Analyzing Functions – Testing for Local Extrema – The First Derivative Test | Lesson 77

(A) Lesson Objectives

a. Introduce the concept of critical points and inflection points
b. Investigate the First Derivative Test
c. Apply the First Derivative Test to analyze functions

(B) New Term – Critical Numbers

a. A ***critical number*** is any ***x*** value in the graph of a function, f(x), at which we can draw a horizontal tangent line (at which the value of the derivative is 0; or where f'(x) = 0)
b. So a ***critical point*** is then obviously an ordered pair/point at which f'(x) = 0

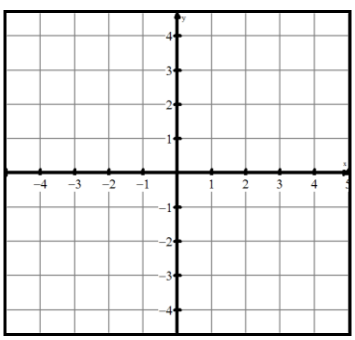
(C) Investigation: Are Critical Points the same as Extrema?

Example 1:

Use BOTH a graphic analysis and an algebraic analysis (calculus) to locate the critical points for $f(x) = 1 - x^2$

Algebraic:

Graphic:



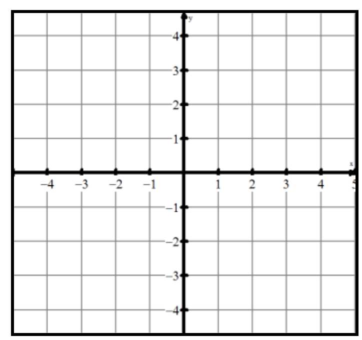
Conclusion:

Example 2:

Use BOTH a graphic analysis and an algebraic analysis (calculus) to locate the critical points for $f(x) = 1 - x^3$

Algebraic:

Graphic:



Conclusion:

New Concept: Inflection Points

(D) Investigating the First Derivative Test – Graphic & Algebraic

a. Use the function $f(x)=\frac{1}{5}x^5-\frac{4}{3}x^3+5$ and determine the critical points of $f(x)$.

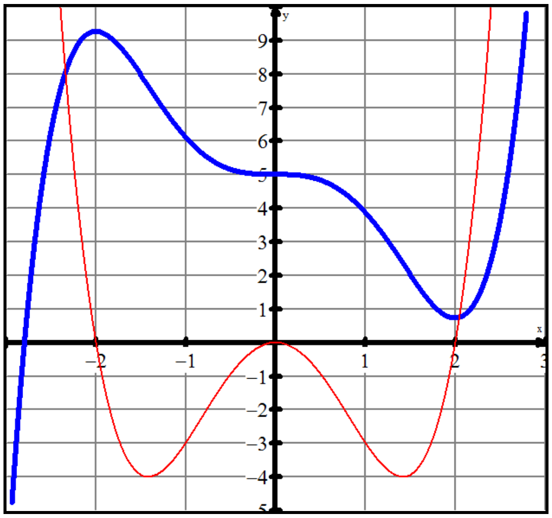
Algebraic:

For $f(x)=\frac{1}{5}x^5-\frac{4}{3}x^3+5$, solve $f'(x)=0$

Graphic:

$f(x)=\frac{1}{5}x^5-\frac{4}{3}x^3+5$ (blue)

$f'(x)=x^4-4x^2$ (red)



Analysis of the Method → Consider what is occurring around x = -2

(i) What type of critical point is at located at x = -2? ______________________________.

(ii) How would you describe the function behaviour BEFORE x = -2? ______________________________.

(iii) How would you describe the function behaviour AFTER x = -2? ______________________________.

(iv) What is true about the values of the derivative of f(x) BEFORE x = -2? ______________________________.

(v) What is true about the values of the derivative of f(x) AFTER x = -2? ______________________________.

(vi) ***CONCLUSION:*** A function has a ______________________________ if the FUNCTION BEHAVIOUR changes from ____________________ to ______________________________. In terms of the derivative, a function has a ____________________ if the DERIVATIVE changes from ______________ to ______________.

Analysis of the Method → Consider what is occurring around x = 2

(i) What type of critical point is at located at x = 2? ______________________________.

(ii) How would you describe the function behaviour BEFORE x = 2? ______________________________.

(iii) How would you describe the function behaviour AFTER x = 2? ______________________________.

(iv) What is true about the values of the derivative of f(x) BEFORE x = 2? ______________________________.

(v) What is true about the values of the derivative of f(x) AFTER x = 2? ______________________________.

(vi) ***CONCLUSION:*** A function has a ______________________________ if the FUNCTION BEHAVIOUR changes from ____________________ to ______________________________. In terms of the derivative, a function has a ____________________ if the DERIVATIVE changes from ______________ to ______________.

Analysis of the Method → Consider what is occurring around x = -0

(i) What type of critical point is at located at x = 0? ____.

(ii) How would you describe the function behaviour BEFORE x = 0? ____________________________________.

(iii) How would you describe the function behaviour AFTER x = 0? _____________________________________.

(iv) What is true about the values of the derivative of f(x) BEFORE x = 0? ______________________________.

(v) What is true about the values of the derivative of f(x) AFTER x = 0? _______________________________.

(vi) ***CONCLUSION:*** A function has a ______________________________________ if the FUNCTION BEHAVIOUR does ____. In terms of the derivative, a function has a ______________________ if the DERIVATIVE does __________________________________.

(E) Summary: The First Derivative Test (FDT)

The First Derivative Test tells us HOW to determine the ***NATURE*** of the critical points of a function. EXPLAIN the FDT in your own words.

(F) **PRACTICE**

Use the function $f(x) = x^3 - 6x^2 + 15$ and determine the critical points of $f(x)$ and then classify all extrema.

Algebraic:

For $f(x) = x^3 - 6x^2 + 15$, solve $f'(x) = 0$

Graphic:

$f(x) = x^3 - 6x^2 + 15$ (blue)

$f'(x) = x^4 - 4x^2$ (red)