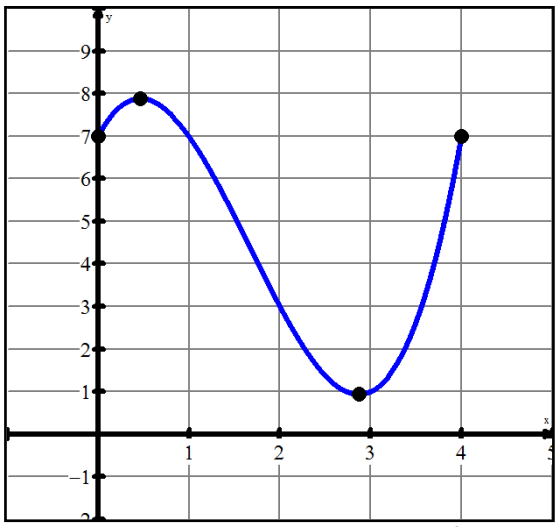
(A) Lesson Objectives

a. Understand what is meant by the terms increasing/decreasing as it relates to functions
b. Use graphic and algebraic methods to determine intervals of increase/decrease
c. Apply the concept of increasing/decreasing functions

(B) Examining the Concept – Increasing & Decreasing Functions

My sister is a published author and has modelled the sales of her new book using a polynomial function $S(m) = m^3 - 5m^2 + 4m + 7$ where S represents the monthly sales in millions of dollars in the first 4 months of the book's release and m represents the number of months since the book was first sold in stores (where $0 \leq m \leq 4$). A graph showing the monthly sales of her book is included:



a. The function is observed to be **DECREASING** for certain values of m.

   i. When do her book sales decrease?

   ii. How do you KNOW?

   iii. Express this interval using interval notation.

   iv. Why might the sale of her book be decreasing?

   v. How could we define what is meant by the term DECREASING FUNCTION?

b. The function is observed to be **INCREASING** for certain values of m

   i. When do her book sales increase?

   ii. How do you KNOW?

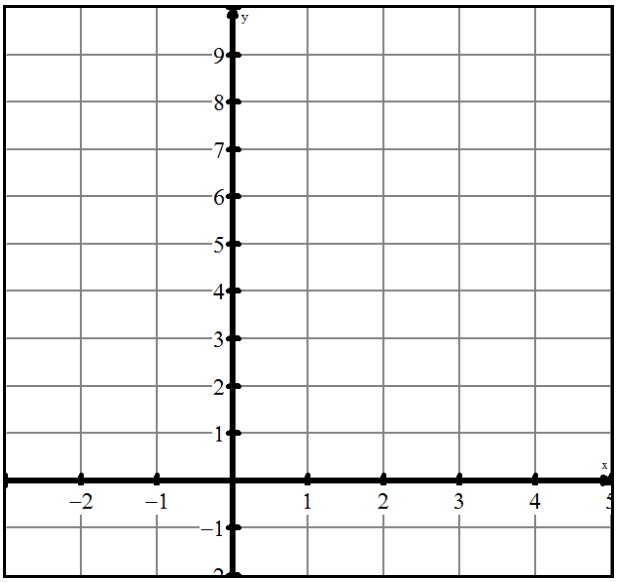
   iii. Express these intervals using interval notation.

   iv. Why might the sale of her book be increasing?

   v. How could we define what is meant by the term INCREASING FUNCTION?

(C) Using a Graphic Approach – Intervals of Increase & Decrease

Example 1: Determine the intervals in which the function $g(x) = x^2 - 2x + 3$ increases and decreases. Our analysis method will INITIALLY be the TI-84, so start by generating the graph on the TI-84.



Now, use the GRAPH to answer the questions:

(i) Where does g(x) increase?

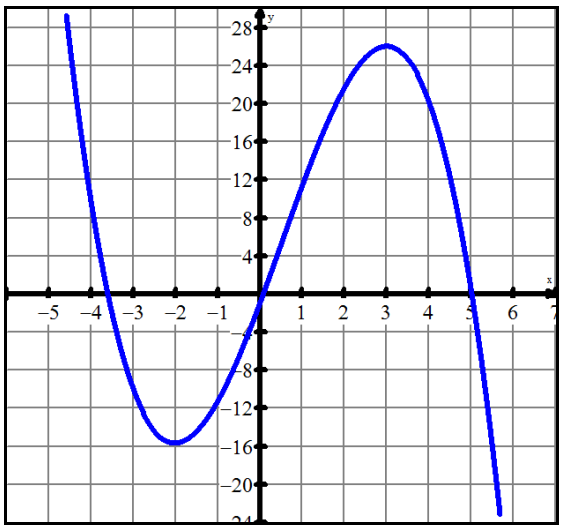
(ii) What KEY POINT has been important in helping you to determine your answer?

(iii) Where does g(x) decrease?

(iv) What KEY POINT has been important in helping you to determine your answer?

Example 2: Determine the intervals in which the function $f(x) = -\frac{2}{3}x^3 + x^2 + 12x - 1$ increases and decreases.

Our analysis method will INITIALLY be the TI-84, so start by generating the graph on the TI-84.



Now, use the GRAPH to answer the questions:

(i) Where does f(x) increase?

(ii) What KEY POINT(S) have been important in helping you to determine your answer?

(iii) Where does f(x) decrease?

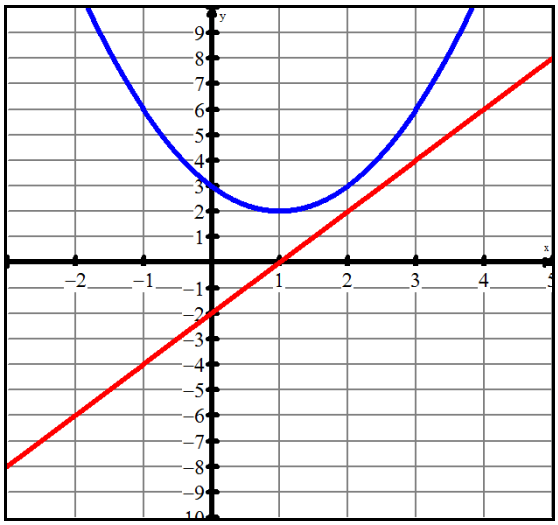
(iv) What KEY POINT(S) have been important in helping you to determine your answer?

CALCULUS CONNECTION:

(i) If we were to draw tangent lines to the curve of g(x) for x values where g(x) is INCREASING, what MUST be true about ALL of these tangent lines?

(ii) What must therefore be true about the DERIVATIVE of g(x) in this interval?

(iii) Here are graphs of g(x) (parabola) and g'(x) (the line). How could we use the graph of g'(x) to help us answer the original question?

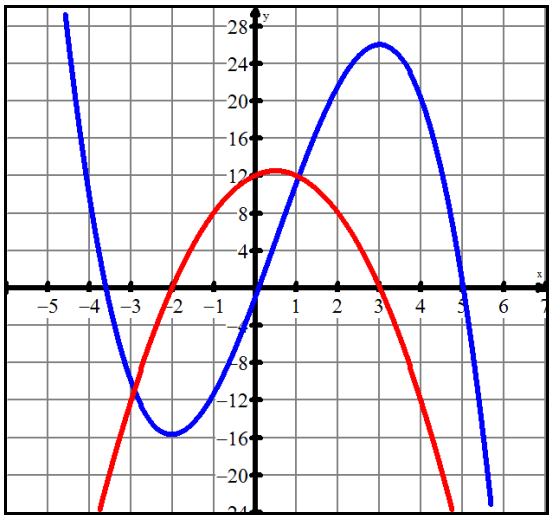


CALCULUS CONNECTION:

(i) If we were to draw tangent lines to the curve of f(x) for x values where f(x) is DECREASING, what MUST be true about ALL of these tangent lines?

(ii) What must therefore be true about the DERIVATIVE of f(x) in these intervals?

(iii) Here are graphs of f(x) (the cubic) and f'(x) (the parabola). How could we use the graph of f'(x) to help us answer the original question?



(D) Using an Algebraic Approach – Derivatives & Intervals of Increase/Decrease

Example 1: Determine the intervals in which the function $g(x) = x^2 - 2x + 3$ increases and decreases. Our analysis method will NOW BE CALCULUS & the derivative of g(x).

STEP #1: Start with your eqn: $g(x) = x^2 - 2x + 3$

STEP 2: Take derivative

WHY am I taking the derivative??

$g'(x) = 2x - 2$

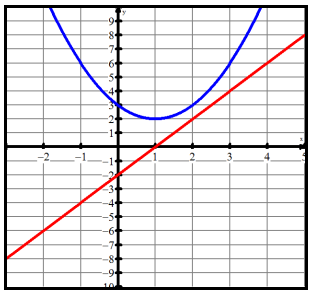
STEP 3a: Solve $g'(x) > 0$

WHAT is so special about $g'(x) = 0$ in the first place?

VISUAL REINFORCEMENT

$2x - 2 > 0$

$x > 1$



WHY am I solving for $g'(x)$ being greater than zero???

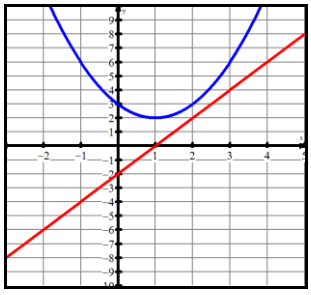
STEP 3b: Solve $g'(x) < 0$

WHY am I solving for $g'(x)$ being less than zero???

VISUAL REINFORCEMENT

$2x - 2 < 0$

$x < 1$



FINAL ANSWER:

(D)Using an Algebraic Approach – Derivatives & Intervals of Increase/Decrease

Example 1: Determine the intervals in which the function $f(x) = -\frac{2}{3}x^3 + x^2 + 12x - 1$ increases and decreases. Our analysis method will NOW BE CALCULUS & the derivative of g(x).

STEP #1: Start with your eqn: $f(x) = -\frac{2}{3}x^3 + x^2 + 12x - 1$

STEP 2A: Take derivative

WHY am I taking the derivative??

$$f'(x) = -2x^2 + 2x + 12$$

STEP 2B: Simplify derivative

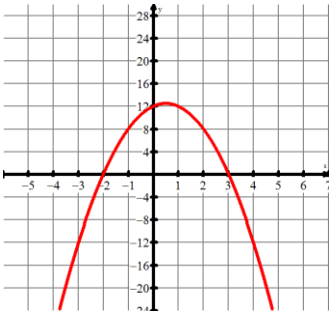
WHY am I rewriting the quadratic in factored form?

$$f'(x) = -2x^2 + 2x + 12$$
$$f'(x) = -2(x^2 - x - 6)$$
$$f'(x) = -2(x-3)(x+2)$$

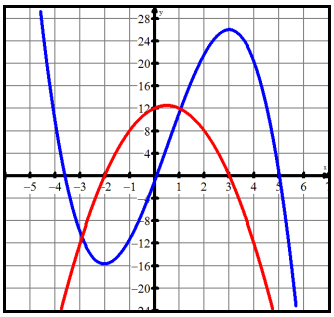
STEP 3A: Solve f'(x) > 0 (use graph rather than algebra)

WHAT is so special about f(x) = 0 in the first place?

VISUAL REINFORCEMENT



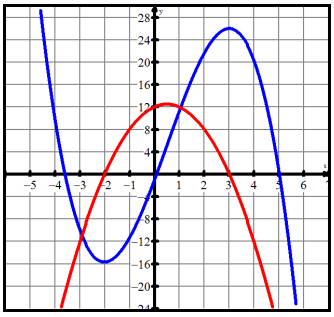
WHY am I solving for f'(x) being greater than zero???



STEP 3B: Solve f'(x) < 0 (use graph rather than algebra)

WHY am I solving for f'(x) being less than zero???

VISUAL REINFORCEMENT



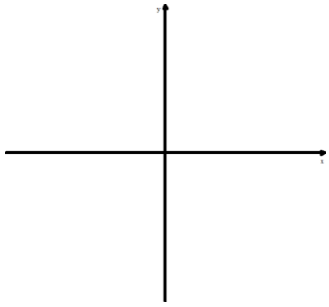
FINAL ANSWER:

(E) Practice the Skill

EXAMPLE 1: Use CALCULUS to determine the intervals of increase and decrease for the function $f(x) = -5x^2 – 20x + 3$

EXAMPLE 2: Use CALCULUS to determine the intervals of increase and decrease for the function $y = x^3 – 12x + 15$

Use the graphing calculator to verify your answer. Include a sketch.



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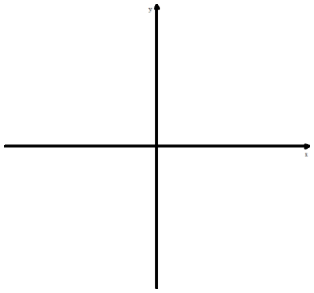

(F) Homework: From Nelson Advanced Functions & Calculus, Chap 4.1, p273, Q1cd, 3, 5bce, 6ef, 7deh, 9