(A) Example from Physics → Motion

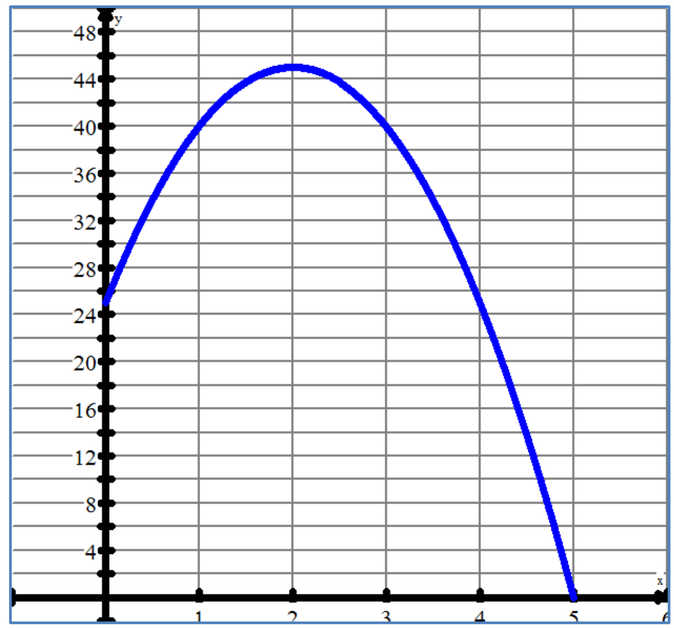
Here is a function showing how the height, in meters, travelled by an object → i.e. $y = h(t) = -5t^2 + 20t + 25$, where t is time in seconds.

Determine the following:

(1) the average rate of change between t = 0 and t = 4. Interpret and comment

(2) the average rate of change between t = 1 and t = 2. Interpret.

(3) the average rate of change between t = 3 and t = 5. Interpret and comment.



(4) Complete the following table of calculations

interval	$\Delta h(t)$	Δt	$\frac{\Delta h(t)}{\Delta t}$
$3 \le t \le 5$			
$3 \le t \le 4$			
$3 \le t \le 3.5$			
$3 \le t \le 3.1$			

interval	$\Delta h(t)$	Δt	$\frac{\Delta h(t)}{\Delta t}$
$1 \le t \le 3$			
$2 \le t \le 3$			
$2.5 \le t \le 3$			
$2.9 \le t \le 3$			

Conclusion → what would you predict the INSTANTANEOUS rate of change to be at $t = 3$ and WHY. Interpret the result

(B) Example from Biology

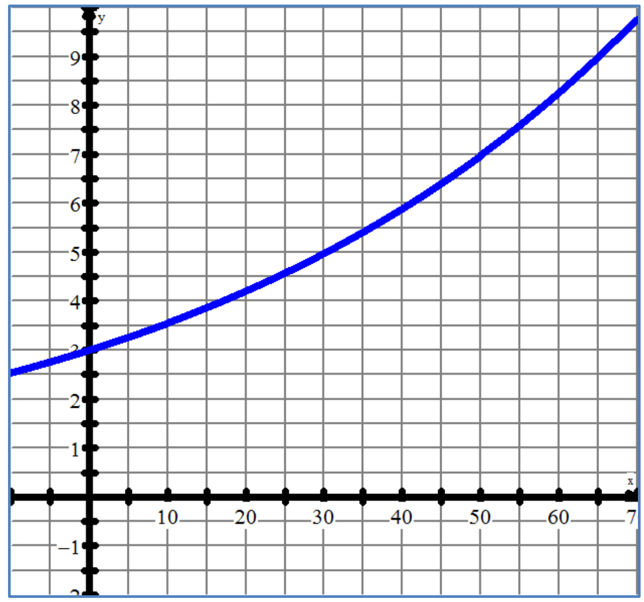
Here is a function modeling the world's population, P(t) in billions of people, in t years in 1960 → $P(t) = 3(1.017)^t$.

Determine the following:

(1) the average rate of change between t = 0 and t = 60. Interpret and comment

(2) the average rate of change between t = 10 and t = 60. Interpret.

(3) the average rate of change between t = 90 and t = 60. Interpret and comment.



(4) Complete the following table of calculations

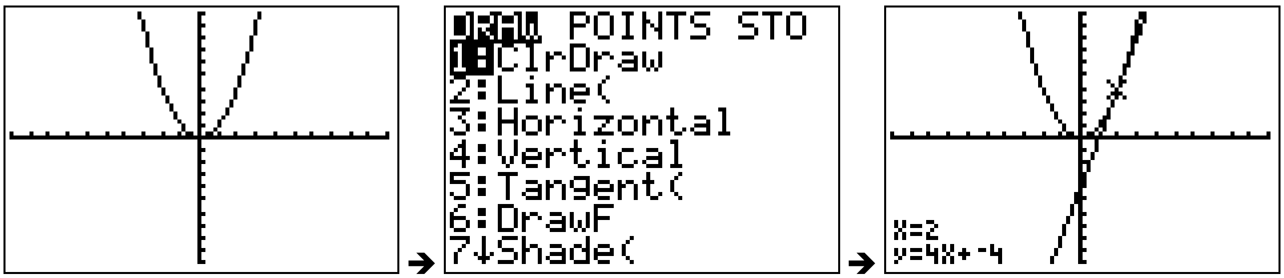
interval	$\Delta P(t)$	Δt	$\frac{\Delta P(t)}{\Delta t}$
$60 \le t \le 80$			
$60 \le t \le 70$			
$60 \le t \le 65$			
$60 \le t \le 61$			

interval	$\Delta P(t)$	Δt	$\frac{\Delta P(t)}{\Delta t}$
$40 \le t \le 60$			
$50 \le t \le 60$			
$55 \le t \le 60$			
$59 \le t \le 60$			

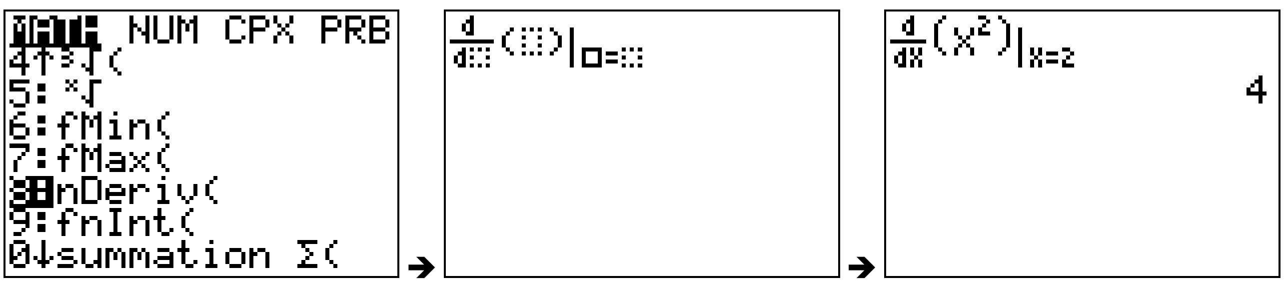
Conclusion → what would you predict the INSTANTANEOUS rate of change to be at *t* = 60 and WHY. Interpret the result

(C) An Easier Way to Predict Instantaneous? ➔ Using the GDC

a. Graphing Tangent Lines ➔ Determine the equation of the tangent line to the function f(x) = x^2 at the point (2,4).



b. Determine the instantaneous rate of change of the function f(x) = x^2 at the point (2,4) ➔ using the nderiv command on the TI-84



c. Practice:

i. For the function f(x) = $3x^2 - 4x + 5$, determine the equation of the tangent line at (1,4).

ii. Given the function f(x) = $6x - x^2$, determine the instantaneous rate of change at x = 4.

iii. A population of mice moves into Mr. S's house. At ***t*** months, the number of mice can be modelled by $P(t) = 100 + 30t + 4t^2$. Determine the rate at which the mouse population is changing is changing when the mouse population has doubled in size.

(D) An Easier Way to Predict Instantaneous ➔ Algebra ➔ Introducing the derivative at a point

a. Given the function $f(x) = x^2$, estimate the instantaneous rate of change at x = 2 by using the following ESTIMATIONS:

i. Using the points (2,f(2)) and (2.01, f(2.01)) ➔ in other words, between (2,4) and (2.01,4.0401)

ii. Using the points (2,f(2)) and (2+h, f(2 + h)) ➔

(E) Mathematical "formula" for a Derivative

a. The equation for a derivative of any function, ***f***(***x***) at the point ***x*** = ***a*** is:

$$f'(a) = \left.\frac{dy}{dx}\right|_{x=a} = \lim_{h\to 0}\left(\frac{f(a+h)-f(a)}{h}\right)$$

b. What does each part of the eqn MEAN??