

(A) Lesson Objectives:

- Introduce patterns/sequences through several practical applications.
- Describe the main concepts associated with sequences.

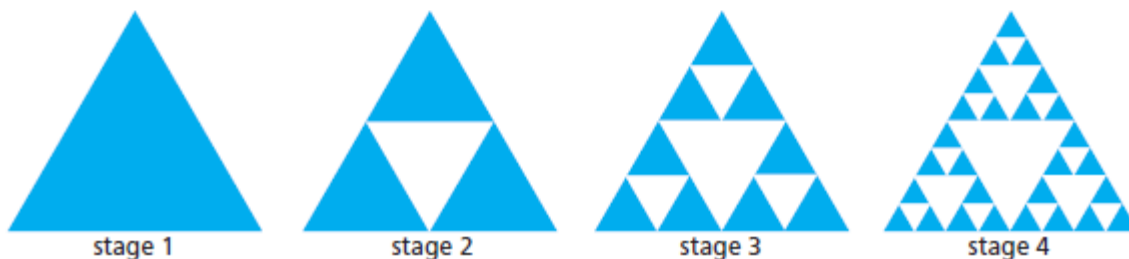
(B) Exploring Patterns and Sequences (From Nelson 11, Chap1.1, p16-17)

Here are six different relationships. Some of them contain a hidden pattern. Try to uncover the hidden pattern.

- Harry sells vacuum cleaners and earns \$400 each month. For each vacuum cleaner he sells, he earns an additional \$110.
- Marka deposits \$500 only once in a savings account that pays 5% interest at the end of each year.
- Andrew drops a rubber ball from a height of 2 m. After each bounce, the height of the ball's bounce is one-half the height of the ball's previous bounce.
- David tosses a coin repeatedly. Each time he tosses the coin, he records 1 if the coin comes up heads and 0 if the coin comes up tails.
- A square number can be represented by an arrangement of dots. Each square number is the total number of dots in each diagram. Here are the first four square numbers:



- In a fractal, a pattern repeats itself in the same shape. The Sierpinski triangle is an example of a fractal. Divide one equilateral triangle into four, congruent triangles by joining the midpoints of the sides of the larger triangle and remove the centre triangle. You can repeat these steps many times. Here are the first four stages of the Sierpinski triangle:



Think, Do, Discuss

1. Copy and complete each table. List the five numbers that you enter for each table separately.

A.

Number of Vacuum Cleaners Sold	1	2	3	4	5
Total Earnings (\$)					

____, ____ , ____ , ____ , ____

B.

At End of Each Year	1	2	3	4	5
Money in Savings Account (\$)					

____, ____ , ____ , ____ , ____

C.

Bounce	1	2	3	4	5
Height of Ball After Each Bounce (m)					

____, ____ , ____ , ____ , ____

D.

Toss	1	2	3	4	5
Heads = 1, Tails = 0					

____, ____ , ____ , ____ , ____

E.

Diagram	1	2	3	4	5
Square Number					

____, ____ , ____ , ____ , ____

F.

Stage	1	2	3	4	5
Total Number of Triangles					

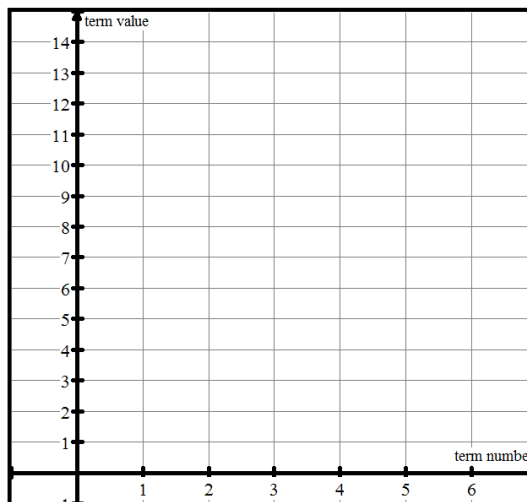
____, ____ , ____ , ____ , ____

- Which of the relationships are predictable? unpredictable?
- If possible, describe in words the pattern in each relationship.
- If possible, use the pattern to find the next three numbers in each list.
- Do any of these relationships come to an end? Explain.
- For each relationship and if possible, find an algebraic expression that describes the value of each term according to its position in the list. Let n represent the position of the term in the list, where n is a natural number, $\{1, 2, 3, \dots\}$.
- Use the algebraic expression you found in step 6 to determine the tenth number in each list.
- If n is the independent variable, then graph each predictable relationship. Can you join the points with a line or a curve? Explain.

(C) Key Ideas - Sequences

- ▶ A sequence is a _____.
- ▶ _____ sequence, meaning that it has _____.
- ▶ _____ sequence, meaning it has _____.
- ▶ Each number in the sequence is called a _____.
- ▶ Each term is numbered and presented in the following notation: _____ is designated as the first term; _____ is referred to as the n th term → For example, in the sequence 3,5,8,2,5,89,4 → $u_4 = 2$ and is referred to as the 4th term.
- ▶ The terms of a sequence may or may not have a specific pattern.
- ▶ If there is a pattern, then we can come up with a _____ to describe every term of the sequence → An algebraic way to express this rule is called _____ of the sequence.
- ▶ A sequence can be **graphically** visualized by plotting on a Cartesian plane, where the position of each term (the number of each term) is plotted on the x axis and the term value is plotted on the y axis.

Term number	Term value
1	2
2	3
3	5
4	8
5	12
6	17

**(D) Examples**

(a) Given the following sequences, find the next four terms:

- (i) 1,5,25,125,625,.... (ii) 6,16,36,66, (iii) 20,14,8,2,-4, (iv) 1,8,27,64,.....

(b) Given a formula, find the first 5 terms

- (i) $u_n = 3n - 2$ (ii) $u_n = n^2 - 1$ (iii) $u_n = (n-2)/(n+2)$ (iv) $u_n = 5^{n-1}$

(c) Find the formula for the n th term (or the general term), then list the next three terms:

(i) 5,10,15,20

(ii) 2,3,4,5,.....

(iii) 1,4,9,16,.....

(iv) 2,4,6,8,.....

(v) -3,-6,-12,-24....

(vi) $1/2, 2/3, 3/4, 4/5, \dots$

(d) Comment upon any pattern you see in these sequences

(i) \$100, \$110, \$120, \$130, \$140, \$150,

(ii) 8, 12, 16, 20, 24,.....

(iii) 11, 9, 7, 5, 3,.....

(e) Given the sequence 8,12,16,20,24,....., develop a formula that you can use to predict the 407th term. EXPLAIN your prediction/formula

(f) Comment upon any pattern you see in the sequences

(i) 2,10,50,250,.....

(ii) 5,-10,20,-40,80,.....

(iii) 6, 0.6, 0.06, 0.006, 0.0006,.....

(iv) 2,4,8,16,32,64,.....

(v) 100, 50, 25, 12.5, 6.25, ...

(g) Given the sequence 3,6,12,24,48,....., develop a formula that you can use to predict the 41st term. EXPLAIN your prediction/formula

(E) **Homework/Resources**

- **HW:**