Quadratics Review Packet

1. (a) Factorize $3x^2 + 13x - 10$. 

(b) Solve the equation $3x^2 + 13x - 10 = 0$. 

Consider a function $f(x) = 3x^2 + 13x - 10$. 

(c) Find the equation of the axis of symmetry on the graph of this function.

(d) Calculate the minimum value of this function.

(Total 8 marks)

2. The perimeter of this rectangular field is 220 m. One side is $x$ m as shown.

(a) Express the width ($W$) in terms of $x$.

(b) Write an expression, in terms of $x$ only, for the area of the field.

(c) If the length ($x$) is 70 m, find the area.

Working:

Answers:
(a) ........................................
(b) ...........................................
(c) ...........................................

(Total 4 marks)
3. The graph of the function \( y = x^2 - x - 2 \) is drawn below.

(a) Write down the coordinates of the point C.

(b) Calculate the coordinates of the points A and B.

\[
\text{Working:}
\]

Answers:
(a) ............................................................
(b) .............................................................

(Total 8 marks)
4. **Diagram 1** shows a part of the graph of \( y = x^2 \).

![Diagram 1](image1)

**Diagrams 2, 3 and 4** show a part of the graph of \( y = x^2 \) after it has been moved parallel to the \( x \)-axis, or parallel to the \( y \)-axis, or parallel to one axis, then the other.

![Diagram 2](image2)

![Diagram 3](image3)

![Diagram 4](image4)

Write down the equation of the graph shown in

(a) **Diagram 2**;

(b) **Diagram 3**;

(c) **Diagram 4**.

(Total 4 marks)
5. A picture is in the shape of a square of side 5 cm. It is surrounded by a wooden frame of width \( x \) cm, as shown in the diagram below.

The length of the wooden frame is \( l \) cm, and the area of the wooden frame is \( A \) cm\(^2\).

(a) Write an expression for the length \( l \) in terms of \( x \).  

(b) Write an expression for the area \( A \) in terms of \( x \).  

(c) If the area of the frame is 24 cm\(^2\), find the value of \( x \).  

(Total 7 marks)

6. A swimming pool is to be built in the shape of a letter L. The shape is formed from two squares with side dimensions \( x \) and \( \sqrt{x} \) as shown.

(a) Write down an expression for the area \( A \) of the swimming pool surface.  

(b) The area \( A \) is to be 30 m\(^2\). Write a quadratic equation that expresses this information.  

(c) Find both the solutions of your equation in part (b).  

(d) Which of the solutions in part (c) is the correct value of \( x \) for the pool? State briefly why you made this choice.
Working:

Answers:

(a) ...................................................
(b) ...................................................
(c) ...................................................
(d) ...................................................
...................................................
...................................................
...................................................
...................................................
(Total 8 marks)
7. The length of one side of a rectangle is 2 cm longer than its width.

(a) If the smaller side is $x$ cm, find the perimeter of the rectangle in terms of $x$.

The perimeter of a square is equal to the perimeter of the rectangle in part (a).

(b) Determine the length of each side of the square in terms of $x$.

The sum of the areas of the rectangle and the square is $2x^2 + 4x + 1$ (cm$^2$).

(c) (i) Given that this sum is 49 cm$^2$, find $x$.

(ii) Find the area of the square.

\[ \text{Working:} \]

\[ \text{Answers:} \]

(a) .................................................

(b) .................................................

(c) (i).........................................

(ii).......................................

(Total 6 marks)
8. (a) Solve the equation $x^2 - 5x + 6 = 0$.

(b) Find the coordinates of the points where the graph of $y = x^2 - 5x + 6$ intersects the $x$-axis.

**Working:**

**Answers:**

(a) ....................................................

(b) ....................................................

(Total 4 marks)
9. The diagram shows the graph of \( y = x^2 - 2x - 8 \). The graph crosses the \( x \)-axis at the point A, and has a vertex at B.

(a) Factorize \( x^2 - 2x - 8 \).

(b) Write down the coordinates of each of these points

(i) A;

(ii) B.

Working:

Answers:

(a) .............................................

(b) (i) .............................................

(ii) .............................................

(Total 4 marks)
10. Consider the graphs of the following functions.

(i) \( y = 7x + x^2; \)
(ii) \( y = (x - 2)(x + 3); \)
(iii) \( y = 3x^2 - 2x + 5; \)
(iv) \( y = 5 - 3x - 2x^2. \)

Which of these graphs

(a) has a y-intercept below the x-axis?
(b) passes through the origin?
(c) does not cross the x-axis?
(d) could be represented by the following diagram?

\[ \text{Working:} \]

\[ \text{Answers:} \]

(a) ..............................................
(b) ..............................................
(c) ..............................................
(d) ..............................................

(Total 8 marks)

11. (a) Sketch the graph of the function \( y = 2x^2 - 6x + 5. \)
(b) Write down the coordinates of the local maximum or minimum of the function.

(c) Find the equation of the axis of symmetry of the function.

(Total 6 marks)

12. The diagram below shows the graph of \( y = c + kx - x^2 \), where \( k \) and \( c \) are constants.

(a) Find the values of \( k \) and \( c \).

(b) Find the coordinates of \( Q \), the highest point on the graph.

\[
\begin{array}{c}
\text{Working:} \\
\text{Answers:} \\
\text{(a) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots} \\
\text{(b) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots} \\
\end{array}
\]

(Total 8 marks)
13. The diagrams below include sketches of the graphs of the following equations where \(a\) and \(b\) are positive integers.

Complete the table to match each equation to the correct sketch.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Sketch</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) (y = ax + b)</td>
<td></td>
</tr>
<tr>
<td>(ii) (y = -ax + b)</td>
<td></td>
</tr>
<tr>
<td>(iii) (y = x^2 + ax + b)</td>
<td></td>
</tr>
<tr>
<td>(iv) (y = x^2 - ax - b)</td>
<td></td>
</tr>
</tbody>
</table>
Working:
14. (a) Solve the following equation for \( x \)

\[ 3(2x + 1) - 2(3 - x) = 13. \]  

(2)

(b) Factorize the expression \( x^2 + 2x - 3 \).  

(2)

(c) Find the positive solution of the equation

\[ x^2 + 2x - 6 = 0. \]  

(2)

Working: 

Answers:

(a) ...................................................

(b) ...................................................

(c) ...................................................

(Total 6 marks)
15. (a) Factorize the expression $x^2 - 25$.

(b) Factorize the expression $x^2 - 3x - 4$.

(c) Using your answer to part (b), or otherwise, solve the equation $x^2 - 3x - 4 = 0$. 

Working:

Answers:

(a) ....................................................

(b) ....................................................

(c) ....................................................

(Total 8 marks)
Mrs. Harvey wants to put a 50 m long fence around her rectangular garden. She only needs to fence in 3 sides because the other side is alongside her house.

The width of the garden is denoted by $x$, and the length by $y$.

(a) Write an expression for $y$ in terms of $x$.

(b) Write an expression for the area, $A$, of the garden, in terms of $x$.

(c) If the area is 200 m$^2$, find the dimensions of the garden.

Working:

Answers:
(a) ..............................................................
(b) ..............................................................
(c) ..............................................................

(Total 8 marks)
The graph of the function \( f(x) = x^2 - 2x - 3 \) is shown in the diagram below.

(a) Factorize the expression \( x^2 - 2x - 3 \).

(b) Write down the coordinates of the points A and B.

(c) Write down the equation of the axis of symmetry.

(d) Write down the coordinates of the point C, the vertex of the parabola.

**Working:**

**Answers:**

(a) ..................................................
(b) ..................................................
(c) ..................................................
(d) ..................................................

(Total 8 marks)

The perimeter of a rectangle is 24 metres.

(a) The table shows some of the possible dimensions of the rectangle. Find the values of \( a, b, c, d \) and \( e \).
<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Width (m)</th>
<th>Area (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>a</td>
<td>10</td>
<td>b</td>
</tr>
<tr>
<td>3</td>
<td>c</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>d</td>
<td>e</td>
</tr>
</tbody>
</table>
20. A small manufacturing company makes and sells \( x \) machines each month. The monthly cost \( C \), in dollars, of making \( x \) machines is given by

\[
C(x) = 2600 + 0.4x^2.
\]

The monthly income \( I \), in dollars, obtained by selling \( x \) machines is given by

\[
I(x) = 150x - 0.6x^2.
\]

(a) Show that the company’s monthly profit can be calculated using the quadratic function

\[
P(x) = -x^2 + 150x - 2600.
\]

(b) The maximum profit occurs at the vertex of the function \( P(x) \). How many machines should be made and sold each month for a maximum profit?

(c) If the company does maximize profit, what is the selling price of each machine?

(d) Given that \( P(x) = (x - 20)(130 - x) \), find the smallest number of machines the company must make and sell each month in order to make positive profit.

(Total 12 marks)
21. The profit \( P \) in Swiss Francs made by three students selling homemade lemonade is modelled by the function

\[
P = -\frac{1}{20}x^2 + 5x - 30
\]

where \( x \) is the number of glasses of lemonade sold.

(a) **Copy** and complete the table below

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>15</td>
<td>90</td>
<td>75</td>
<td>50</td>
<td></td>
<td>50</td>
<td>75</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

(b) On graph paper draw axes for \( x \) and \( P \), placing \( x \) on the horizontal axis and \( P \) on the vertical axis. Use suitable scales. Draw the graph of \( P \) against \( x \) by plotting the points. Label your graph.

(c) **Use your graph** to find

(i) the maximum possible profit;  
(ii) the number of glasses that need to be sold to make the maximum profit;  
(iii) the number of glasses that need to be sold to make a profit of 80 Swiss Francs;  
(iv) the amount of money initially invested by the three students.

(d) The three students Baljeet, Jane and Fiona share the profits in the ratio of 1:2:3 respectively. If they sold 40 glasses of lemonade, calculate Fiona’s share of the profits.

(Total 15 marks)