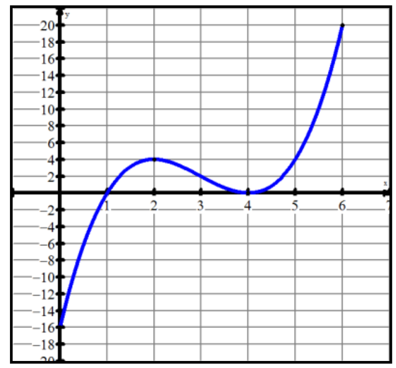
Analyzing Functions – Maximum & Minimum Points | Lesson 75

(A) Lesson Objectives

a. Understand what is meant by the term “extrema” as it relates to functions
b. Use graphic and algebraic methods to determine extrema of a function
c. Apply the concept of extrema of functions

(B) Examining the Concept – Extrema



My sister is a published author and has modelled the profits from the sales of her new book using a polynomial function $P(m) = m^3 - 9m^2 + 24m - 16$ where P represents the monthly profit in thousands of dollars in the first 6 months of the book’s release and m represents the number of months since the book was first sold (where $0 \leq m \leq 6$). A graph showing the monthly profit of her book is included.

a. Two **EXTREME VALUES** occur at the ENDPOINTS of the domain interval, at $m = 0$ and at $m = 6$.

i. Evaluate P(0) and interpret the meaning of this point in the context of the problem.

ii. The function value at $m = 0$ will be referred to now as the **ABSOLUTE MINIMUM VALUE** of the function on the domain of $0 \leq m \leq 6$. Explain what this idea of an ABSOLUTE minimum means.

iii. Evaluate P(6) and interpret the meaning of this point in the context of the problem.

iv. The function value at $m = 6$ will be referred to now as the **ABSOLUTE MAXIMUM VALUE** of the function on the domain of $0 \leq m \leq 6$. Explain what this idea of an absolute maximum means.

b. Two other **EXTREME VALUES** occur WITHIN the domain interval, at $m = 2$ and at $m = 4$.

i. Evaluate P(2) and interpret the meaning of this point in the context of the problem.

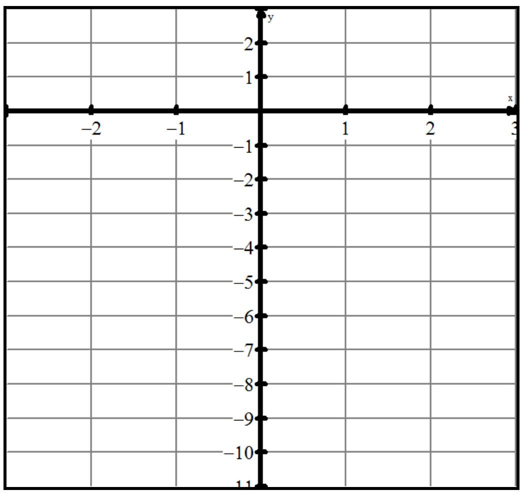
ii. The function value at $m = 2$ will be referred to now as the **LOCAL MAXIMUM VALUE** of the function on the domain of $0 \leq m \leq 6$. Explain what this idea of an LOCAL maximum means.

iii. Evaluate P(4) and interpret the meaning of this point in the context of the problem.

iv. The function value at $m = 4$ will be referred to now as the **LOCAL MINIMUM VALUE** of the function on the domain of $0 \leq m \leq 6$. Explain what this idea of an absolute minimum means.

(C) Using a Graphic Approach – Extrema and Extreme Values

Example 1: On $\{x \in R | -2 \leq x \leq 3\}$, determine the extreme values of the function g(x) = 2 – x – x^2. Our analysis method will INITIALLY be the TI-84, so start by generating the graph on the TI-84.



Now, use the GRAPH to answer the questions:

(i) Where does g(x) appear to have extreme values?

(ii) Use the TI 84 to determine the extreme points.

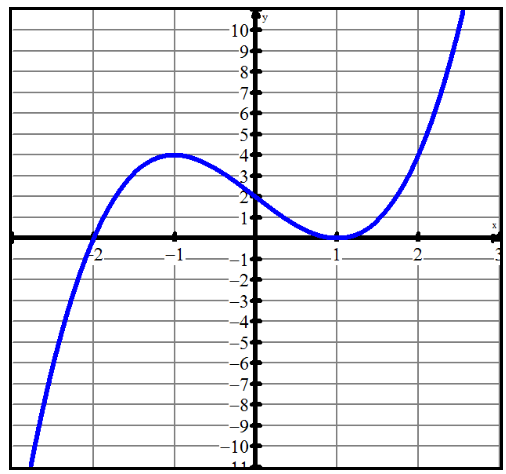
(iii) Classify the extreme values as local, absolute, maximum, minimum

CALCULUS CONNECTION:

(i) If we were to draw tangent lines to the curve of g(x) for x values where g(x) has LOCAL EXTREMA, what MUST be true about ALL of these tangent lines?

(ii) What must therefore be true about the DERIVATIVE of g(x) at these points?

Example 2: On $\{x \in R | -2.5 \leq x \leq 2.5\}$, determine the extreme values of the function $f(x) = x^3 - 3x + 2$. Our analysis method will INITIALLY be the TI-84, so start by generating the graph on the TI-84.



Now, use the GRAPH to answer the questions:

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CALCULUS CONNECTION:

(i) If we were to draw tangent lines to the curve of f(x) for x values where f(x) has LOCAL EXTREMA, what MUST be true about ALL of these tangent lines?

(ii) What must therefore be true about the DERIVATIVE of f(x) at these points?

(D)Using an Algebraic Approach – Extrema and Extreme Values

Example 1: Determine & classify the extreme values of the function g(x) = $x^2 + 6x + 7$ on $\{x \in R | -5 \leq x \leq 0\}$. Our analysis method will NOW BE CALCULUS & the derivative of g(x).

STEP 1: Start with your eqn: g(x) = $x^2 + 6x + 7$

STEP 2: Evaluate g(x) at the "endpoints"

WHY am I evaluating the function at x = -5 and x = 0?

g(-5) = $(-5)^2$ + 6(-5) + 7 = 2

g(-5) = $(0)^2$ + 6(0) + 7 = 7

STEP 2: Take derivative

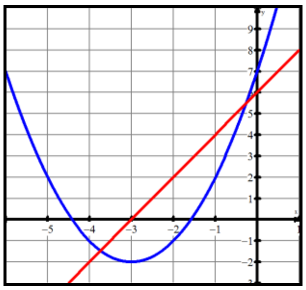
WHY am I taking the derivative??

g'(x) = 2x + 6

STEP 3: Solve g'(x) = 0

WHAT is so special about g'(x) = 0 in the first place?

VISUAL REINFORCEMENT



2x + 6 = 0

2x = -6

x = -3

FINAL ANSWER:

(D)Using an Algebraic Approach – Extrema and Extreme Values

Example 2: Determine & classify the extreme values of the function $f(x) = -\frac{4}{3}x^3 + 3x^2 + 4x + 1$ on $\{x \in R \mid -1 \le x \le 3\}$. Our analysis method will NOW BE CALCULUS & the derivative of f(x).

STEP 1: Evaluate f(x) at the "endpoints"

$$f(x) = -\frac{4}{3}(-1)^3 + 3(-1)^2 + 4(-1) + 1 = \frac{4}{3}$$

$$f(x) = -\frac{4}{3}(3)^3 + 3(3)^2 + 4(3) + 1 = 4$$

WHY am I evaluating the function at x = -1 and x = 3?

STEP 2A: Take derivative

$$f'(x) = -4x^2 + 6x + 4$$

WHY am I taking the derivative??

STEP 2B: Simplify derivative

$$f'(x) = -4x^2 + 6x + 4$$
$$f'(x) = -2(2x^2 - 3x - 2)$$
$$f'(x) = -2(2x+1)(x-2)$$

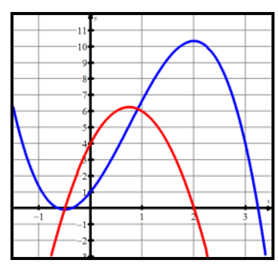
WHY am I rewriting the quadratic in factored form?

STEP 3: Solve f'(x) = 0

$$f'(x) = 0$$
$$0 = -2(2x+1)(x-2)$$
$$\therefore 2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$$
$$\therefore x - 2 = 0 \Rightarrow x = 2$$

WHAT is so special about f'(x) = 0 in the first place?

VISUAL REINFORCEMENT



STEP 2: Evaluate f(x) at the "extrema"

$$f(2) = -\frac{4}{3}(2)^3 + 3(2)^2 + 4(2) + 1 = 10\frac{1}{3}$$

$$f(-0.5) = -\frac{4}{3}\left(-\frac{1}{2}\right)^3 + 3\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 1 = -\frac{1}{12}$$

WHY am I evaluating the function at x = -½ and x = 2?

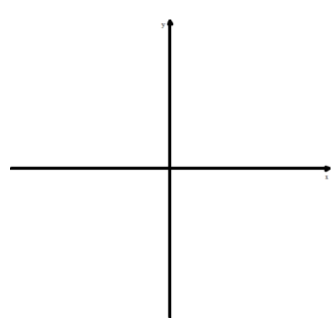
FINAL ANSWER:

(E) Practice the Skill

EXAMPLE 1: Use CALCULUS to determine & classify the extrema of f(x) = $-5x^2 - 20x + 3$ on $\{x \in R | -4 \le x \le 1\}$

EXAMPLE 2: Use CALCULUS to determine & classify the extrema of y = $2x^3 + 3x^2 - 12x$ on $\{x \in R | -2 \le x \le 2\}$

Use the graphing calculator to verify your answer. Include a sketch.



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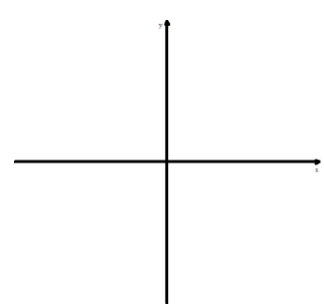

(F) Homework: From Nelson Advanced Functions & Calculus, Chap 4.2, p283, Q1, 3, 4, 6abcf, 8