

Quiz #4 – Logic

Total: 51 Marks

Answer all of the following questions on lined binder paper. . **Show all work clearly.** Draw and label all tables where appropriate.

1. Write the negation for each proposition.
- (a) We enjoy playing hockey. [1 mark]
- (b) Grass is blue. [1 mark]

2. Given p : Samantha eats chocolate
 q : Sebastian drinks milk
- (a) Write a conjunction using propositions p and q . [1 mark]
- (b) Write a disjunction using propositions p and q . [1 mark]

3. (a) Complete the truth table below for
 p : $x < 5$
 q : x is a prime number

p	q	$p \wedge q$	$\neg p$	$\neg p \vee (p \wedge q)$
T	T			
T	F			
F	T			
F	F			

[4 marks]

4. The following propositions are given.
- p : The car is green.
 q : The car is not red.
 r : The car is a sports car.
- (a) Use p , q , r and logic notation to represent the following sentences as logic statements.
- (i) The car is red or green. [1 mark]
- (ii) The car is a sports car if and only if it is red and not green. [1 mark]
- (b) Write the following logic statement as a sentence in words:
 $(\neg r \wedge q) \Rightarrow p$ [2 marks]

5. Consider the following statement:
 If Lisa is in Singapore then she is in Asia. [1 mark]
- (a) Write the converse of this statement.
- (b) (i) Write the inverse of this statement. [1 mark]
 (ii) Is the inverse always true? Explain your answer. [2 marks]

6. (a) Complete the truth table given below. [3 marks]

p	q	$\neg p$	$p \Rightarrow q$	$q \Rightarrow \neg p$
T	T			
T	F			
F	T			
F	F			

- (b) If p is true, what can you say about $(p \Rightarrow q) \wedge (q \Rightarrow \neg p)$? [1 mark]

7. (a) Complete the truth table below for the logic statement:

$$(p \vee q) \vee (\neg p \wedge \neg q)$$

[4 marks]

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg p \wedge \neg q$	$(p \vee q) \vee (\neg p \wedge \neg q)$
T	T					
T	F					
F	T					
F	F					

- (b) Describe the compound proposition $(p \vee q) \vee (\neg p \wedge \neg q)$.

[1 mark]

8. The following propositions p, q, and r are given:

p: x is a multiple of 5

q: x is a multiple of 3

r: x is a factor of 90

where $x \in \mathbb{N}$.

- (a) (i) Write a sentence, in words, for the statement $(q \vee r) \wedge \neg p$. [2 marks]

- (ii) Use p, q, r and logic notation to write a statement for the following:

If x is a factor of 90 then it is either a multiple of 5 or it is not a multiple of 3.

[2 marks]

- (b) (i) Use truth tables to determine the truth values of each of the following two statements:

$$(q \vee r) \wedge \neg p$$

$$r \Rightarrow (p \vee \neg q)$$

Write the first three columns of your truth table in the following format.

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

[6 marks]

- (ii) List the combinations of truth values of p, q and r that make the statement $(q \vee r) \wedge \neg p$ true.

[3 marks]

- (iii) Write down a possible value of x for each of the combinations of truth values from part (ii) which make the statement $(q \vee r) \wedge \neg p$ true. [3 marks]

- (c) (i) Draw up a truth table to determine the conditions for equivalence between the two statements in part (b)(i). [2 marks]

- (ii) When the equivalence is true describe, in words, the conditions on the value of x. [2 marks]

Answers to Test #8 – Logic

1. (a) Not all negation is in this test. (Some negation is not in this test.)
 (b) Pigs do not fly.

2. (a) Samantha eats chocolate and Sebastian drinks milk.
 (b) Samantha eats chocolate or Sebastian drinks milk.

3. (a)

p	q	$p \wedge q$	$\neg p$	$\neg p \vee (p \wedge q)$
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

- (b) (i) p is F and q is T so the result is TRUE.
 (ii) p is T and q is F so the result is FALSE.

4. (a) (i) $\neg q \vee p$
 (ii) $r \Leftrightarrow (\neg q \wedge \neg p)$
 (b) If the car is not a sports car and not red then the car is green.

5. (a) If Lisa is in Asia then she is in Singapore.
 (b) (i) If Lisa is not in Singapore then she is not in Asia.
 (ii) No, the inverse is not always true.
 If Lisa is not in Singapore then she could be in Manila, which is still in Asia.
 $T \Rightarrow F$ is False.

6. (a)

p	q	$\neg p$	$p \Rightarrow q$	$q \Rightarrow \neg p$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

- (b) If p is true then $(p \Rightarrow q) \wedge (q \Rightarrow \neg p)$ is false.

Answers to Test #8 – Logic (Continued)

7. (a)

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg p \wedge \neg q$	$(p \vee q) \vee (\neg p \wedge \neg q)$
T	T	F	F	T	F	T
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

(b) $(p \vee q) \vee (\neg p \wedge \neg q)$ is a tautology.

8. p: I like a subject
 q: I work hard at it
 r: I fail

Given: $\neg p \Rightarrow \neg q$, $\neg q \Rightarrow r$, $\neg r$

Conclusion: p

Show $[(\neg p \Rightarrow \neg q) \wedge (\neg q \Rightarrow r) \wedge (\neg r)] \Rightarrow p$

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg p \Rightarrow \neg q$	$\neg q \Rightarrow r$	$(\neg p \Rightarrow \neg q) \wedge (\neg q \Rightarrow r)$
T	T	T	F	F	F	T	T	T
T	T	F	F	F	T	T	T	T
T	F	T	F	T	F	T	F	F
T	F	F	F	T	T	T	T	T
F	T	T	T	F	F	T	T	F
F	T	F	T	F	T	F	T	F
F	F	T	T	T	F	F	F	F
F	F	F	T	T	T	T	T	T

p	q	r	$(\neg p \Rightarrow \neg q) \wedge (\neg q \Rightarrow r) \wedge r$	$[(\neg p \Rightarrow \neg q) \wedge (\neg q \Rightarrow r) \wedge r] \Rightarrow p$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

This is a tautology, so the argument is valid.

(Note: Could make r: I pass, then need to show $[(\neg p \Rightarrow \neg q) \wedge (\neg q \Rightarrow \neg r) \wedge r] \Rightarrow p$)

Answers to Test #8 – Logic (Continued)

9. (a) (i) x is a multiple of 3 or a factor of 90 and x is not a multiple of 5.
 (ii) $r \Rightarrow (p \vee \neg q)$

(b) (i)

p	q	r	$(q \vee r)$	$\neg p$	$(q \vee r) \wedge \neg p$	$\neg q$	$(p \vee \neg q)$	$r \Rightarrow (p \vee \neg q)$
T	T	T	T	F	F	F	T	T
T	T	F	T	F	F	F	T	T
T	F	T	T	F	F	T	T	T
T	F	F	F	F	F	T	T	T
F	T	T	T	T	T	F	F	F
F	T	F	T	T	T	F	F	T
F	F	T	T	T	T	T	T	T
F	F	F	F	T	F	T	T	T

(ii) $(q \vee r) \wedge \neg p$ is true in three cases.

Case 1: $p = F, q = T, r = T$

Case 2: $p = F, q = T, r = F$

Case 3: $p = F, q = F, r = T$

(iii) Case 1: $x = 3$ (answers will vary)

Case 2: $x = 12$ (answers will vary)

Case 3: $x = 2$ (answers will vary)

(c) (i)

p	q	r	$(q \vee r) \wedge \neg p$	$r \Rightarrow (p \vee \neg q)$	$((q \vee r) \wedge \neg p) \Leftrightarrow (r \Rightarrow (p \vee \neg q))$
T	T	T	F	T	F
T	T	F	F	T	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	F	T	F

(ii) Case 1: $p = F, q = T, r = F$

x must not be a multiple of 5, must be a multiple of 3, and must not be a factor of 90.

Case 2: $p = F, q = F, r = T$

x must not be a multiple of 5, must not be a multiple of 3, and must be a factor of 90.