

Lesson 45 – Intro to Logic - Propositions

(A) Logic: Definition

- The study of correct reasoning
- So how do we distinguish correct reasoning → this is the role of logic as it prescribes the rules for correct reasoning
- The main thing we shall study are the principles that govern the validity of arguments
- Our look into logic will begin with a study of LANGUAGE

(B) Propositions

- A proposition is a statement that can be either true or false, but not both. Comments, opinions or questions are NOT propositions
- For example, “Today is Friday” is a proposition. This statement can be true or false, but not both.
- It is common to define a shorthand notation for propositions:
 - Let P be the proposition “Today is Friday.”
- If the statement is true, then P has truth value true. If it is false, then P has truth value false.
- It is also common notation to use a “place filler” in a proposition.
 - For example let P(x) be “x is an odd number.” Then P(x) is a proposition depending on x.

(C) Negations

- These represent one operation that we can do with propositions in order to develop mathematical ideas and arguments
- The negation of a proposition is its opposite.
- Notation → For example, let P denote “Today is Friday”. Then the negation of P, written $\neg P$, is “Today is not Friday.” Notice that P and $\neg P$ cannot both have the same truth value

P: today is Friday	$\neg P$: today is not Friday	$\neg(\neg P)$:

(D) Exercises 15A.1 and 15A.2

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(E) Compound Propositions

- a. Now we consider working with 2 (or more) propositions
- b. Therefore, we define **two operations** on propositions. Again these operations are intuitive based on every day vernacular. The operations are “and”, denoted “ \wedge ”, and “or”, denoted “ \vee ”

- c. Conjunction: a new proposition formed when two propositions are joined with the word “**and**”. In order for this new proposition $P \wedge Q$ to be true, both P and Q must be true.
- d. Disjunction: a new proposition formed when two propositions are joined with the word “**or**”. In order for this new proposition $P \vee Q$ to be true either P must be true or Q must be true, not necessarily both.

- e. For example, let P be “ $x < 4$ ” and Q be “ $x > 2$.”
 - i. Then $P \wedge Q$ is the proposition “ $x < 4$ and $x > 2$.”
 - ii. $P \vee Q$ is the proposition “ $x < 4$ or $x > 2$.”

- f. Disjunctions can be either inclusive or exclusive.
 - i. Inclusive means that the disjunction is true when **one or both** propositions are true
Ex. \rightarrow P: Kay’s eyes are blue Q: Kay’s hair is blond Is $P \vee Q$ possible?
 - ii. Exclusive means that the disjunction is true when **only one of** the propositions is true.
Ex \rightarrow P: I travel all the way to work by car Q: I travel all the way to work by bike
Is $P \vee Q$ possible? So use the symbol $P \vee Q$

(F) Conjunctions and Disjunctions Illustrated on Venn Diagram

(G) Exercises 15B