Modeling with Exponential Equations: **Lesson 16**

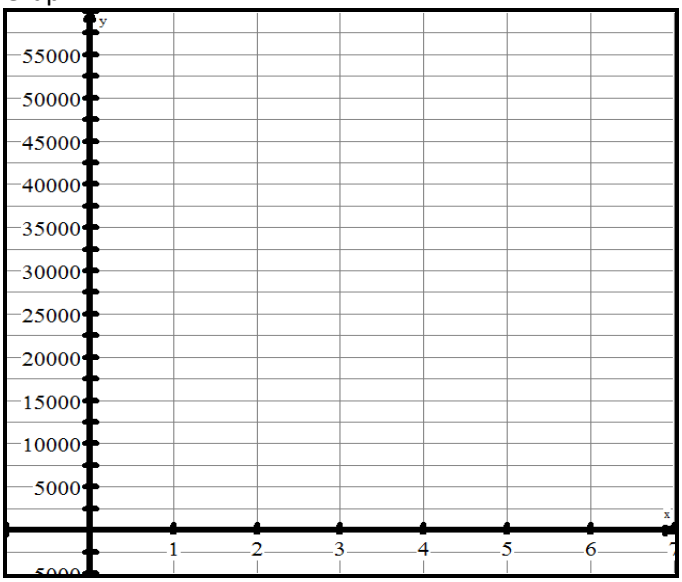
(A) Lesson Objectives:

a. Write exponential equations to model real world applications
b. Use multiple representations to solve the exponential equations that arise from real world applications

(B) Review ➔ An Exponential equation has the form $\mathbf{Y = Y_0 (b)_x}$ where $\mathbf{Y_0}$ = initial value, **b** is the growth factor/common ratio. (It turns out that **b** = **1 + r** , where **r** is the decimal value of % increase given).

(C) Opening Exploration ➔ Mr Santowski has been given a new job contract. He will earn $40,000 per year and get a 6% raise per year for the next 5 years

Graph:



DEFINE YOUR VARIABLES, then complete the tables

Data Table:

x						
y						

(a) Write an equation for Mr. S's salary.

(b) What does the y-intercept represent?

(c) I would like Mr. S's salary to be modelled with a linear relation. HOW would you change the original info so that a linear model can be used?

(d) What would my salary be in 8 years?

(e) After how many years would my salary be $70,000?

(f) What assumption are you making as you answer Qd,e?

(D) Examples – CLEARLY show your METHOD for getting the answers.

a. A colony of 1,000 ants can increase by 15% in a month.

   i. How many ants will be in the colony after 10 months?

   ii. How long will it take to get 7,500 ants in the colony?

b. A population of 10 hamsters will triple every year.

   i. What will be the population after 4 years?

   ii. How long will it take to get 1,500 hamsters?

   iii. Determine the WEEKLY growth rate for the hamsters.

c. A baby weighing 7 pounds at birth may increase in weight by 11% per month.

   i. How much will the baby weigh after 1 year?

   ii. When will the baby weigh 18 pounds?

   iii. Determine the approximate DAILY rate of growth for this infant.

d. A deposit of $1500 in an account pays interest 7.25% compounded annually.

   i. What is the account balance after 8 years?

   ii. When will the value of the account be double its original value?

(E) Examples – CLEARLY show your METHOD for getting the answers.

a. A colony of 100,000 ants is infected by a virus and decreases by 12% in a month.
   i. How many ants will be in the colony after 10 months?
   ii. How long will it take to get 25,000 ants in the colony?
   iii. Determine the DAILY death rate for the ant colony.

b. A sample of 100 g radioactive plutonium-238 has a half-life of 87.7 years, so it will exponentially decay every year.
   i. Determine the YEARLY decay rate for plutonium.
   ii. What amount will remain after 400 years?
   iii. How long will it take to eliminate 95% of the plutonium?

c. A balance of $150,000 in an account loses value at a rate of 3.25% annually.
   i. What is the account balance after 5 years?
   ii. When will the value of the account be half its original value?

(F) Homework Links:

a. http://mrsantowski.tripod.com/2011IntegratedMath2/HW/Nelson12ExponentialApps.pdf
b. http://mrsantowski.tripod.com/2011IntegratedMath2/HW/M11SB061.pdf

(G) COMPOUND INTEREST: Example to Investigate

a. When my oldest son, Alexander, was born, my wife and I invested $5,000 in an education fund for him. The education fund is earning 8% interest every year. You will develop an answer to my questions
   i. How much this investment is worth when Alexander starts university at the age of 19 years old?
   ii. When has the investment tripled its value?

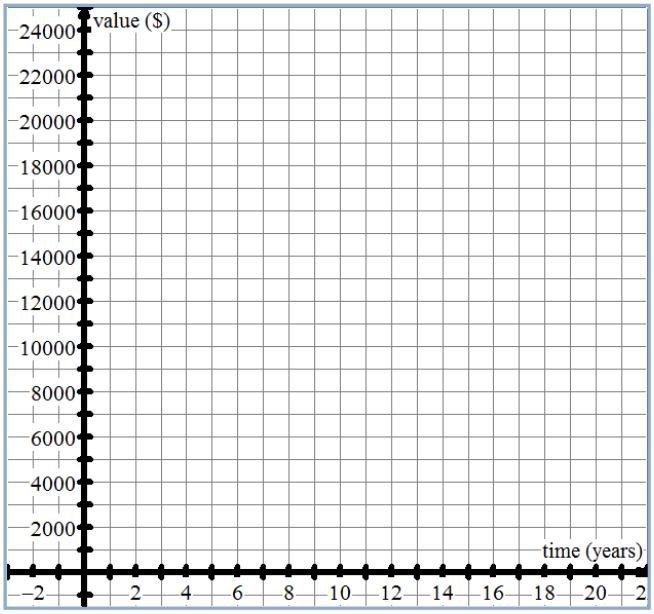
Visualization: time line

Data table

X (time in years)	Y (value in $)

Eqn

Graph



(a) Value when Alex is 19 ➔ (write eqn)

(b) When has the investment tripled in value ➔ (write eqn)

(c) What assumption are you making ➔

(H) Compound Interest Formula

b. When the interest is paid out at the END OF EVERY YEAR, we say that the compounding conditions are "compounded annually" at ***x***%/a.

c. In this case, our general formula for exponential equations is $Y = Y_0(b)^x$ will become "modified/rewritten/represented as $A = P(1 + r)^t$. The PARAMETERS (A, P, r, t) mean:

A ➔	P ➔	r ➔	t ➔

(I) Examples

d. Martina invests $5000 in a savings account that pays 5.25%/a, compounded annually. She does not make another deposit.
   i. Determine the amount in the account after 20 years.
   ii. Determine when the account has a balance of $12,500.

e. Natalie invests $18 000 at 8%/a, compounded annually.
   i. Determine the value of the investment after four years.
   ii. Find the interest earned at this time.

f. Determine the present value of an investment that will be worth $5000 in ten years. The interest rate is 4.8%/a, compounded annually.

(J) Examining Changes in the Compounding Conditions

When interest is "paid" to the investor, it DOES NOT HAVE TO BE ANNUALLY!!!. What if an investor (like me) wants the interest paid MORE FREQUENTLY? How does this change the value of an investment?? How does it change the formula that I can use to predict future values?

Let's go back to my first example:

g. When my oldest son, Alexander, was born, my wife and I invested $5,000 in an education fund for him. The education fund is earning 8% interest every year. You will develop an answer to my questions
   i. How much this investment is worth when Alexander starts university at the age of 19 years old?
   ii. When has the investment tripled its value?

Now I will have 4 investment options that you will investigate:

OPTION A ➔ 8%/a compounded semi-annually

OPTION B ➔ 8%/a compounded quarterly

OPTION C ➔ 8%/a compounded monthly

OPTION D ➔ 8%/a compounded daily

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Option you are investigating

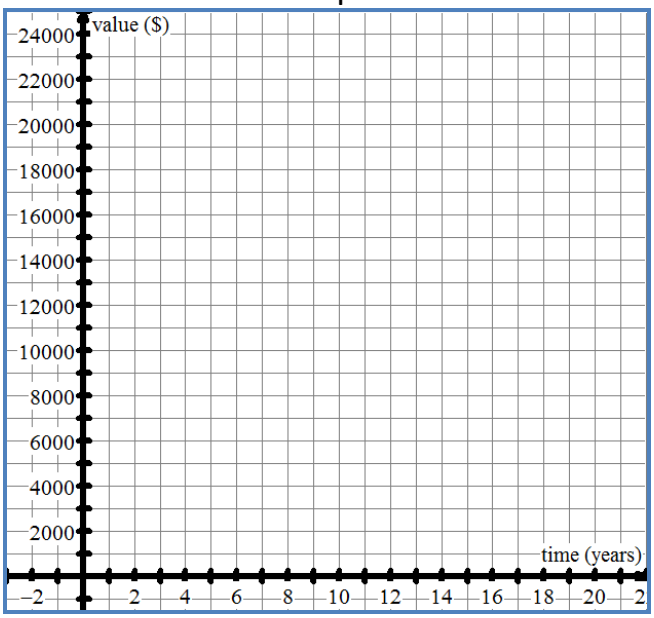
Visualization: time line

Data table

X (time in years)	Y (value in $)

Eqn

Graph



(a) Value when Alex is 19 ➔ (write eqn)

(b) When has the investment tripled in value ➔ (write eqn)

(c) What assumption are you making ➔

(K) Summary

h. Does the value of my investment for Alex change in value given the different compounding conditions? Any ideas as to WHY/WHY NOT?

i. Does the time taken to triple my investment change given the different compounding conditions? Any ideas as to WHY/WHY NOT?

j. Does the formula I use to predict future values change given the different compounding conditions?

(L) Examples ➔ For each situation, determine: (i) the amount (value of the investment) (ii) the interest earned

(a) $4000 borrowed for 4 years at 3%/a, compounded annually

(b) $7500 invested for 6 years at 6%/a, compounded monthly

(c) $15 000 borrowed for 5 years at 2.4%/a, compounded quarterly

(d) $28 200 invested for 10 years at 5.5%/a, compounded semi-annually

(e) $850 financed for 1 year at 3.65%/a, compounded daily

(f) $2225 invested for 47 weeks at 5.2%/a, compounded weekly

(M) Internet Resources

a. Video #1: from PatrickJMT ➔ http://www.youtube.com/watch?v=B3IdfBcXrLA
b. Video #2: from PatrickJMT ➔ http://www.youtube.com/watch?v=3vN-6DA79N0

c. Reading & Examples From PurpleMath ➔ http://www.purplemath.com/modules/expofcns4.htm

(N) Homework

d. From the Nelson 11 Textbook, S1.8,p 70, Q7,8,9,10,11