Lesson 54 – Probability of Compound Events – Visualizations

(A) Overview

We will continue our investigation into probability by considering COMPOUND EVENTS:

Def'n:

(B) Probabilities to be Calculated – AND/OR

1. We can determine p(A) AND p(B) ➔

2. We can determine p(A) OR p(B) ➔

(C) HOW to calculate probabilities

1. **Algebraically** ➔ we can develop formulas to use in probability questions
2. **Visualizations** ➔ we can draw various representations that help us to see outcomes and sample spaces which include:
    i.
    ii.
    iii.
    iv.

(D) Classifying Compound Events:

To help us to eventually develop algebraic formulas for calculating probabilities of compound events, we need to understand that there are different TYPES of compound events:

1. Independent ➔
2. Dependent ➔
3. Mutually exclusive ➔

(E) Examples of Working with Visualizations and Compound Probability Calculations

<table>
<tr><td colspan="3">Ex 1. If two coins are tossed, what is the probability of getting exactly two heads?</td></tr>
<tr><td>Grid diagram:</td><td>Lists:</td><td>Tree Diagram:</td></tr>
<tr><td colspan="3">Ex 2. If three coins are tossed, what is the probability of getting exactly two heads?</td></tr>
<tr><td>Grid diagram:</td><td>Lists:</td><td>Tree Diagram:</td></tr>
<tr><td colspan="3">Ex 3. If three coins are tossed, what is the probability of getting at most two heads??</td></tr>
<tr><td colspan="2">Lists:</td><td>Tree Diagram:</td></tr>
</table>

(F) Examples of Working with Visualizations and Compound Probability Calculations

1. A box contains three green marbles (G), five blue marbles (B) and eight orange marbles (O). Use a tree diagram (or create lists) to help you visualize or analyze and then calculate the probabilities of the following events:

(a) that the first two picked will both be green if the first marble is ***replaced*** after its colour is noted.

(b) that the first two picked will both be green if the first marble is ***NOT replaced*** after its colour is noted.

(c) Should the answers for Q(a) and Q(b) be the same? Why or why not?

(d) Selecting a blue marble OR a green marble with your first pick.

(e) NOT selecting a blue marble with the first pick

(f) Selecting an orange OR green marble with the first pick

Now we will assume replacement and make 2 picks, so how probable is:

(g) Selecting a blue marble AND then a green marble.

(h) Selecting a green marble given that your first pick had been blue.

(i) Selecting at least one orange marble in your first two picks.

(j) NOT selecting an orange marble .

(k) Selecting an orange marble AND a blue marble.

(l) Selecting an orange marble OR a blue marble.

(m) Selecting a green marble given that your first marble was NOT blue.

(G) Examples of Working with Visualizations and Compound Probability Calculations

1. A shuffled deck of cards is placed face-down on the table. It contains 3 hearts (H), 4 diamonds (D), 5 clubs (C) and 2 spades (S). Use a tree diagram (or create lists) to help you visualize or analyze and then calculate the probabilities of the following events:

i. the top two cards are both spades.
ii. The top 2 cards are EITHER hearts OR spades.
iii. The top two cards are clubs AND diamonds.
iv. The top two cards are NOT red.
v. The top two cards are NEITHER spades NOR hearts.

Lesson 54 – Probability of Compound Events – Visualizations

FROM ➔ http://www.cwu.edu/~chueh/Math130WS8Solution.pdf

1. A couple plans to have two children. For each child it is equally likely to be a boy or a girl.
    a. Represent the sample space for this problem by a tree diagram.
    b. What is the probability that they end up with two boys?
    c. What is the probability that they end up with at least one boy?
    d. What is the probably that they end up with two boys, given that they end up with at least one boy.
    e. Is the event that they end up with two boys independent of the event that they end up with at least one boy? Explain with a probability equation.

2. The Mathematics Department has 7 professors available from which three will be chosen at random to teach sections 1, 2, and 3 of Math 130. No professor will teach more than one of the three sections. Five of these professors are male and two are female. We are interested in certain questions involving the gender of the two professors.
    a. Make a tree diagram representing the sample space for the experiment of selecting the three professors. Label the outcomes by letters and compute the probability for each outcome.
    b. What is the probability that exactly one female professor is chosen?
    c. What is the probability that at least one female professor is chosen?
    d. What is the probability that two female professors are chosen, given that at least one female professor is chosen?
    e. What is the probability that exactly two male professors are chosen, given that at least one female professor is chosen?

FROM ➔ http://www.ibmaths.com/free/ws/statsprob/conditional%20trees.pdf

3. A computer program generates random questions in arithmetic that children have to answer within a fixed time. The probability of the question being answered correctly is 0.8. Whenever a question is answered correctly, the next question generated is more difficult, and the probability of a correct answer being given is reduced by 0.1. Whenever a question is answered wrongly, the next question is of the same standard, and the probability of a correct answer remains unchanged.
    a. Find the probability that the second question is answered correctly. [2]
    b. Find the probability that the second question is answered correctly given that the third question is answered correctly. [4]

4. A box contains 20 chocolates, of which 15 have soft centres and 5 have hard centres. Two chocolates are taken at random, one after the other. Calculate the probability that,
    c. both chocolates have soft centres, [2]
    d. one of each chocolate is taken, [2]
    e. both chocolates have hard centres, given that the second chocolate has a hard centre. [4]

5. A student who is traveling by train through Europe decides not to buy a ticket for the train. The train is traveling through both France and Germany and as such he may be asked to show his ticket to either the French or the German train guard. The probability of being caught without a ticket by the German guard is 0.8, and the probability of getting caught by the French guard is 0.3. The probabilities of each are independent from one another.
    f. Draw a tree diagram to represent the information in the paragraph above. [3]
    g. Calculate the probability of being caught by both guards. [2]
    h. Given that the student has been caught by one of the guards only, find the probability that he was caught by,
        i. the French guard,
        ii. the German guard. [4]

6. A bus serving a number of outlying villages is due to arrive in a particular village at 10 o'clock. Past experience tells the people waiting in the village for the bus that the probability of the service being cancelled on any day is 0.05, and that, when it runs, the probability of the bus being more than 10 minutes late is 0.1. Using a tree diagram, or otherwise, find the conditional probability that the service has been cancelled, given that at 10 minutes past 10 the bus has not arrived in the village. [6]

http://natna.info/English/Teaching/MTH23-Fall2008/materials/lecture13-compound_events.pdf

http://www.gcsemathstutor.com/pdf/ws-info-probability/ws-ip-treediagrams01-pw.pdf

http://www.mathsisfun.com/data/probability-tree-diagrams.html

http://www.regentsprep.org/regents/math/algebra/APR4/PracTre.htm

http://nrich.maths.org/7288