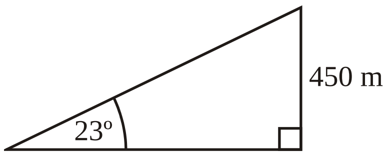
Triangles Review Packet-Answers

1. (a)



(M1) (C1)

Note: *All three (23°, x, 450) must be labelled and in correct position for (M1)*

(b) $\tan 23° = \frac{450}{x}$ (M1)

Note: *Follow through from candidate's diagram*

$x = \frac{450}{\tan 23°}$ (M1)

$x = 1060.13...$

$x = 1060$ (3 s.f.) (A1) (C3)

[4]

2. (a) $C\hat{A}B = 180 - 2 \times 23°$ (M1)

$= 134°$ (A1) (C2)

(b) $\frac{AB}{\sin 23°} = \frac{15}{\sin 134°}$ (M1)

Note: *Follow through with candidate's answer from (a)*

$AB = \frac{15 \sin 23°}{\sin 134°}$

$AB = 8.147702831...$

$= 8.15$ (3 s.f.) (A1) (C2)

[4]

3. (a) $l = \sqrt{8^2 + 8^2}$ (M1)

$= \sqrt{128}$

$= 11.3$ (3 s.f.) (A1)

(b) $L = \sqrt{\sqrt{128}^2 + 8^2}$ **OR** $L = \sqrt{11.3^2 + 8^2}$ (allow ft from (a)) (M1)

$= \sqrt{128 + 64}$ **OR** $= \sqrt{127.69 + 64}$

$= 13.9$ (3 s.f.) **OR** $= 13.8$ (3 s.f.) (A1)

[4]

4. (a) $AC^2 = CD^2 + AD^2 - 2 \times AD \times CD \times \cos(C\hat{D}A)$ (M1)

$= 80^2 + 30^2 - 2 \times 30 \times 80 \times \cos(60°)$ (A1)

$AC^2 = 4900$ (A1)

so AC = 70 m (units not required) (A1) (C4)

(b) $\dfrac{50}{\sin(30°)} = \dfrac{70}{\sin(\hat{ABC})}$ (M1)

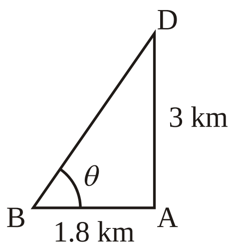
$\sin(\hat{ABC}) = \dfrac{1}{2} \times \dfrac{70}{50} = \dfrac{7}{10}$ (M1)(A1)

$\hat{ABC} = \sin^{-1}\left(\dfrac{7}{10}\right) = 44.4°$ (A1) (C4)

Note: *Accept 44° 25′ ± 1′.*

[8]

5. (a)



(A3) (C3)

Note: *Award (A1) for AB = 1.8, (A1) for AD = 3, (A1) for $\hat{A}$ = 90°*

(b) $\sqrt{3^2 + 1.8^2} = \sqrt{12.24}$ (3.50 (3s.f)) (M1)(A1) (C2)

(c) $\tan\theta = \dfrac{3}{1.8}$ (M1)

$\theta = 59.0°$ (or 1.03 radians) (A2) (C3)

[8]

6. (a) $BC = \sqrt{48^2 + 57^2 - 2(48)(57)\cos 117°}$ (or equivalent) (M1)

≈ 89.7 m (3 s.f.) (A1)

(b) Area of $\Delta ABC = \dfrac{1}{2}ab\sin C = \dfrac{1}{2}(48)(57)\sin 117°$ (M1)

$= 1220\ \text{m}^2$ (3 s.f.) (A1)

[4]

7. **Note on use of radians:**

In this question the height now becomes 13.0609… (though this value is not required).
The volume will be 298.
If marks were lost for radian use earlier, then 298 can receive full ft marks, but note that 299 is the result of premature rounding of the height to 13.1 above and so must be awarded (A0).
If marks were **not** lost due to radian use (e.g. question not completed), then the first three marks can be awarded for working but the answer of 298 receives (A0).

$\frac{\text{height}}{5.7} = \tan 42°$, (M1)

therefore height =5.7 tan 42° (= 5.1323...cm) (A1)
or (G2)

Volume of prism = $\frac{5.7\tan(42°)\times 5.78}{2}$

= 117 cm^3 (3 s.f.) (A1)
or (G2)

__Note:__ The only departures from the substituted volume formula allowed are those where the 5.7tan(42) is replaced with a value that the candidate seems to believe is the height. e.g. 5.7 repeated is a possibility. In such cases, award (M1)(A0).

[4]

8. (a) XM = 2 (A1) (C1)

(b) DM = $\sqrt{(9+4)} = \sqrt{13}$ (= 3.61) (M1)(A2) (C3)

(c) tanDMX = $\frac{3}{2}$ (M1)(A1)

__Note:__ Award (M1) for the correct angle, (A1) for the correct ratio.

angle DMX = 56.3° (A2) (C4)
OR
sin DMX = $\frac{3}{3.61}$ (M1)(A1)
angle DMX = 56.2° (A2)
OR
cos DMX = $\frac{2}{3.61}$ (M1)(A1)
angle DMX = 56.4° (A2)

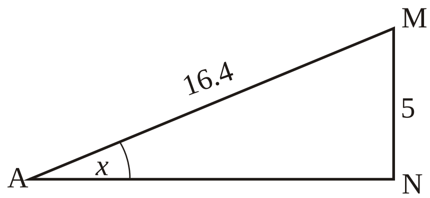
__Note:__ Accept correct answer given in radians, or degrees, minutes and seconds.

[8]

9. (a) (i) AF = $\sqrt{(12^2+5^2)}$ = 13 (A1)

(ii) AM = $\sqrt{(13^2+10^2)}$ = 16.4 (M1)(A1) 3

(b)



(M1)

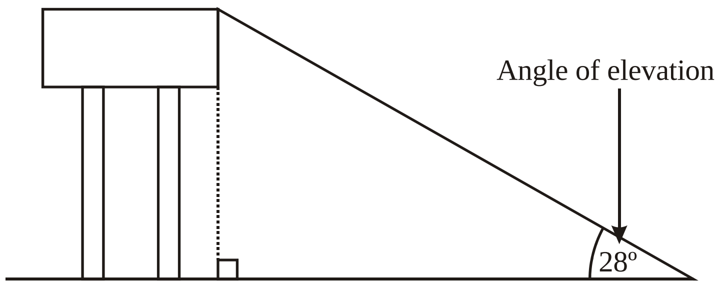
Note: *Award (M1) for correct angle.*

$\sin \hat{A} = \frac{5}{16.4}$ (M1)

$\hat{A} = 17.8°$ (A1) 3

[6]

10. (a)



(A1)

(b) $x = \frac{26.5}{\tan 28°}$ (or equivalent, allow follow-through from part (a)) (M1)

= 49.83925... (A1)

= 50 m (correct to nearest metre) (A1)

[4]

11. $\sin 28° = \frac{BE}{8}$ (M1)

$8 \times \sin 28° = BE$

$F\hat{B}C = 28°$ (M1)

$\cos 28° = \frac{BF}{5}$ (M1)

$5\cos 28° = BF$

Altitude of C = $8\sin 28° + 5\cos 28°$

= 8.170510467

= 8.17 cm (3 s.f) (A1)

[4]

12. (a) AC = 19 – 11 = 8 (M1)

$6^2 = 5^2 + 8^2 - 2(5)(8)\cos BAC$ (M1)

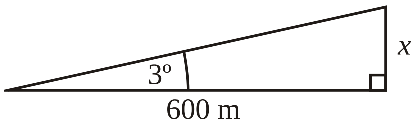
$\Rightarrow B\hat{A}C = 48.5°$ (3 s.f.) (A1) 3

(b) Area = $\left(\frac{1}{2}\right)(5)(8)\sin B\hat{A}C$ (M1)

$= 15.0\text{ cm}^2$ (3 s.f.) (allow **ft** from part (a)) (A1) 2

[5]

13. (a)



(M1)

$\tan 3° = \frac{x}{600}$

$x = 600 \tan 3°$

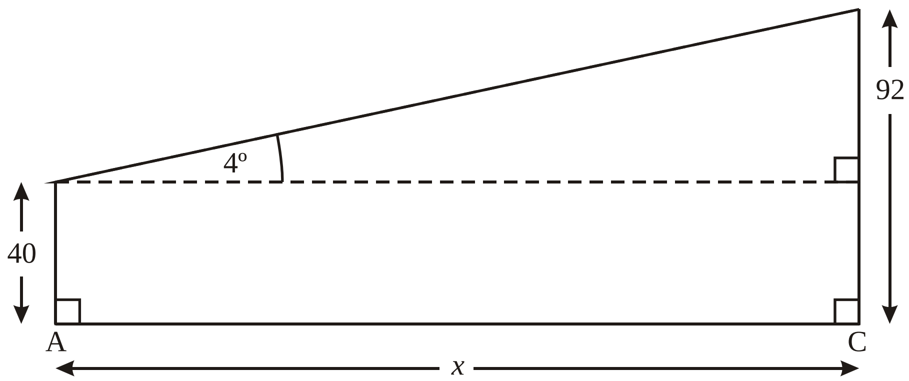
$x = 31.4447$

$x = 31.4$ m (A1)

Therefore, height = 40 m + 31.4 m

= 71.4 m (A1) 3

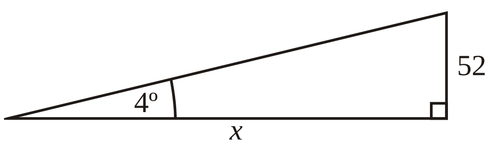
(b) (i)



(A1)

Note: *For (A1) the candidate must have the 40, the 92 and the 4° in the appropriate place.*

(ii)



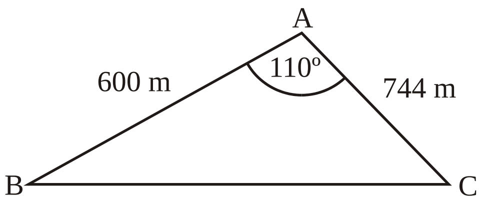
(A1)

$\tan 4° = \frac{52}{x}$ (M1)

$x = \frac{52}{\tan 4°}$

$x = 743.6346453 = 744$ m (A1) 4

(c) (i)



$BC^2 = 600^2 + 744^2 - 2 \times 600 \times 744\cos 110°$ (M1)

$BC^2 = 1218891.584$ (or 1218198.119) (A1)

BC = 1104.034231 (or 1103.720...)

BC = 1104 (to the nearest metre) (A1)

(ii) $\frac{\sin c}{600} = \frac{\sin 110°}{1104}$ (M1)

$\sin c = \frac{600 \times \sin 110°}{1104}$ (M1)

$c = 30.710635°$

$c = 30.7°$ (3 s.f.) (A1)

(iii) area = $\frac{1}{2} \times 600 \times 744\sin 110°$ (M1)

= 209739.393

= 210000 m^2 (3 s.f.) (A1) 8

[15]

14. (a) $BD^2 = 15^2 + 20^2 - 2 \times 15 \times 20 \times \cos 110°$ (M1)(A1)

Award (M1) for using the cosine rule, award (A1) for correct substitution.

$BD^2 = 830.212$

BD = 28.8 (A1)

OR

BD = 28.8 (G3) 3

(b) $\frac{28.81}{\sin C} = \frac{22}{\sin 30°}$ (M1)(A1)

C = 40.9° (G1)

OR

C = 40.9° (G3) 3

(c) BD = 30 (A1) 1

(d) $\frac{30}{\sin C} = \frac{22}{\sin 30°}$ (M1)

C = 43.0° (A1)

OR

C = 43.0° (G2) 2

(e) Percentage error = $\frac{43.0 - 40.9}{40.9} \times 100$ (M1)(A1)

= 5.13 % (A1) 3

[12]

15. (a) $\sin 50° = \frac{EP}{750}$ (M1)

EP = 575 m (A1)

$\cos 50° = \frac{RP}{750}$ (M1)

RP = 482 m (A1)

Total distance = 750 + 575 + 482

= 1807 = 1810 m (A1) 5

(b) (i) $EM^2 = 500^2 + 400^2 - 2 \times 500 \times 400 \times \cos 115°$ (M1)(A1)

$EM = \sqrt{579047}$ (M1)

= 761 (AG)

(ii)

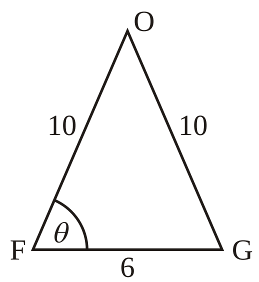
$d = 900 + 761 = 1661$ m (A1)

$s = 2$ m / s

$t = \frac{1661}{2} = 830.5 = 831$ s $\left[\text{or } \frac{900 + \sqrt{579047}}{2} = 830 \text{ s}\right]$ (M1)(A1) 6

[11]

16. ***Note:*** *Throughout this question watch out for and accept other alternative approaches e.g. in part (a)(i), the cosine formula may not necessarily be used.*



(a) (i) $\theta = \text{O}\hat{\text{F}}\text{G} = \arccos\left(\frac{10^2 + 6^2 - 10^2}{(2)(10)(6)}\right)$ (M2)

Notes: *Award (M1) for any correct method (formulae).*
Award (M1) for substituting correctly in the formula used.

$\Rightarrow \theta = 72.5°$ (3 s.f.) (A1) 3

Note: *Award (A1) for correct answer only.*

(ii) h = slant height or shortest distance from O to FG = $3 \tan\theta$ (M1)

= 9.53939... = 9.54 m (3 s.f.)

Note: *Follow through with candidate's θ*

(iii) Area of ΔOFG = $\frac{1}{2}(10)(6)(\sin\theta)$ (M2)

therefore total surface area of roof = $4 \times \frac{1}{2}(10)(6)(\sin\theta)$

= 114.4727... = 114 m^2 (3 s.f.) (A1) 3

Notes: *Award (M1) for using any correct method (formulae).*
Award (M1) for substituting correctly in the formulae used.
Follow through with candidate's θ. Accept 115 m^2.

(iv) Angle between slant height (line) and plane EFGH = $\arccos\left(\frac{3}{h}\right)$ (M1)

= 71.7° (3 s.f.) (A1) 2

Note: *Follow through with candidate's h.*

(v) H = Height of tower from base to O

= $40 + \sqrt{h^2 - 3^2}$ (M1)

= 49.055385... = 49.1 m (3 s.f.) (A1) 2

Note: *Follow through with candidate's h*

(b) Height (BP) = $\frac{6\sin 79°}{\sin(90° - 79°)}$ (M1)

= 30.9 m (3 s.f.) (A1) 2

[14]

17. (a) V = $\frac{1}{2} \times \frac{4}{3}\pi r^3$ For using $\frac{4}{3}\pi r^3$ (with or without $\frac{1}{2}$) (M1)

= $\frac{2}{3} \times \pi \times 3^3$ For using $\frac{1}{2}$ (their sphere formula) (M1)

= 18π cm^3 (AG) 2

(b) V = $\frac{2}{3} \times 18\pi$ For using $\frac{2}{3}$ × their answer to (a) (M1)

= 12π (A1)

$12\pi = \frac{1}{3}\pi \times 3^2 \times h$ For equating the volumes (M1)

$\frac{36\pi}{9\pi} = h\left(\frac{113.097}{28.27}\right)$ (A1)

h = 4 cm (AG) 4

(c) $l^2 = 4^2 + 3^2$ For using Pythagoras theorem (M1)
l = 5 (A1) 2

(d) For identifying the correct angle (M1)

$\tan\theta = \frac{4}{3}$ or $\sin\theta = \frac{4}{5}$ or $\cos\theta = \frac{3}{5}$ (M1)

θ = 53.1° (0.927 radians) (A1) 3

(e) For summing volume of cone and hemisphere. (M1)

Volume = $12\pi + 18\pi$

$= 30\pi$ cm^3 (94.2 cm^3)

For multiplying the volume by 0.6 (M1)

Weight = $0.6 \times 30\pi$

= 56.5g (A1) 3

(f) Surface area of cone = πrl

$= \pi \times 3 \times 5 = 15\pi$ (M1)(A1)

Surface area of a hemisphere = $\frac{1}{2} \times 4\pi r^2$

$= \frac{1}{2} \times 4 \times \pi \times 3^2$

$= 18\pi$ (M1)(A1)

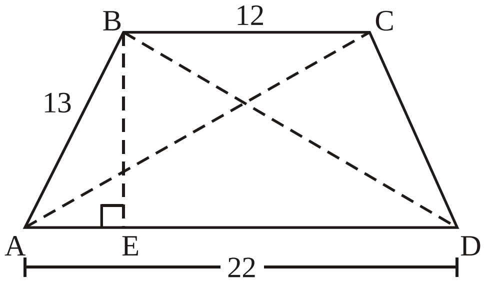
Total surface area = $15\pi + 18\pi$

= 103.67

= 104 cm2 (A1) 5

[19]

18.



(a) $22 - 12 = 10$ (R1)

Therefore, AE = $\frac{10}{2} = 5$ (R1)(AG)

Also allow $12 + 2(5) = 22$. (R2) 2

(b) $13^2 = 5^2 + BE^2$ (M1)

BE = $\sqrt{169-25}$

= 12 cm (A1)

Also allow just an answer 12 (Pythagorean triple) (C2) 2

(c) (i) $\tan B\hat{A}E = \frac{12}{5}$ (accept any other correct ratio) (M1)

= 2.4

$B\hat{A}E$ = 67.4° (3 s.f.) (A1)

(ii) $B\hat{C}D$ = 180 – 67.4 (A1) 3

= 113° (3 s.f.)

(d)	$CA^2 = BD^2 = 13^2 + 22^2 - 2(13)(22) \cos 67.4°$	(M1)	
	$= 433.183$	(M1)	
	CA = 20.8 (3 s.f.)	(A1)	
	OR		
	ED = 17	(M1)	
	$CA^2 = BD^2 = 12^2 + 17^2 = 433$	(M1)	
	Therefore, CA = 20.8 cm (3 s.f)	(A1)	3
	Accept 20.9		

[10]