Lesson 47 – Implication

(A) Definition

a. A compound statement formed using an if …. then …. connection ➔ so, if *p*, then *q*.
b. The connection can also be formed if several other ways:

   i. *p* only if *q*
   ii. *p* is sufficient for *q*
   iii. *q* is necessary for *p*

c. *p* is referred to as the ***antecedent*** and *q* is referred to as the ***consequent***

(B) Notation:

The notation used will be $p \Rightarrow q$

(C) Truth Values of Implication

a. Example used to demonstrate truth values:
   i. Let *p* be the proposition "I get a raise in pay"
   ii. Let *q* be the proposition "I will buy you a ring"
b. So our implication is "If I get a raise in pay, then I will buy you a ring"
c. And so our truth values are as follows:

p (get a pay raise)	q (buy a ring)	Interpretation of $p \Rightarrow q$	Truth values
I DO get a pay raise ➔ T	I DO buy you a ring ➔ T	I DO get a pay raise which implies that I DO buy you a ring	T
I DO get a pay raise ➔ T	I DO NOT buy you a ring ➔ F	I DO get a pay raise, but I DO NOT buy you a ring	F
I DO NOT get a pay raise ➔ F	I DO buy you a ring ➔ T	I DO NOT get a pay raise but I DO buy you a ring	T
I DO NOT get a pay raise ➔ F	I DO NOT buy you a ring ➔ F	I DO NOT get a pay raise which implies that I DO NOT buy you a ring	T

So to summarize:

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

d. Examples ➔ Complete Cirrito, Exercises 14.1.4 on pg 422
e. Examples ➔ complete Haese and Harris, Exercise 15E, p507, Q1-4

(D)Equivalence

a. Two statement are equivalent if one implies the other and vice versa ➔ so we are looking at the conjunction to two implications ➔ if p then q AND if q then p

b. So we have $(p \Rightarrow q) \wedge (q \Rightarrow p)$ and our statement will now be notated as $p \Leftrightarrow q$ and we will use the statement "if and only if" to denote equivalence.

c. For example, let p be the proposition that "I will pass the Semester exam" and let q be the proposition that "the exam is easy"
   i. So the implication $p \Rightarrow q$ is interpreted as "if I pass the semester exam, then the exam was easy"
   ii. And the implication $q \Rightarrow p$ is interpreted as "if the exam is easy, then I will pass the semester exam"
   iii. So then $p \Leftrightarrow q$ is read as "I will pass the exam if and only if the exam is easy"

d. Then our truth values for this statement will be:

p (pass exam)	q (easy exam)	Interpretation of $p \Rightarrow q$	Interpretation of $q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$
Pass ➔T	Easy exam ➔ T	If I pass, then it was an easy exam ➔ T	If it was an easy exam, then I do pass ➔ T	T
Pass ➔T	Not an easy exam➔ F	If I pass, then it was not an easy exam ➔ F	If it was not an easy exam, then I do pass ➔ T	F
Fail ➔ F	Easy exam ➔ T	If I fail, then it was an easy exam ➔ T	If it was an easy exam, then I do fail ➔ F	F
Fail ➔ F	Not an easy exam ➔ F	If I fail, then it was not an easy exam ➔ T	If it was not an easy exam, then I do fail ➔ T	T

So to summarize:

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(E) Examples ➔ complete Haese and Harris, Exercise 15E, p507, Q4,5

(F) Implications ➔ Other Considerations (Converses, Inverses, Contrapositives)

a. ***Converse***: the converse of the statement $p \Rightarrow q$ is the statement $q \Rightarrow p$. For example:

   i. Let p be: the triangle is isosceles
   ii. Let q be: two angles of the triangle are equal
   iii. So $p \Rightarrow q$ would mean ➔ If the triangle is isosceles, then two angles of the triangle are equal
   iv. And $q \Rightarrow p$ would mean ➔ If two angles of a triangle are equal, then the triangle is isosceles.

   v. Let p be: $x > 10$ (i.e I pick any number more than 10)
   vi. Let q be: $x > 4$ (i.e. I pick any number more than 4)
   vii. So $p \Rightarrow q$ would mean ➔ If $x > 10$, then $x > 4$ ➔ (i.e. if I pick any number more than 10, then I have picked a number more than 4)
   viii. And $q \Rightarrow p$ would mean ➔ If $x > 4$, then $x > 10$ ➔ (i.e if I pick any number more than 4, then I have picked a number more than 10)

b. ***Inverse***: the inverse of the statement $p \Rightarrow q$ is the statement $\neg p \Rightarrow \neg q$. For example:

   i. Let p be: the triangle is isosceles
   ii. Let q be: two angles of the triangle are equal
   iii. So $\neg p \Rightarrow \neg q$ would mean ➔ If the triangle is NOT isosceles, then two angles of the triangle are NOT equal
   iv. And to review: $q \Rightarrow p$ would mean ➔ If two angles of a triangle are equal, then the triangle is isosceles.

   v. Let p be: $x > 10$ (i.e I pick any number more than 10)
   vi. Let q be: $x > 4$ (i.e. I pick any number more than 4)
   vii. So $\neg p \Rightarrow \neg q$ would mean ➔ If x is NOT greater 10, then x is NOT greater 4 ➔ (i.e. if I pick any number that is NOT more than 10, then I have picked a number is NOT more than 4)
   viii. And to review: $q \Rightarrow p$ would mean ➔ If $x > 4$, then $x > 10$ ➔ (i.e if I pick any number more than 4, then I have picked a number more than 10)

   ix. Now: Use truth tables to show that $\neg p \Rightarrow \neg q$ and $q \Rightarrow p$ are logically equivalent

c. ***Contrapositives***: the contrapositive of the statement $p \Rightarrow q$ is the statement $\neg q \Rightarrow \neg p$. For example:

i. Let p be: the triangle is isosceles
ii. Let q be: two angles of the triangle are equal
iii. So $\neg q \Rightarrow \neg p$ would mean ➔ If two angles of a triangle are NOT equal, then the triangle is NOT isosceles
iv. And to review: $p \Rightarrow q$ would mean ➔ If the triangle is isosceles, then two angles of the triangle are equal.

v. Let p be: $x > 10$ (i.e I pick any number more than 10)
vi. Let q be: $x > 4$ (i.e. I pick any number more than 4)
vii. So $\neg q \Rightarrow \neg p$ would mean ➔ If $x \leq 4$, then $x \leq 10$ ➔ (i.e. if I pick any number that is less than or equal to 4, then I have picked a number is less than or equal to 10)
viii. And to review: $p \Rightarrow q$ would mean ➔ If $x > 10$, then $x > 4$ ➔ (i.e if I pick any number more than 10, then I have picked a number more than 4)

ix. Now: Use truth tables to show that $\neg q \Rightarrow \neg p$ and $p \Rightarrow q$ are logically equivalent

(G) Examples ➔ complete Haese and Harris, Exercise 15F, p509-10, Q1-6