Lesson 56 – Conditional Probability

(A) ***Opening Example #1:*** A survey of 500 adults asked about college expenses. The survey asked questions about whether or not the person had a child in college and about the cost of attending college. Results are shown in the table below.

	Cost Too Much	Cost Just Right	Cost Too Low
Child in College	0.30	0.13	0.01
Child not in College	0.20	0.25	0.11

Suppose one person is chosen at random:

a. ***Given that*** the person has a child in college, what is the probability that he or she ranks the cost of attending college as "cost too much"?

b. Find the probability that a person thinks college costs are just right ***given that*** they have a child in college.

c. Find the probability that the person thinks college costs are too low ***given that*** they do not have a child in college.

d. Find the probability that the person does not have a child in college ***given that*** they think college costs are too low.

(B) ***Opening Example #2:*** Take a card from a normal deck, without looking at it.

a. What is the probability you drew a spade?_____

b. Now assume that you peeked just a little and know that the card is black. What is the probability you drew a spade?_____

c. Assume you didn't peek but your friend did and tells you that the card is not a heart. What is the probability you drew a spade?_____

d. The proper notation for the last question is $P(\text{spade} \mid \text{not a heart})$. This means "the probability of a spade given that the card is not a heart."

e. The symbol for a *known* or *given* condition is a _____ and the given fact determines the _____of the probability.

(C) Definition

a. The conditional probability of an event B in relationship to an event A is the probability that event B occurs given that event A has already occurred.

b. Notation ➔ $P(B|A)$ is the conditional probability that B occurs given that you have knowledge that A has occurred.

c. FORMULA ➔

d. Independence ➔

(D)Conditional Probability & Tree Diagrams: Examples:

a. Ex 1. Isabel goes to school by one of two routes, A or B. The probability of going by route A is 30%. If she goes by route A, the probability of being late is 5% and if she goes by route B, the probability of being late is 10%

   i. Draw a tree diagram

   ii. Find the probability that Isabel is late for school.

   iii. Given that she is late for school, find the probability that she went to school using route A

b. Ex2. Five percent of my students suffer from a terrible malady called Lazybrain(LB). A blood test detects LB accurately 90% of the time. Yusuke is told that his blood test is positive for LB. Yusuke hopes that this is a "false positive" and he actually doesn't have Lazybrain.

   i. Draw the tree with all the probabilities. Indicate which branches are the *false positive, false negative, correct positive,* and *correct negative.*
   ii. Find the probability that Yusuke is OK even though his blood test was positive.

c. Ex 3. Widgets are made in Factories K and P. Factory K makes 60% of all widgets and Factory P makes the other 40%. K has a 20% defective rate and P has a 25% defective rate.

   i. What is the probability that a defective widget came from K?
   ii. What is the probability that a defective widget came from P?
   iii. Who has the lower defective rate?_____
   iv. Who is most likely to have made a defective widget?_____
   v. Explain the paradox_____

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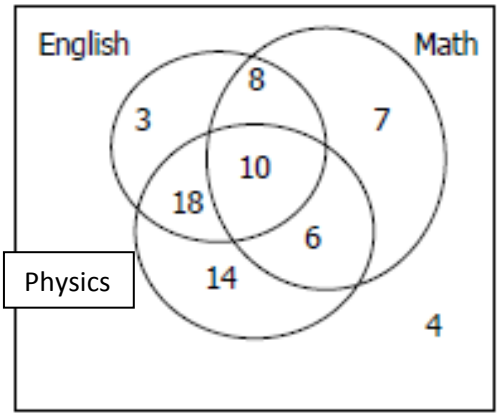
(E) Conditional Probability & Venn Diagrams: Examples

a. Use the Venn Diagram given at the right to find the following:



i. P(Physics |English)

ii. P(Math and English | Physics)

iii. P(English | Math or Physics)

b. Ex 1. A small school has 24 boys graduating. Half of them are funny and 7 are good dancers. Eight of them are neither funny nor good dancers. One boy is selected at random. Translate the following into conditional probability notation, then find the probabilities.

i. Probability he is a good dancer given that he is funny.
ii. Probability he is funny given that he is a good dancer.
iii. If he is a good dancer, what is the probability he is not funny?
iv. If he is not a good dancer, what is the probability he is funny?

(F) Conditional Probability & Formulas ➔ the formula that we can use is $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$

a. The probability that event A occurs is .63. The probability that event B occurs is .45. The probability that both events A and B occur is .18. Find the following:

i. $P(A|B)$
ii. $P(B|A)$
iii. Are the events independent?

3. $P(A) = 0.7, P(B) = 0.4, P(A \text{ I } B) = 0.25$

4. $P(A) = 0.45, P(B) = 0.8, P(A \text{ I } B) = 0.3$

5. $P(A) = 0.61, P(B) = 0.18, P(A \text{ I } B) = 0.07$

6. $P(A) = 0.2, P(B) = 0.5, P(A \text{ I } B) = 0.2$

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Conditional Probability Worksheet II

1. Suppose you throw a pair of fair 6-sided dice. One is white and the other is black. Let T=total showing on both dice, and B = number showing on the black die.

a) Find $P(T=5 \mid B=2)$ b) Find $P(B=2 \mid T=5)$

2. Jar A has 4 red and 5 black candies. Jar B has 6 red and 2 black candies. A fair die is rolled and jar A is selected if a number divisible by 3 comes up, otherwise, Jar B is selected. One candy is drawn from the jar.

a) What is the probability you selected Jar A and got a red candy?

b) What is the probability you selected Jar B and got a red candy?

c) What is the probability you got a red candy?

d) Suppose a red candy is drawn, what is the probability it came from jar A?

e) What is the probability Jar B was selected if a black candy is drawn?

3. Box X contains 2 red and 3 white marbles. Box Y contains 1 red and 3 white marbles. A marble is randomly chosen from Box X and put into box Y. A marble is then randomly chosen from box Y.

a) What is the probability that the marble chosen from box Y is red?

b) If the marble from Y is red, what is the probability that the marble moved from X was white?

4. In a school 65% of students like pork chops. Two students from the school are picked at random. Given that at least one of them likes pork chops, what is the probability that both like pork chops?

5. The probability of winning a game is 0.8. You play the game 3 times.

a) What is the probability that you win all 3 games?

b) What is the probability that you win at least once?

c) If you win at least once, what is the probability that you have 3 wins?

6. Two boxes contain coins. One box has 1 silver and 3 gold coins, the other box has 2 silver coins and 1 gold. A box is randomly chosen, and a coin is randomly selected. The coin is silver. If a second coin is selected from the same box, what is the probability that it will be silver also?

7. Donald, the quarter back, has 2 wide receivers. He throws to Goofy three out of five plays and Goofy drops the ball 90% of the time. Donald throws to Pluto two out of fie plays and Pluto is able to catch the ball 70% of the time.

a) Find the probability that the ball is dropped by either Goofy or Pluto.

b) If a ball is dropped, then the probability that it as Goofy who dropped it is_____

c) Given that the pass is caught, what is the probability that it was Goofy who caught it?

8. You have to travel from here to Grandma's house. Half the time you go by mule, 1/10 of the time you walk, and you fly 0.4 of the time. The mule has a 10% chance of completing the trip, while the plane has a 0.9 probability of completing the trip. You have a 1% chance of completing the trip by walking.

a) What is the probability you walk and do not complete the trip?

b) What is the probability that you make it to Grandma's house?

c) What is the probability that you took the mule given that you did not make it?

Answers:

1. a) $\frac{1}{6}$ b) $\frac{1}{4}$

2. a) 14.8% b) 50% c) 64.8% d) 22.9% e) 47.4%

3. a) $\frac{7}{25}$ b) $\frac{3}{7}$

4. 48.15%

5. a) 0.512 b) 0.992 c) 0.516

6. $\frac{4}{11}$

7. a) 0.66 b) 0.818 c) 0.176

8. a) 9.9% b) 41.1% c) 76.4%