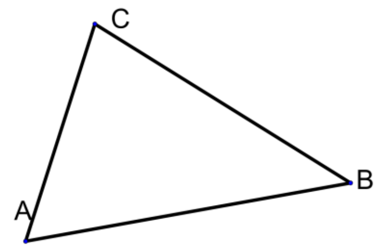
Lesson 40 – Sine Law & The Ambiguous Case

Integrated Math 10 - Santowski

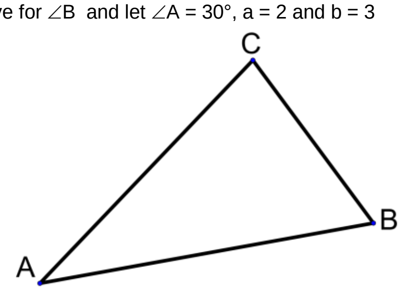
(A) Exploration #1

- Solve for ∠B and let ∠A = 30°, a = 3 and b = 2



(A) Exploration #2

- Solve for ∠B and let ∠A = 30°, a = 2 and b = 3



(A) Exploration #3

- Determine the measure of an angle whose sine ratio is 0.75
- Solve the equation sin(*x*) = 0.75 for *x*
- Solve the equation $x = \sin^{-1}(0.75)$
- What is the difference in meaning amongst these 3 questions??
- Explain why it is IMPOSSIBLE to solve $\sin^{-1}(1.25) = x$

Lesson Objectives

- Understand from a geometric perspective WHY the ambiguous case exists
- Understand how to identify algebraically that their will be 2 solutions to a given sine law question
- Solve the 2 triangles in the ambiguous case
- See that the sine ratio of a acute angle is equivalent to the sine ratio of its supplement

(A) Exploration #4 - Constructions

- Draw a long “baseline’ and label POINT A near one end
- Now, use this baseline to construct a triangle wherein:

[illegible]

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(A) Exploration #5 - Constructions

- Draw a long “baseline’ and label POINT A near one end
- Now, use this baseline to construct a triangle wherein:

[illegible]

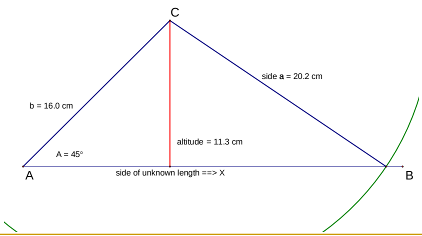
a = 2.5 cm	a = 5 cm	a = 7 cm	a = 10 cm	a = 13 cm

(B) Considerations with Sine Law

- If you are given information about non-right triangle and you know 2 angles and 1 side, then ONLY one triangle is possible and we never worry in these cases
- If you know 2 sides and 1 angle, then we have to consider this “ambiguous” case issue
  - If the side opposite the given angle IS THE LARGER of the 2 sides ➔ NO WORRIES
  - **If the side opposite the given angle IS THE SHORTER of the 2 sides ➔ ONLY NOW WILL WE CONSIDER THIS “ambiguous” case**
- WHY????

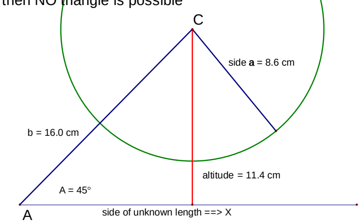
Case #1 – if a > b

If: (i) a > b, then ONE OBTUSE triangle is possible



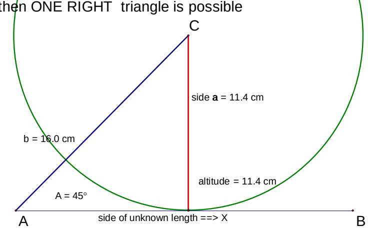
Case #2 - if a < b

If: (i) a < b AND
(ii) b sinA (altitude) > a,
then NO triangle is possible



Case #3 – if a < b

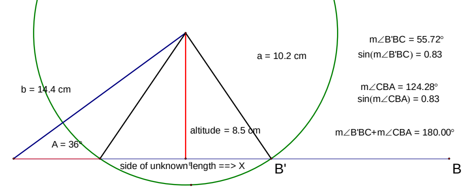
If: (i) a < b AND
(ii) b sinA (altitude) = a,
then ONE RIGHT triangle is possible



Case #4 – the Ambiguous Case

If: (i) a < b, AND
(ii) if bsinA < a
then TWO triangles are possible

sin(m∠CB'B) = 0.83
m∠CB'B = 55.72°



m∠B'BC = 55.72°
sin(m∠B'BC) = 0.83

m∠CBA = 124.28°
sin(m∠CBA) = 0.83

m∠B'BC+m∠CBA = 180.00°

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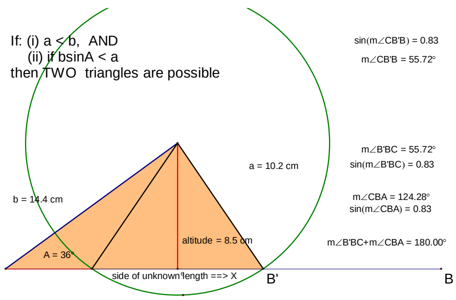

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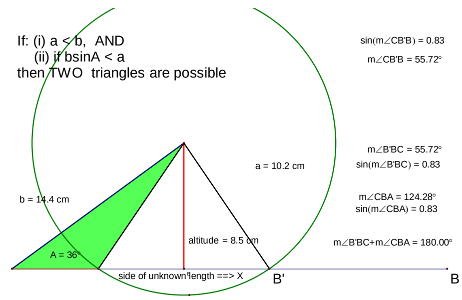
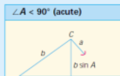
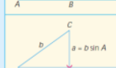
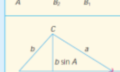


Summary

- Case 1 ➔ if we are given 2 angles and one side ➔ proceed using sine law **(ASA)**
- Case 2 ➔ if we are given 1 angle and 2 sides and the side opposite the given angle is LONGER ➔ proceed using sine law
- if we are given 1 angle and 2 sides and the side opposite the given angle is SHORTER ➔ proceed with the following "check list"
- Case 3 ➔if the product of "bsinA > a", NO triangle possible
- Case 4 ➔ if the product of "bsinA = a", ONE triangle
- Case 5 ➔ if the product of "bsinA < a" TWO triangles
- RECALL that "bsinA" represents the altitude of the triangle

Summary

∠A < 90° (acute)	Conditions	Number and Type of Triangles Possible
	$a < b \sin A$	no triangle
	$a = b \sin A$	one right triangle
	$b \sin A < a < b$	two triangles—one acute, one obtuse
	$a \geq b$	one triangle

Examples of Sine Law

- if ∠ A = 44º and ∠ B = 65º and b=7.7 find the missing information.

Examples of Sine Law

- if ∠ A =44.3º and a=11.5 and b=7.7 find the missing information.

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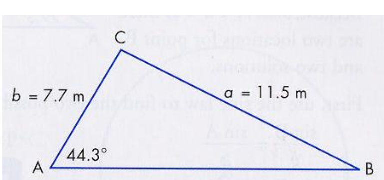
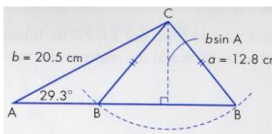


Examples of Sine Law

- if ∠ A=29.3º and a=12.8 and b = 20.5

Examples of Sine Law

- if ∠ A=29.3 and a=12.8 and b = 20.5
- All the other cases fail, because bsinA<a<b 10<a (12.8)<20.5, which is true.
- Then we have two triangles, solve for both angles



Examples of Sine Law

- Solve triangle *PQR* in which ∠ *P* = 63.5° and ∠ *Q* = 51.2° and *r* = 6.3 cm.

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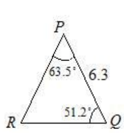

Examples of Sine Law

- ex. 1. In ΔABC, ∠ A = 42°, a = 10.2 cm and b = 8.5 cm, find the other angles
- ex. 2. Solve ΔABC if ∠ A = 37.7°, a = 30 cm, b = 42 cm

Examples of Sine Law

- ex. 1. In Δ ABC, ∠ A = 42°, a = 10.2 cm and b = 8.5 cm, find the other angles
- First test ➔side opposite the given angle is longer, so no need to consider the ambiguous case ➔ i.e. a > b ➔ therefore only one solution

- ex. 2. Solve Δ ABC if ∠ A = 37.7, a = 30 cm, b = 42 cm
- First test ➔ side opposite the given angle is shorter, so we need to consider the possibility of the "ambiguous case" ➔ a < b ➔ so there are either 0,1,2 possibilities.
- So second test is a calculation ➔ Here a (30) > b sin A (25.66), so there are two cases

Homework

- HW

- Nelson 11 Chapter 6.1, p511, Q5 & 6