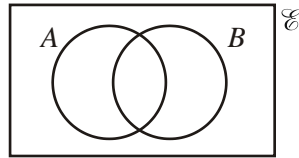


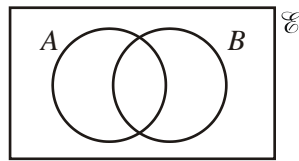
**Sets and Logic Review Packet from November Questions**

1. In each of the Venn diagrams, shade the region indicated.

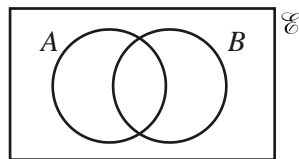
(a)  $A \cap B$



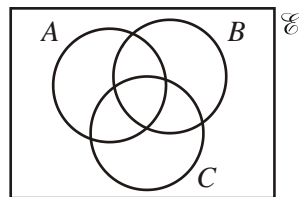
(b) The complement of  $(A \cap B)$



(c) The complement of  $(A \cup B)$



(d)  $A \cup (B \cap C)$



*Working:*

**(Total 4 marks)**

2. You may choose from three courses on a lunchtime menu at a restaurant.

- $s$ : you choose a salad,
- $m$ : you choose a meat dish (main course),
- $d$ : you choose a dessert.

You choose a **two** course meal which **must** include a main course and either a salad or a dessert, but not both.

- (a) Write the sentence above using logic symbols. (2)
- (b) Write in words  $s \Rightarrow \neg d$ . (2)
- (c) Complete the following truth table. (2)

$s$	$d$	$\neg s$	$\neg s \Rightarrow d$
T	T		
T	F		
F	T		
F	F		

*Working:*

*Answers:*

- (a) .....
- (b) .....
- .....
- .....

**(Total 6 marks)**

3. Write down the values for **a**, **b**, **c**, **d**, **e** and **f** from the table below:

$p$	$q$	$\neg p$	$p \wedge q$	$p \vee q$	$p \underline{\vee} q$	$p \Rightarrow q$	$p \Leftrightarrow q$
T	T	<b>a</b>			<b>d</b>		
T	F		<b>b</b>				<b>f</b>
F	T			<b>c</b>			
F	F					<b>e</b>	

(Total 6 marks)

4. Consider the following logic statements:

$p$ :  $x$  is a factor of 6

$q$ :  $x$  is a factor of 24

(a) Write  $p \Rightarrow q$  in words.

(1)

(b) Write the converse of  $p \Rightarrow q$ .

(1)

(c) State if the converse is true or false and give an example to justify your answer.

(2)

(Total 4 marks)

5. Complete the Truth Table for the compound proposition  $(p \wedge \neg q) \Rightarrow (p \vee q)$ .

$p$	$q$	$\neg q$	$(p \wedge \neg q)$	$(p \vee q)$	$(p \wedge \neg q) \Rightarrow (p \vee q)$
T	T	F	F		
T	F	T	T		
F	T	F		T	
F	F		F	F	

Working:

(Total 8 marks)

6. Consider the following logic statements:

$p$ : the train arrives on time

$q$ : I am late for school

(a) Write the expression  $p \Rightarrow \neg q$  as a logic statement.

(b) Write the following statement in logic symbols:

"The train does not arrive on time and I am not late for school."

(c) Complete the following truth table.

$p$	$q$	$\neg p$	$\neg q$	$p \Rightarrow \neg q$	$\neg p \wedge \neg q$
T	T	F	F	F	F
T	F	F	T	T	–
F	T	T	F	–	–
F	F	T	T	T	T

(d) Are the two compound propositions  $(p \Rightarrow \neg q)$  and  $(\neg p \wedge \neg q)$  logically equivalent?

<p><i>Working:</i></p>	<p><i>Answers:</i></p> <p>(a) .....</p> <p>.....</p> <p>(b) .....</p> <p>.....</p> <p>(d) .....</p>
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(Total 8 marks)

7. Two logic propositions are given.

$p$ : Paula eats chocolates.  
 $q$ : Paula watches television.

Write in words

(a)  $p \wedge \neg q$ ;

(b)  $p \vee q$ ;

(c)  $q \Rightarrow \neg p$ .

<p><i>Working:</i></p>	<p><i>Answers:</i></p> <p>(a) .....</p> <p>.....</p> <p>.....</p> <p>(b) .....</p> <p>.....</p> <p>.....</p> <p>(c) .....</p> <p>.....</p> <p>.....</p>
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**(Total 6 marks)**

8. Two propositions  $p$  and  $q$  are defined as follows.

$p$ : Jones passed this course

$q$ : Smith passed this course

(a) Write in symbolic form

(i) neither Jones nor Smith passed the course;

(ii) it is not the case that Jones and Smith both passed the course.

(b) Complete the following truth table for the logic statement  $\neg p \vee q$ .

$p$	$q$	$\neg p$	$\neg p \vee q$
T	T		
T	F		
F	T		
F	F		

Working:

Answers:

(a) (i) .....

(ii) .....

(Total 4 marks)

9. (a) Solve  $2x + 3 = 5$ .
- (b) Consider the logic statements.

$p: 2x + 3 = 5$        $q: x^2 = x$

The compound proposition  $2x + 3 = 5 \Rightarrow x^2 = x$  is given.  
Is this compound proposition true?

- (c) Write down the converse of this compound proposition.
- (d) Give an example to show that the converse is false.

<p><i>Working:</i></p>	<p><i>Answers:</i></p> <p>(a) .....</p> <p>(b) .....</p> <p>(c) .....</p> <p>(d) .....</p>
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**(Total 8 marks)**

10. Consider two propositions  $p$  and  $q$ . Complete the truth table below for the compound proposition.

$$(p \wedge \neg q) \Rightarrow (\neg p \vee q)$$

$p$	$q$	$\neg p$	$\neg q$	$p \wedge \neg q$	$\neg p \vee q$	$(p \wedge \neg q) \Rightarrow (\neg p \vee q)$
T	T	F	F	F	<b>(d)</b>	T
T	F	F	T	<b>(b)</b>	F	<b>(f)</b>
F	T	<b>(a)</b>	F	<b>(c)</b>	T	<b>(g)</b>
F	F	T	T	F	<b>(e)</b>	<b>(h)</b>

*Working:*

**(Total 8 marks)**



11. Consider the following statements.

$p$ : students work hard  
 $q$ : students will succeed

(a) Write the following proposition in symbols using  $p$ ,  $q$  and logical connectives only.

*If students do not work hard, then they will not succeed.*

(b) Complete the following truth table, relating to the statement made in part (a), and decide whether the statement is logically valid.

$p$	$q$			
T	T			
T	F			
F	T			
F	F			

*Working:*

*Answers:*

(a) .....

(b) .....

**(Total 8 marks)**

12. Let  $p$  and  $q$  be the statements

$p$ : you watch the music TV channel  
 $q$ : you like music

(a) Consider the following logic statement.

*If you watch the music TV channel then you like music.*

- (i) Write down in words the inverse of the statement.
- (ii) Write down in words the converse of the statement.

(4)

(b) Construct truth tables for the following statements:

- (i)  $p \Rightarrow q$ .
- (ii)  $\neg p \Rightarrow \neg q$ .
- (iii)  $p \vee \neg q$ .
- (iv)  $\neg p \wedge q$ .

(4)

(c) Which of the statements in part (b) are logically equivalent?

(1)

(Total 9 marks)

13. The propositions  $p$ ,  $q$  and  $r$  are defined as follows:

$p$ : this is a good course  
 $q$ : the course is worth taking  
 $r$ : the grading is lenient

(a) Write a symbolic statement for each of the following sentences.

- (i) *If this is a good course, then it is worth taking.*
- (ii) *Either the grading is lenient, or the course is not worth taking.*

(2)

(b) Write the following argument using  $p$ ,  $q$ ,  $r$  and logic symbols or connectives only.

*If this is a good course, then it is worth taking. Either the grading is lenient, or the course is not worth taking. But the grading is not lenient. Therefore, this is not a good course.*

(2)

(Total 4 marks)

14. If each of the following compound propositions is true, what conclusions can be made?

(a)  $x < 7$  or  $x \geq 3$ , and  $x \neq 7$

(b)  $p = 3$  if and only if  $q = 5$ , and if  $q \neq 5$  then  $r \neq 12$ .

*Working:*

*Answers:*

(a) .....

(b) .....

**(Total 4 marks)**

15. The propositions  $p$  and  $q$  are defined as follows:

$p$ : you have understood this topic

$q$ : you will be able to do this question

(a) Write the following proposition in symbols using  $p$ ,  $q$  and logical connectives only.

*“You have understood this topic, or you will not be able to do this question.”*

(b) Explain, in words only, what the following symbolic proposition represents:

$$(p \wedge \neg q) \Rightarrow \neg p.$$

*Working:*

*Answers:*

(a) .....

(b) .....

**(Total 4 marks)**

16. Three propositions are defined as follows:

$p$ : The oven is working.

$q$ : The food supply is adequate.

$r$ : The visitors are hungry.

(a) Write one sentence, in words only, for each of the following logic statements.

(i)  $q \wedge r \wedge \neg p$  (2)

(ii)  $\neg r \vee (p \wedge q)$  (2)

(b) Write the sentence below using only the symbols  $p$ ,  $q$  and logic connectives.

*"If the oven is working and the food supply is adequate then the oven is working or the food supply is adequate."*

(2)

(c) A tautology is a compound statement which is always true. Use a truth table to determine whether or not your answer to part (b) is a tautology.

**Hint:** Begin by writing the first two columns of your truth table in the following format:

$p$	$q$
T	T
T	F
F	T
F	F

(3)

(Total 9 marks)

17. Let

$\mathcal{U} = \{\text{positive integers less than 15}\};$

$X = \{\text{multiples of 2}\};$

$Y = \{\text{multiples of 3}\}.$

(a) Show, in a Venn diagram, the relationship between the sets  $\mathcal{U}$ ,  $X$  and  $Y$ .

(1)

(b) List the elements of:

(i)  $X \cap Y$  (1)

(ii)  $X \cap \mathcal{C}Y$ . (2)

(c) Find the **number of elements** in the complement of  $(X \cup Y)$ .

(2)

(Total 6 marks)

18. In a club with 60 members, everyone attends either on Tuesday for Drama ( $D$ ) or on Thursday for Sports ( $S$ ) or on both days for Drama and Sports.

One week it is found that 48 members attend for Drama and 44 members attend for Sports and  $x$  members attend for both Drama and Sports.

- (a) (i) Draw and **label fully** a Venn diagram to illustrate this information. (3)
- (ii) Find the number of members who attend for both Drama and Sports. (2)
- (iii) Describe, in words, the set represented by  $(D \cap S)'$ . (2)
- (iv) What is the probability that a member selected at random attends for Drama only or Sports only? (3)

The club has 28 female members, 8 of whom attend for both Drama and Sports.

- (b) What is the probability that a member of the club selected at random
- (i) is female and attends for Drama only or Sports only? (2)
- (ii) is male and attends for both Drama and Sports? (2)
- (Total 14 marks)

19. Three propositions  $p$ ,  $q$  and  $r$  are defined as follows:

$p$ : the water is cold.      $q$ : the water is boiling.      $r$ : the water is warm.

(a) Write one sentence, in words, for the following logic statement:

$$(\neg p \wedge \neg q) \Rightarrow r$$

(b) Write the following sentence as a logic statement using symbols only.

*"The water is cold if and only if it is neither boiling nor warm"*

*Working:*

*Answers:*

(a) .....

(b) .....

**(Total 4 marks)**

20. Given a universal set  $U = \{\text{cars}\}$ ,  $S = \{\text{sports cars}\}$ ,  $G = \{\text{green sports cars}\}$ .

(a) Draw a Venn diagram to illustrate this information.

**(3)**

(b) Shade the set  $S \cap G'$  on your diagram.

**(1)**

(c) Write in words the meaning of  $S \cap G'$ .

**(2)**

**(Total 6 marks)**

21. The universal set  $U$  is defined as the set of positive integers less than 10. The subsets  $A$  and  $B$  are defined as:

$$A = \{\text{integers that are multiples of 3}\}$$

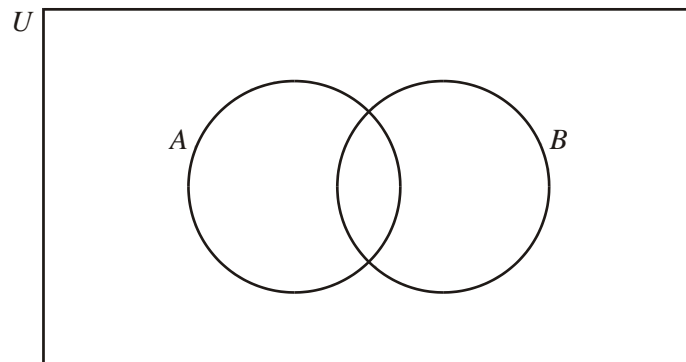
$$B = \{\text{integers that are factors of 30}\}$$

(a) List the elements of

(i)  $A$ ;

(ii)  $B$ .

(b) Place the elements of  $A$  and  $B$  in the appropriate region in the Venn diagram below.



*Working:*

*Answers:*

(a) (i) .....

(ii) .....

**(Total 4 marks)**

22. In a survey of 52 students it was found that 30 study Spanish and 15 have computers. Seven of the students who study Spanish also have computers.

(a) **Copy** and complete this table.

	Study Spanish	Do not study Spanish	Total
Have computers			
Do not have computers			
Total			52

(3)

(b) Draw and **label fully** a Venn diagram to illustrate this information. Use  $U$  to represent the set of all students surveyed,  $S$  the set of students who study Spanish and  $C$  the set of students who have computers.

(2)

(c) Describe, in words, the set represented by  $C \cup S'$ .

(2)

(d) Find  $n(C \cup S')$ .

(1)

A student is selected at random to attend a computer workshop given in Spanish.

(e) What is the probability that the student

(i) has a computer and studies Spanish?

(2)

(ii) has a computer but does not study Spanish?

(2)

(iii) has a computer if he/she studies Spanish?

(2)

(Total 14 marks)



23. A committee  $U$  has three sub-committees: research  $R$ , finance  $F$  and purchasing  $P$ . No member belongs to both finance and purchasing sub-committees. Some members belong to both research and purchasing committees. All members of the finance sub-committee also belong to the research sub-committee.

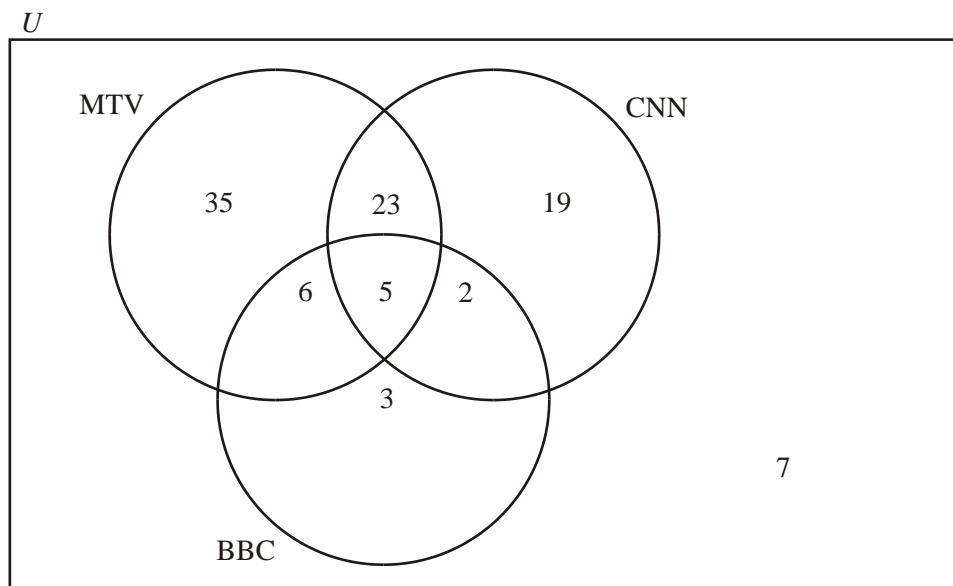
Draw a Venn diagram, showing the relationship between the sets  $U$ ,  $R$ ,  $F$  and  $P$ .

*Answer:*



(Total 4 marks)

24. 100 students were asked which television channel (MTV, CNN or BBC) they had watched the previous evening. The results are shown in the Venn diagram below.

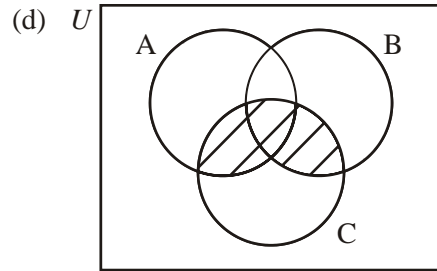
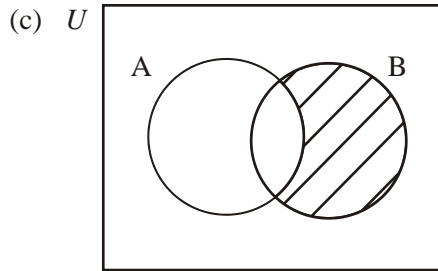
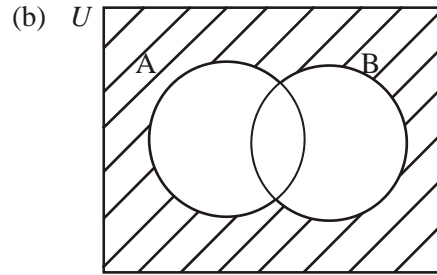
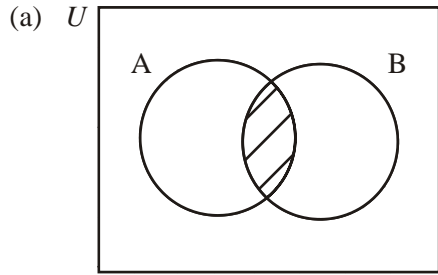


From the information in the Venn diagram, write down the number of students who watched

- (a) both MTV and BBC;
- (b) MTV or BBC;
- (c) CNN and BBC but not MTV;
- (d) MTV or CNN but not BBC.

(Total 4 marks)

25. Write down an expression to describe the shaded area on the following Venn diagrams:



*Working:*

*Answers:*

- (a) .....
- (b) .....
- (c) .....
- (d) .....

**(Total 8 marks)**

26. A poll was taken of the leisure time activities of 90 students.

60 students watch TV ( $T$ ), 60 students read ( $R$ ), 70 students go to the cinema ( $C$ ).

26 students watch TV, read **and** go to the cinema.

20 students watch TV and go to the cinema only.

18 students read and go to the cinema only.

10 students read and watch TV only.


(a) Draw a Venn diagram to illustrate the above information.

(b) Calculate how many students

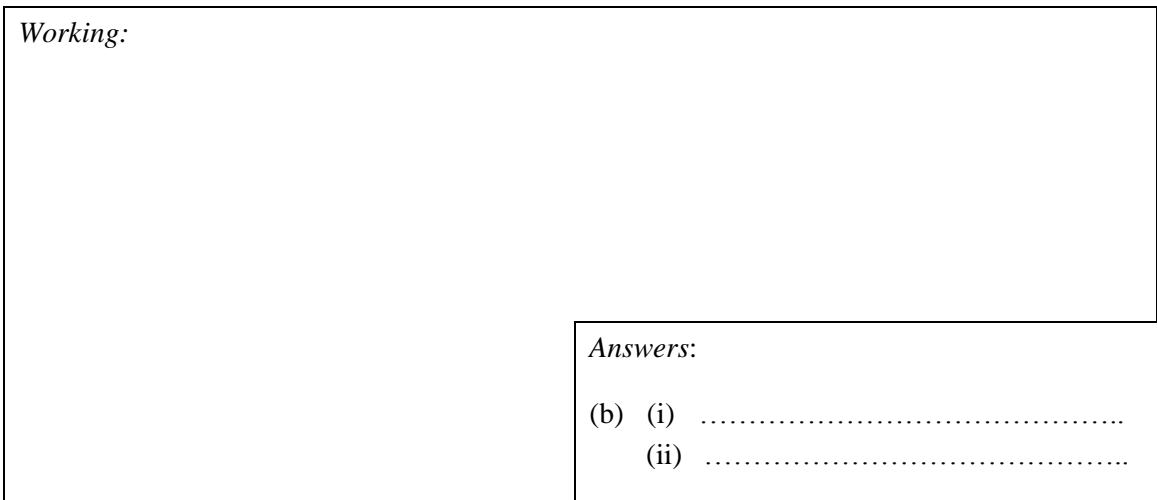
(i) only watch TV;

(ii) only go to the cinema.

*Diagram:*



*Working:*



*Answers:*

(b) (i) .....

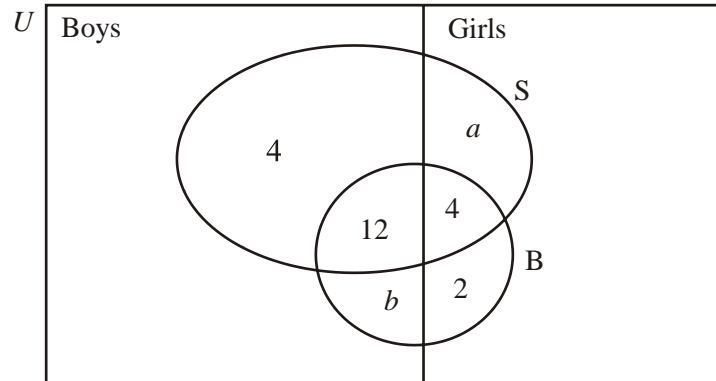
(ii) .....

**(Total 8 marks)**

27. Children in a class of 30 students are asked whether they can swim (S) or ride a bicycle (B).

There are 12 girls in the class. 8 girls can swim, 6 girls can ride a bicycle and 4 girls can do both.

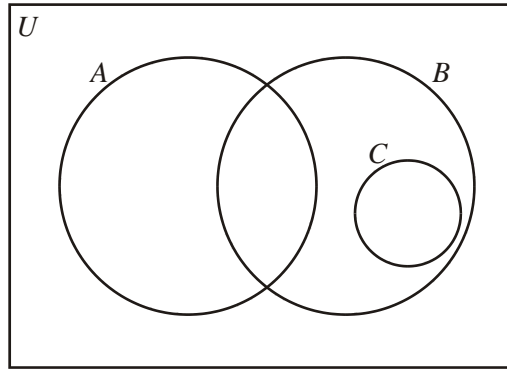
16 boys can swim, 13 boys can ride a bicycle and 12 boys can do both. This information is represented in a Venn diagram.



- (a) Find the values of  $a$  and  $b$ . (2)
  
- (b) Calculate the number of students who can do neither. (2)
  
- (c) Write down the probability that a student chosen at random can swim. (2)
  
- (d) Given that the student can ride a bicycle, write down the probability that the student is a girl. (2)

(Total 8 marks)

28. The following Venn Diagram shows the sets  $U$ ,  $A$ ,  $B$  and  $C$ .



State whether the following statements are true or false for the information illustrated in the Venn Diagram.

- (a)  $A \cap C = \emptyset$
- (b)  $C \cup B = C$
- (c)  $C \subset (A \cup B)$
- (d)  $A \subset C'$

*Working:*

*Answers:*

(a) .....

(b) .....

(c) .....

(d) .....

**(Total 8 marks)**

29. On a particular day 100 children are asked to make a note of what they drank that day.

They are given three choices: water (W), coffee (C) or fruit juice (F)

1 child drank only water.

6 children drank only coffee.

8 children drank only fruit juice.

5 children drank all three.

7 children drank water and coffee only.

53 children drank coffee and fruit juice only.

18 children drank water and fruit juice only.

(a) Represent the above information on a Venn Diagram.

(4)

(b) How many children drank none of the above?

(2)

(c) A child is chosen at random. Find the probability that the child drank

(i) coffee;

(ii) water or fruit juice but not coffee;

(iii) no fruit juice, given that the child did drink water.

(4)

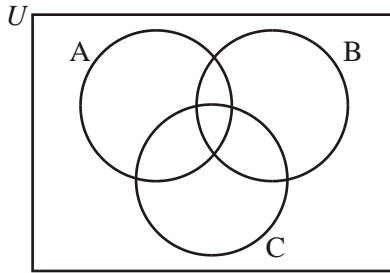
(d) Two children are chosen at random. Find the probability that both children drank all three choices.

(3)

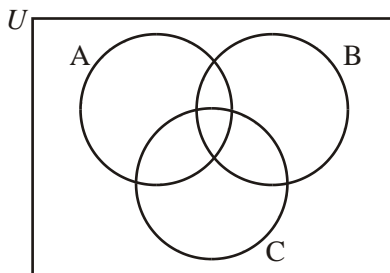
**(Total 13 marks)**

30. Shade the given region on the corresponding Venn Diagram.

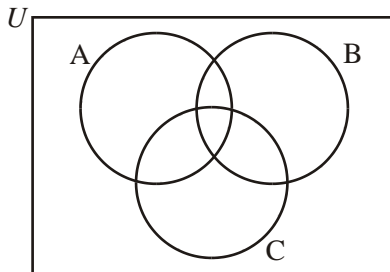
(a)  $A \cap B$



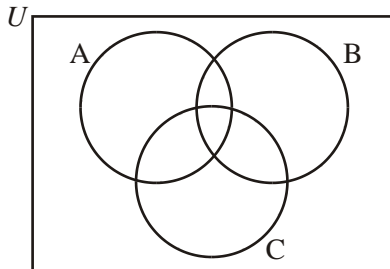
(b)  $C \cup B$



(c)  $(A \cup B \cup C)'$



(d)  $A \cap C'$



(Total 8 marks)

31. A school jazz band contains three different musical instruments — saxophone (S), clarinet (C) and drums (D). Students in the band are able to play one, two or three different instruments. In a class of 40 IB students, 25 belong to the jazz band. Out of these 25

- 3 can play all three instruments
- 5 can play the saxophone and clarinet **only**
- 5 can play **at least** the clarinet and drums
- 7 can play **at least** the saxophone and drums
- 16 can play the saxophone
- 12 can play the clarinet.

- (a) Draw a Venn Diagram and clearly indicate the numbers in each region. (5)
  
- (b) Show that the number of students who play the drums **only** is 5. (2)
  
- (c) Find the probability that a student chosen at random from the IB class plays only the saxophone. (2)
  
- (d) Find the probability that a student chosen at random from the IB class plays either the clarinet or drums or both. (2)
  
- (e) Given that a student plays the saxophone, find the probability that he also plays the clarinet. (3)

**(Total 14 marks)**



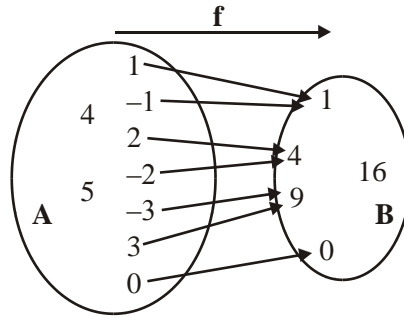
32. Given  $\mathbb{Z}$  the set of integers,  $\mathbb{Q}$  the set of rational numbers,  $\mathbb{R}$  the set of real numbers.

- (a) Write down an element that belongs to  $\mathbb{R} \cap \mathbb{Z}$ .
- (b) Write down an element that belongs to  $\mathbb{Q} \cap \mathbb{Z}'$ .
- (c) Write down an element that belongs to  $\mathbb{Q}'$ .
- (d) Use a Venn diagram to represent the sets  $\mathbb{Z}$ ,  $\mathbb{Q}$  and  $\mathbb{R}$ .

<p><i>Working:</i></p>	<p><i>Answers:</i></p> <p>(a) .....</p> <p>(b) .....</p> <p>(c) .....</p> <p>(d) .....</p>
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**(Total 6 marks)**

33.



The diagram shows a function  $f$ , mapping members of set A to members of set B.

- (a) (i) Using set notation, write down all members of the domain of  $f$ .
- (ii) Using set notation, write down all members of the range of  $f$ .
- (iii) Write down the equation of the function  $f$ .

The equation of a function  $g$  is  $g(x) = x^2 + 1$ . The domain of  $g$  is  $\mathbb{R}$ .

- (b) Write down the range of  $g$ .

**(Total 6 marks)**

34. The universal set  $U$  is the set of integers from 1 to 20 inclusive.

$A$  and  $B$  are subsets of  $U$  where:  
 $A$  is the set of even numbers between 7 and 17.  
 $B$  is the set of multiples of 3.

List the elements of the following sets:

- (a)  $A$ ; (1)
- (b)  $B$ ; (1)
- (c)  $A \cup B$ ; (2)
- (d)  $A \cap B'$ . (2)

<i>Working:</i>	<i>Answers:</i> (a) ..... (b) ..... (c) ..... (d) .....
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**(Total 6 marks)**

35. A school offers three activities, basketball ( $B$ ), choir ( $C$ ) and drama ( $D$ ). Every student must participate in at least one activity.

16 students play basketball only.

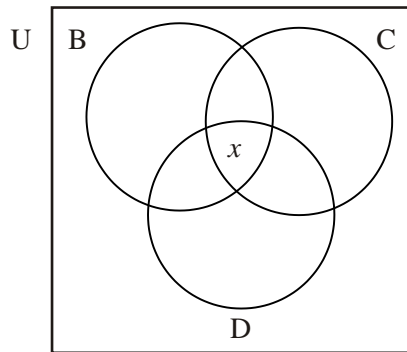
18 students play basketball and sing in the choir but do not do drama.

34 students play basketball and do drama but do not sing in the choir.

27 students are in the choir and do drama but do not play basketball.

(a) Enter the above information on the Venn diagram below.

(2)



99 of the students play basketball, 88 sing in the choir and 110 do drama.

(b) Calculate the number of students  $x$  participating in all three activities.

(1)

(c) Calculate the total number of students in the school.

(3)

*Working:*

*Answers:*

(b) .....

(c) .....

**(Total 6 marks)**