Lesson 36 – Problem Solving with Quadratic Relations

(A) Lesson Objectives:

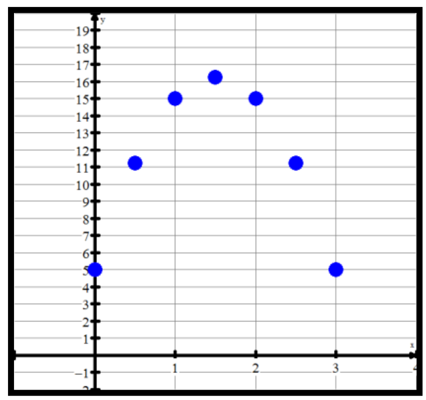
a. Consolidate skills learned in previous lessons ➔ quad algebra, quad graphs, quad relations & data
b. Review the key strategies involved in analyzing data of quad relations & writing equations
c. Review the key features if the graphs of quad functions and related them to working with word problems involving quad relations
d. Solve quad word problems using the TI-84 as the "tool"

(B) Review of Strategies in Data Analysis – Example of Equation Writing

Ex 1. A ball is thrown from the first floor walkway near the FAT down to the HS track. The relationship between the flight time of the ball (in seconds) and the height of the ball (in meters) is shown below:

Time (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
Height (m)	5.0	11.25	15.0	16.25	15.0	11.25	5.0

a. Create a scatterplot of the data and draw the quadratic curve of best fit

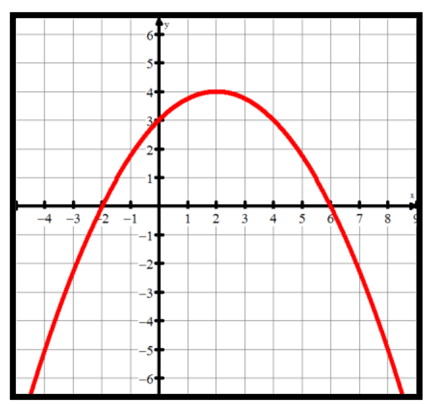


b. Estimate the coordinates of the vertex & then write the equation modeling this data.

c. Estimate the coordinates of the x-intercepts & then write the equation modeling this data.

d. Prove that the 2 equations are actually describing the same relation!

Ex 2. From the graph provided, determine an equation that represents the graphed relation using the strategies listed below:



a. Estimate the coordinates of the vertex & then write the equation of this quadratic relation.

b. Estimate the coordinates of the x-intercepts & then write the equation of this quadratic relation.

c. Prove that the 2 equations are actually describing the same relation!

(C) Key Features of the Graphs of Quadratic Relations

(i)	(ii)	(iii)	(iv)	(v)	(vi)

(D) Determining the Key Features ➔ Using the TI-84 "Tool"

a. For the quadratic function $f(x) = 2x^2 + 5x - 4$, use the TI-84 to determine:

b. Sketch and label these key factors on the graph

(i)	(ii)	(iii)	(iv)	(v)	(vi)

(E) Relating Features to Applications – In Class Detailed Example

Mr. S, has analyzed the population data for the town of Mathville and he has determined that the population can be modelled using the quadratic function $P(t) = -0.2t^2 + 10t + 200$, where P is the town's population in thousands and t is time in years since 1998.

a. What was the town's population in 2003? Round to 3 sig figs (if necessary).

b. When was the town's population 255,000? State the year(s) and month(s).

c. When did the town's population exceed 300,000? State the year(s) and month(s).

d. When did the town's population reach its maximum? State the year(s) and month(s).

e. What was the town's population in 1998?

f. Determine the zeroes/roots/x-intercepts of the quadratic function? Where are they and what do they represent in the context of this question?

g. Is a quadratic function suitable for modelling a town's population? Why or why not?

h. What would the equation $P(-10) = -0.2(-10)^2 + 10(-10) + 200 = 80$ mean and represent in the context of the question?

i. What domain would this quadratic function have? Why?

j. What range would this quadratic function have? Why?

(F) Further Examples to Practice

12. A baseball is thrown from the top of a building and falls to the ground below. Its path is approximated by the relation $h = -5t^2 + 5t + 30$, where h is the height above ground in metres and t is the elapsed time in seconds.

(a) How tall is the building?
(b) When will the ball hit the ground?
(c) When does the ball reach its maximum height?
(d) How high above the building is the ball at its maximum height?

14. A computer software company models the profit on its latest game using the relation $P = -2x^2 + 28x - 90$, where x is the number of games it produces in hundred thousands and P is the profit in millions of dollars.

(a) What is the maximum profit the company can earn?
(b) How many games must it produce to earn this profit?
(c) What are the break-even points for the company?

16. The path of a shot put is given by $h = -0.0502(d^2 - 20.7d - 26.28)$ where h is the height and d is the horizontal distance in metres.

(a) Rewrite the relation in the form $h = a(d - s)(d - t)$ where s and t are the zeros of the relation.
(b) What is the significance of s and t in this question?

18. **Thinking, Inquiry, Problem Solving:** Soundz Inc. makes CD players. Last year, accountants modelled the company's profit by $P = -5x^2 + 60x - 135$. Over the course of the year, in an effort to become more efficient, Soundz Inc. restructured its operation, eliminating some employees and reducing costs. This year, accountants are using $P = -7x^2 + 70x - 63$ to project the company's profit. In both models, P is the profit in hundreds of thousands of dollars and x is the number of CD players made, in hundreds of thousands. Was Soundz Inc.'s restructuring effective? Justify your answer.

5. Mirna's Fashion store determined that each relation below modelled expected revenue for an article of clothing. R is the revenue in dollars and x is the amount of the change in price. Solve for x. Interpret each answer in the context of the question.

(a) Sweatshirts $R = (20 - 2x)(30 + 3x)$ when $R = \$594$

(b) Suits $R = (200 - 25x)(9 + 3x)$ when $R = \$2250$

(c) Pants $R = (30 - 4x)(16 + 2x)$ when $R = \$396$

12. **Knowledge and Understanding:** Suppose that the population of a town is described by $P = 0.16t^2 + 7.2t + 100$, where P is the population in thousands and t is the time in years, with $t = 0$ representing the year 2000.

(a) What will the population be in 2010?

(b) What was the population in 1995?

(c) When will the population reach 52 000?

(d) Will the population ever reach zero under this model? Explain.

13. A model rocket is shot straight up from the roof of a school. The height at any time t is approximated by the model $H = 15 + 23t - 5t^2$, where H is the height in metres and t is the time in seconds.

(a) What is the height of the school?

(b) How long does it take for the rocket to pass a window 10 m above the ground?

(c) When does the rocket hit the ground?

(d) What is the maximum height the rocket reaches above the roof of the school?

15. The safe stopping distance, d, in metres, for a boat travelling at v km/h in calm water is determined to be $d = 0.002(2v^2 + 10v + 3000)$.

(a) What is the safe stopping distance if the speed is 12 km/h?

(b) What is the initial speed of the boat if it takes 15 m to stop?

14. **Communication:** Water from a fire hose is sprayed on a fire 15 m up the side of a wall. The equation $H = -0.011x^2 + x + 1.6$ models the height of the jet of water and the horizontal distance from the nozzle in metres. What is the farthest distance back from the building that a firefighter could stand and still reach the fire? Explain. Include a diagram with your explanation.

8. The world production of gold from 1970 to 1990 can be modelled by $G = 1492 - 76t + 5.2t^2$, where G is the number of tonnes of gold and t is the number of years since 1970 ($t = 0$ for 1970, $t = 1$ for 1971, etc.).
Source: Energy, Mines and Resources Canada.

(a) During this period, when was the minimum amount of gold mined?

(b) What was the least amount of gold mined in one year?

(c) How much gold was mined in 1985?

9. From 1960 to 1990, the average number of cigarettes C smoked in one year by each Canadian over 18 can be modelled by $C = 4024.5 + 51.4t - 3.1t^2$, where t is the number of years since 1960 ($t = 0$ for 1960). Source: Health Canada.

(a) When was cigarette consumption highest during this period?

(b) What was the average per capita consumption of cigarettes in the peak year?

(c) Since the late 1960s, cigarette packages have carried health warnings. Does the model indicate that this had any effect on cigarette consumption? Explain.

10. From 1995 to 1999, the average ticket price for a regular movie theatre (all ages) can be modelled by the $C = 0.06t^2 - 0.27t + 5.36$, where C is the price in dollars and t is the number of years since 1995 ($t = 0$ for 1995).
Source: Canadian Motion Picture Theatres and Drive-Ins.

(a) When were ticket prices at their lowest during this period?

(b) What was the average ticket price in 1998?

(c) What does the model predict the average ticket price will be in 2010?

12. A movie theatre can accommodate a maximum of 450 moviegoers in a day. The theatre operators have changed admission prices on several occasions to find out how price affects attendance, daily revenue, and profit. Using the formula *profit* = *revenue* − *expenses*, and after reviewing their data, the operators found they could express the relation between profit P and ticket price t as $P = t(450 - 30t) - 790$.

(a) What does the expression $450 - 30t$ represent? What does the 790 represent?

(b) What is the ticket price that maximizes the daily profit? What is the maximum profit? About how many tickets will be sold at this price?

(c) Use graphing technology to find the break-even ticket price and the required minimum ticket sales.

13. **Application:** In question 12, the theatre operators forgot to consider the revenue from concession sales. They expect this to be $3.50 per ticket. This changes the profit equation to $P = (t + 3.5)(450 - 30t) - 790$.

(a) What ticket price will maximize this new profit equation? What will that profit be?

(b) Use graphing technology to find the new break-even ticket price and the required minimum ticket sales.

12. **Knowledge and Understanding:** The cost C, in dollars, of operating a concrete-cutting machine is modelled by $C = 2.2n^2 - 66n + 655$, where n is the number of minutes the machine is run. How long must the machine run for the operating cost to be at a minimum? What is the minimum cost?

13. The annual budget B, in $billions, for the National Aeronautics and Space Administration (NASA) is approximated by the quadratic relation $B = -0.1492x^2 + 1.8058x + 9$, where x is the number of years since 1988.

(a) Use the model to predict NASA's budget in 2001.

(b) When does the model indicate that NASA's budget was about $12 billion?

(c) In what year was NASA's budget at its maximum?

15. The algebraic relation $d = 0.0056s^2 + 0.14s$ models the relation between a vehicle's stopping distance d, in metres, and its speed s, in kilometres per hour.

(a) What is the fastest you could drive and still be able to stop within 80 m?

(b) What is the stopping distance for a car travelling at 120 km/h?

(c) Estimate the length of an average car. How many car lengths does the stopping distance in (b) correspond to?