Name: _____ Date: _____ Math Studies IBS2

Lesson 50 – Review – Logic

Answer all of the following questions on lined binder, unless otherwise indicated. **Show all work clearly**. Draw and label all tables where appropriate.

1. Write the negation of each proposition.
   (a) All teachers like chocolate.
   (b) Monday is not a holiday.
2. Given *p*: John reads books.
   q: John reads magazines.
   (a) Write a conjunction using propositions *p* and *q*.
   (b) Write a disjunction using propositions *p* and *q*.
3. (a) Complete the following truth table on this page for:
   p: $x > 3$
   q: $x^2 > 9$

p	q	$\neg p$	$\neg p \vee q$
T	T		
T	F		
F	T		
F	F		

   (b) Using the results of part (a), is $\neg p \vee q$ true or false when
   (i) $x > 3$ and $x^2 \not> 9$?
   (ii) $x \not> 3$ and $x^2 > 9$?
4. Given the following propositions:
   p: the person is male
   q: the person is over 16 years old
   r: the person is tall
   Write each of the following statements in simple English.
   (a) $\neg p \wedge q$
   (b) $p \vee \neg r$
   (c) $\neg(\neg p)$
   (d) $q \wedge \neg r$
   (e) $\neg(p \vee q)$
   Write each of the following statements using logic symbols.
   (f) the person is a short female
   (g) the person is neither short nor over 16 years old
   (h) the person is a short male over 16 years old

5. Given the following propositions:

a: N is a multiple of 2

b: N is a multiple of 3

c: $N \geq 25$

Give an example of a number N for which the following statements are true.

(a) $b \wedge \neg a \wedge c$

(b) $\neg(a \vee b) \wedge c$

6. Given the propositions:

p: the man fell

q: the man was pushed

Write the following sentences as compound propositions:

(a) The man fell if the man was pushed.

(b) The man fell only if the man was pushed.

(c) The man fell if and only if the man was pushed.

(d) If the man was not pushed then the man did not fall.

7. (a) Complete the following truth tables on this paper.

(i)

p	q	$\neg q$	$p \vee \neg q$
T	T		
T	F		
F	T		
F	F		

(ii)

p	q	$p \wedge q$	$\neg p$	$(p \wedge q) \wedge \neg p$
T	T			
T	F			
F	T			
F	F			

(iii)

p	$\neg p$	$p \vee \neg p$
T		
F		

This question continues on the next page.

7. (a) Continued

(iv)

p	q	$\neg p$	$\neg p \wedge q$	$\neg q$	$p \vee \neg q$	$\neg(p \vee \neg q)$	$(\neg p \wedge q) \Leftrightarrow \neg(p \vee \neg q)$
T	T						
T	F						
F	T						
F	F						

(b) Using your answers from part (a), state which of the compound propositions are
   (i) tautologies
   (ii) contradictions

8. Draw up a truth table to determine whether the statements $(p \wedge q) \Rightarrow r$ and $p \wedge (q \Rightarrow r)$ are logically equivalent.

9. Is the following a valid argument? Support your answer.
   If the weather is sunny, the plane arrives on time. If the plane arrives on time, we will be able to ski today. The weather is sunny. Therefore we will be able to ski today.

Answers to Review Test #4 – Logic

1. (a) Not all teachers like chocolate. (b) Monday is a holiday.

2. (a) John reads books and magazines. (b) John reads books or magazines.

3. (a)

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

(b) False (c) True

4. (a) The person is not male and is over 16 years old.
   (b) The person is male or not tall.
   (c) The person is male.
   (d) The person is over 16 years old and not tall.
   (e) The person is not male or over 16 years old.
   (f) $\neg p \wedge \neg r$ (g) $r \wedge \neg q$ (h) $\neg r \wedge p \wedge q$

Answers Continued

5. (a) 27 (b) 25 Note – There are many possible answers.

6. (a) $q \Rightarrow p$ (b) $p \Rightarrow q$ (c) $p \Leftrightarrow q$ (d) $\neg q \Rightarrow \neg p$

7. (a)

(i)

p	q	$\neg q$	$p \vee \neg q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

(ii)

p	q	$p \wedge q$	$\neg p$	$(p \wedge q) \wedge \neg p$
T	T	T	F	F
T	F	F	F	F
F	T	F	T	F
F	F	F	T	F

(iii)

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

(iv)

p	q	$\neg p$	$\neg p \wedge q$	$\neg q$	$p \vee \neg q$	$\neg(p \vee \neg q)$	$(\neg p \wedge q) \Leftrightarrow \neg(p \vee \neg q)$
T	T	F	F	F	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	F	F	T	T
F	F	T	F	T	T	F	T

(b) (i) $p \vee \neg p$ and $(\neg p \wedge q) \Leftrightarrow \neg(p \vee \neg q)$ are tautologies

(ii) $(p \wedge q) \wedge \neg p$ is a contradiction

Answers Continued

8.

p	q	r	$p \wedge q$	$(p \wedge q) \Rightarrow r$	$q \Rightarrow r$	$p \wedge (q \Rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	F
F	T	F	F	T	F	F
F	F	T	F	T	T	F
F	F	F	F	T	T	F

Therefore, $(p \wedge q) \Rightarrow r$ and $p \wedge (q \Rightarrow r)$ are not logically equivalent.

9. p: the weather is sunny

q: the plane arrives on time

r: we will be able to ski today

Given: $p \Rightarrow q$ and $q \Rightarrow r$ and p

Conclusion: r

Prove $((p \Rightarrow q) \wedge (q \Rightarrow r) \wedge p) \Rightarrow r$ is a tautology.

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge p$	$((p \Rightarrow q) \wedge (q \Rightarrow r) \wedge p) \Rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

Therefore the argument is valid.