

(b) When has the investment tripled in value ➔ (write eqn)

(c) What assumption are you making ➔

## *(G)<u>Summary</u>*

h. Does the value of my investment for Alex change in value given the different compounding conditions? Any ideas as to WHY/WHY NOT?

i. Does the time taken to triple my investment change given the different compounding conditions? Any ideas as to WHY/WHY NOT?

j. Does the formula I use to predict future values change given the different compounding conditions?

## (H) Examples ➔ For each situation, determine: (i) the amount (value of the investment) (ii) the interest earned

(a) $4000 borrowed for 4 years at 3%/a, compounded annually

(b) $7500 invested for 6 years at 6%/a, compounded monthly

(c) $15 000 borrowed for 5 years at 2.4%/a, compounded quarterly

(d) $28 200 invested for 10 years at 5.5%/a, compounded semi-annually

(e) $850 financed for 1 year at 3.65%/a, compounded daily

(f) $2225 invested for 47 weeks at 5.2%/a, compounded weekly

## (I) Internet Resources

a. Video #1: from PatrickJMT ➔ http://www.youtube.com/watch?v=B3IdfBcXrLA
b. Video #2: from PatrickJMT ➔ http://www.youtube.com/watch?v=3vN-6DA79N0

c. Reading & Examples From PurpleMath ➔ http://www.purplemath.com/modules/expofcns4.htm

## (J) Homework

d. From the Nelson 11 Textbook, Chap 1.8, p 70, Q7,8,9,10,11