(A) Lesson Objectives

a. Review algebra skills related to perfect square trinomials
b. Introduce the idea of "completing the square" and practice the skill when a = 1 and also when a $\neq$ 1
c. Extend the algebraic process of C/S to solving equations using the C/S method
d. Solve word problems using the C/S method to find max/min points

(B) Skills Review – Perfect Square Trinomials

(NOTE: a trinomial is an "expression" with three terms and in this lesson all trinomials will be quadratic in nature (as they have the standard form of $ax^2 + bx + c$)

Expand $(x + 3)^2$ Expand $(x - 2)^2$ Expand $(x + 5)^2$ Expand $(x + h)^2$

Factor $x^2 + 8x + 16$ Factor $x^2 - 6x + 9$ Factor $x^2 + 14x + 49$ Factor $x^2 + 2cx + c^2$

(a) What do we MEAN when we use the term "perfect square trinomial?

(b) Graph several "perfect square trinomials. What do you notice?

(c) What value does ***c*** have so that the trinomial is a "perfect square trinomial"?

$x^2 + 4x + \boldsymbol{c}$ $x^2 + 8x + \boldsymbol{c}$ $x^2 - 20x + \boldsymbol{c}$

(d) For what value of b (where b > 0) is the trinomial a "perfect square trinomial"?

$x^2 + \boldsymbol{b}x + 64$ $x^2 + \boldsymbol{b}x + 81$ $x^2 + \boldsymbol{b}x + 1$

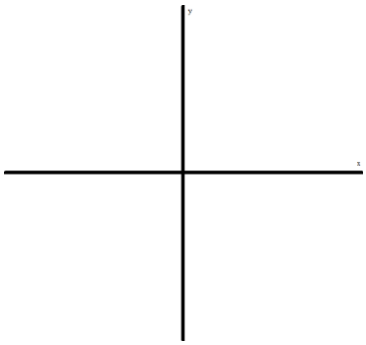
(C) Skill Application – Completing the Square ➔ a = 1

Example #1: Convert the equation $f(x) = x^2 + 8x + 15$ from standard form to vertex form.	**Why Did I Do That????**
$f(x) = x^2 + 8x + 15$	STEP 1A ➔ Why is there a +16 here now?
STEP 1: $f(x) = (x^2 + 8x + 16 - 16) + 15$ **1A** **1B**	STEP 1B ➔ Why is there a – 16 also included?
STEP 2: $f(x) = (x + 4)^2 - 16 + 15$ **2A**	STEP 2A ➔ Where did the $(x + 4)^2$ come from?
STEP 3: $f(x) = (x + 4)^2 - 1$ **3A**	STEP 3A ➔ Where did the – 1 come from?
Practice #1: Convert the equation $f(x) = x^2 - 10x + 15$ from standard form to vertex form.	Practice #2: Identify the transformations of $f(x) = x^2$ if the "new" equation is $f(x) = x^2 - 7x + 2$.
EXTENSION: Are there another ways that I can work out the vertex form of an equation if I am given the standard form (i.e ways that DON'T involve the completing the square method?	

(D)Skill Extension – Solving Equations Using the Square Root Method

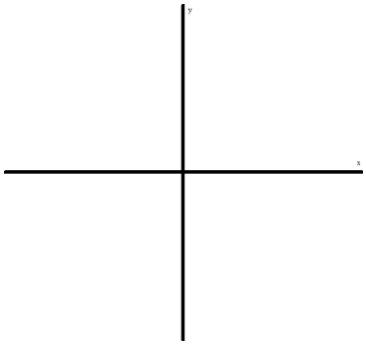
Example 1: Solve $(x-2)^2 - 4 = 0$ algebraically.

Verify the solution to $(x-2)^2 - 4 = 0$ graphically. Include a labelled sketch.



Example 2: Solve $x^2 - 12x + 27 = 0$ using the c/s method.

Verify the solution to $x^2 - 12x + 27 = 0$ graphically. Include a labelled sketch.



Is there another algebraic method that we could have used for Ex 2?

Ex 3: Solve $x^2 - 6x + 4 = 0$ using the c/s method.

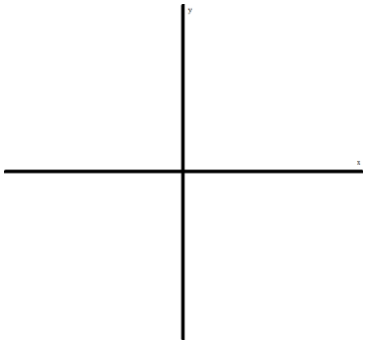
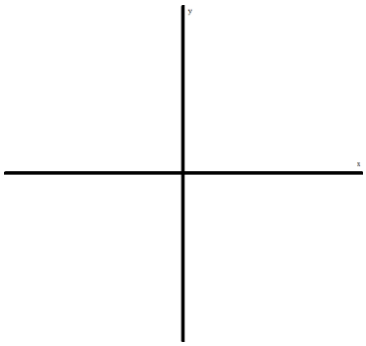
Ex 4: Solve $x^2 - 6x + 15 = 0$ using the c/s method

(E) Skill Application – Completing the Square ➔ a ≠ 1

Example #1: Convert the equation $f(x) = 2x^2 + 24x + 15$ from standard form to vertex form.	**Why Did I Do That????**
STEP 1: $f(x) = 2(x^2 + 12x) + 15$ **1A**	STEP 1A ➔ Where did the 2 & 12 come from?
STEP 2: $f(x) = 2(x^2 + 12x + 36 - 36) + 15$ **2A** **2B**	STEP 2A ➔ Why is there a +36 here now? STEP 2B ➔ Why is there a –36 also included?
STEP 3: $f(x) = 2(x^2 + 12x + 36) - 72 + 15$ **3A**	STEP 3A ➔ Where did the –72 come from?
STEP 3: $f(x) = 2(x + 6)^2 - 57$ **4A** **4B**	STEP 4A ➔ Where did the $(x + 6)^2$ come from? STEP 4B ➔ Where did the – 57 come from?

Practice #1: Convert the equation $f(x) = \frac{1}{2}x^2 - 2x + 3$ from standard form to vertex form.

(F) Skill Extension – Solving Equations Using the Square Root Method

Example 1: Solve $\frac{1}{4}(x-2)^2 - 4 = 0$ algebraically.	Verify the solution to $\frac{1}{4}(x-2)^2 - 4 = 0$ graphically. Include a labelled sketch. 
Example 2: Solve $3x^2 + 12x - 12 = 0$ using the c/s method.	Verify the solution to $3x^2 + 12x - 12 = 0$ graphically. Include a labelled sketch. 

(G) Applications of Quadratic Equations & Completing the Square ➔ Finding Optimal Values

a. A model rocket is fired directly upwards with an initial velocity of 300 m/s. Its height, in meters, can be modeled by the equation $h(t) = -5t^2 + 300t$, where t is time in seconds.

i. Use the C/S method to determine what maximum height the rocket reaches and at what time the rocket reaches its optimal height.

ii. There is another ALGEBRAIC method that you could have used to answer the previous question. Provide this alternate solution.

b. The sum of two numbers is 26 and the sum of their squares is a minimum. Find the 2 numbers.

(H) Homework:

from the Nelson 10 textbook, Chap 4.6, page 390, Q6abc, 8ae, 9af, 10, 19, 16