IM2 Unit 4 Lesson 4: Exponent Laws

Exponent Laws

Definition of the terms in an exponential equation: $b^x = p$

- b is the base (of the exponent)
- x is the exponent
- p is the power (the result of repeatedly multiplying b by itself, x number of times, or a base raised to an exponent)

Example: In $2^3 = 8$, the base is 2, the exponent is 3 and the power is 8. This can be read as the following:

- "Two cubed is 8."
- "Two to the exponent 3 is 8."
- "Two to the 3 is 8."
- "Eight is the third power of 2."
- BUT it CANNOT be read as: "Two to the power 3 is 8." (The power is NOT 3 - the power is 8 and the EXPONENT is 3!)

EXPONENT LAWS (Part 1):

1. Exponent of zero: Any base raised to an exponent of zero (or the zeroeth power of any base) is ALWAYS equal to one.
   - $b^0 = 1$
   - One exception is 0^0; this is a non-unique or **indeterminate** value.

Examples:

2. Negative exponent: When a base is raised to a negative exponent, reciprocate the base and raise the result to the positive exponent.
   - $b^{-x} = \frac{1}{b^x}$, $b \neq 0$ (why can't b equal zero?)

Examples:

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3. Multiplication of like bases:

When multiplying (2 or more) like bases, keep the base and ADD the exponents.

- $b^x \cdot b^y = b^{x+y}$

4. Division of like bases:

When dividing like bases, keep the base and SUBTRACT the exponents.

- $\frac{b^x}{b^y} = b^{x-y}$ (as long as $b \neq 0$)

5. Power of a product:

If a single term is being raised to an exponent, then the exponent applies to each *factor* of the single term.

- $(ab)^x = a^x b^x$

6. Power of a quotient:

If a fraction is being raised to an exponent, then the exponent applies to both the numerator and the denominator of the fraction.

- $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}, b \neq 0$ $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}, \ b \neq 0$

  (why can't b equal zero?)

7. Power of a power:

When a power (such as b^x) is being raised to another (outer) exponent, the result is called a power of a power. In this case, keep the base and multiply exponents.

- $(b^x)^y = b^{xy}$

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Exercises:

1. Identify the parts of an exponential equation. State the base, the exponent and the power for each.

   a) $(-4)^3 = -64$ c) $e^2 = p$ e) $27^{\frac{1}{3}} = 3$

   b) $2^{-5} = \frac{1}{32}$ d) $j^0 = 1$ f) $16^{\frac{1}{2}} = 4$

2. Use the exponent laws to write each expression with a single, simplified base.

   a) $x^4 \cdot x^5 \cdot x^9$ c) $\frac{x^{12}}{x^4}$ e) $\frac{a}{a^{-5}}$ g) $\frac{(k^a)^b \cdot k^{3ab}}{k^{7ab}}$

   b) $x^4 \cdot x^{-5}$ d) $\frac{a^{10}}{a^{14}}$ f) $(g^7)^{20}$ h) $\left(\sqrt{x}\right)^6$

3. Evaluate (simplify as a number) the following.

   a) -3^2 f) $\left(\frac{-2}{5}\right)^2$ k) $(3x)^2$

   b) $(-3)^2$ g) $\left(\frac{-2}{5}\right)^{-2}$ l) $(3x^3)^2$

   c) -3^{-2} h) $\left[\left(\frac{-2}{5}\right)^{-2}\right]^{-1}$ m) 2^{3^2}

   d) $(-3)^{-2}$ i) $-\left(\frac{-2}{5}\right)^2$ n) $\left(100^{\frac{1}{2}} - 36^{\frac{1}{2}}\right)^2$

   e) $(3^{-2} + 3^{-3})^{-1}$ j) $\left(\frac{-2}{5}\right)^3$ o) $\left(\frac{2y^{-1}}{3x}\right)^2$

Name____________________

More Properties of Exponents

Date________________ Period____

Simplify. Your answer should contain only positive exponents.

1) $(x^{-2}x^{-3})^4$

2) $(x^4)^{-3} \cdot 2x^4$

3) $(n^3)^3 \cdot 2n^{-1}$

4) $(2v)^2 \cdot 2v^2$

5) $\dfrac{2x^2y^4 \cdot 4x^2y^4 \cdot 3x}{3x^{-3}y^2}$

6) $\dfrac{2y^3 \cdot 3xy^3}{3x^2y^4}$

7) $\dfrac{x^3y^3 \cdot x^3}{4x^2}$

8) $\dfrac{3x^2y^2}{2x^{-1} \cdot 4yx^2}$

9) $\dfrac{x}{(2x^0)^2}$

10) $\dfrac{2m^{-4}}{(2m^{-4})^3}$

11) $\dfrac{(2m^2)^{-1}}{m^2}$

12) $\dfrac{2x^3}{(x^{-1})^3}$

13) $(a^{-3}b^{-3})^0$

14) $x^4y^3 \cdot (2y^2)^0$

15) $ba^4 \cdot (2ba^4)^{-3}$

16) $(2x^0y^2)^{-3} \cdot 2yx^3$

17) $\dfrac{2k^3 \cdot k^2}{k^{-3}}$

18) $\dfrac{(x^{-3})^4x^4}{2x^{-3}}$

19) $\dfrac{(2x)^{-4}}{x^{-1} \cdot x}$

20) $\dfrac{(2x^3z^2)^3}{x^3y^4z^2 \cdot x^{-4}z^3}$

21) $\dfrac{(2pm^{-1}q^0)^{-4} \cdot 2m^{-1}p^3}{2pq^2}$

22) $\dfrac{(2hj^2k^{-2} \cdot h^4j^{-1}k^4)^0}{2h^{-3}j^{-4}k^{-2}}$

Name: ____________________________________ Date: __________________

Zero and Negative Exponents
Algebra 1 Homework

Skills

For problems 1 through 36, rewrite without zero or negative exponents.

1. $4^{-3} =$
2. $-5^{-2} =$
3. $5^{0} =$
4. $10^{-2} =$
5. $-4^{-3} =$
6. $2^{-4} =$
7. $\dfrac{1}{2^{-2}} =$
8. $\dfrac{1}{4^{0}} =$
9. $(-3)^{-2} =$
10. $3x^{0} =$
11. $5x^{-4} =$
12. $\dfrac{x^{5}}{y^{-3}} =$
13. $\dfrac{a^{-4}}{b^{-3}} =$
14. $-2x^{0}y^{-2} =$
15. $2^{-3} =$
16. $\left(16x^{2}y^{-5}\right)^{0} =$
17. $-3^{0} =$
18. $8x^{0}y^{-3} =$
19. $(-3)^{-3} =$
20. $\left(\dfrac{1}{2}\right)^{-1} =$
21. $\left(\dfrac{1}{2}\right)^{-2} =$
22. $\left(\dfrac{1}{3}\right)^{-1} =$
23. $1^{-6} =$
24. $(-5)^{0} =$
25. $(-1)^{-2} =$
26. $-2^{-1} =$
27. $(-2)^{-1} =$
28. $(-2)^{-2} =$
29. $\left(-2^{-2}\right)^{-1} =$
30. $\dfrac{2x^{-3}y^{2}}{4x^{-4}y^{-1}} =$
31. $a^{3}b^{-4} =$
32. $\dfrac{a^{-2}}{b^{4}} =$
33. $\dfrac{x^{2}}{2y^{-3}} =$
34. $\dfrac{-3x^{3}}{y^{-4}} =$
35. $\dfrac{x^{0}y^{-3}}{z^{2}} =$
36. $2x^{-1}y^{-4} =$

Use the STORE feature on your calculator to help evaluate the following.

37. y^{-3} for $y = 2$
38. y^{-3} for $y = \dfrac{1}{2}$
39. $2x^{-4}y^{-1}$ for $x = 2,\ y = \dfrac{1}{3}$
40. $(x+3)^{-2}$ for $x = -4$
41. x^{-y} for $x = -2, y = 2$
42. $\left(x^{4}y^{2}\right)^{0}$ for $x = \dfrac{4}{3}, y = -\dfrac{2}{7}$
43. $x^{y}x^{-y}$ for $x = \dfrac{2}{5}, y = -\dfrac{4}{3}$

Reasoning

Fill in the missing $\square$ for each of the following.

44. $\frac{1}{9} = 3^{\square}$

45. $4^{-2} = \frac{1}{\square}$

46. $\frac{1}{25} = \square^{-2}$

47. $\frac{\square}{2} = 2^{-1}$

48. $6^{-2} = \frac{1}{\square}$

49. $10^{\square} = \frac{1}{10,000}$

50. $\frac{1}{81} = 3^{\square}$

51. $\frac{1}{64} = 4^{\square}$

Write the answer to each of the following as a single number.

52. $[-1+(5+2)^0]^3 =$

53. $\left[\frac{1}{2}+(3-1)^{-1}\right]^2 =$

54. $\left[3^{-1}+\frac{8}{3}\right]^{-3} =$

55. Evaluate each of the following products:

(a) $2^3 \cdot 2^{-3} =$

(b) $5^2 \cdot 5^{-2} =$

(c) $10^{-4} \cdot 10^4 =$

(d) $x^a \cdot x^{-a} =$

56. Which of the following is correct?

(a) $2x^{-3} = \frac{1}{2x^3}$

(b) $2x^{-3} = \frac{2}{x^3}$

Explain why the other choice is incorrect.

True or False

57. $\left(\frac{1}{2}\right)^{-1} = 2$

58. $\left(\frac{4}{3}\right)^{-1} = -\frac{4}{3}$

59. $(-2)^{-2} = \frac{1}{4}$

60. $\frac{-2x^{-3}y^2}{a^3x^2} = \frac{-2y^2}{a^3x^5}$

61. $3^7 \cdot 3^{-4} = 27$

62. $\left(a^{-2}\right)^{-3} = \frac{1}{a^6}$

63. $(-4)^0 = 0$

64. $2^{-3} \cdot 2^3 \cdot 2^0 = 2$

65. $\frac{x^2y^{-1}}{x^{-3}y^2} = \frac{x^5}{y^3}$

Find the value of x that makes each statement true.

66. $2^x \cdot 2^4 = 2^{12}$

67. $5^{-2} \cdot 5^x = 5^9$

68. $\left(4^x\right)^2 = 4^{10}$

69. $3^{x-2} = 27$

70. $\left(4^2 \cdot 3^{-2} \cdot 5^4\right)^x = 1$

71. $2^{2x+6} = \frac{1}{4}$