

Answers to Calculus Review Packet

1. (a) 30 (A1)

(b) $f'(x) = 3x^2 - 6x - 24$ (A1)(A1)(A1)

Note: Award (A1) for each term. Award at most (A1)(A1) if extra terms present.

(c) $f'(1) = -27$ (M1)(A1)(ft)(G2)

Note: Award (M1) for substituting $x = 1$ into their derivative.

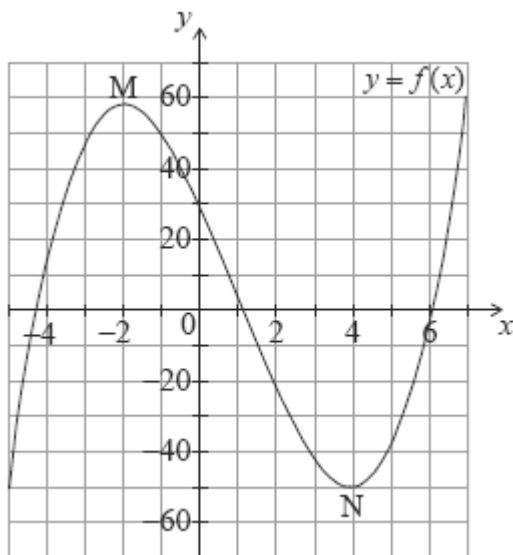
(d) (i) $f'(x) = 0$
 $3x^2 - 6x - 24 = 0$ (M1)
 $x = 4; x = -2$ (A1)(ft)(A1)(ft)

Notes: Award (M1) for either $f'(x) = 0$ or $3x^2 - 6x - 24 = 0$ seen. Follow through from their derivative. Do not award the two answer marks if derivative not used.

(ii) M(-2, 58) accept $x = -2, y = 58$ (A1)(ft)
 N(4, -50) accept $x = 4, y = -50$ (A1)(ft)

Note: Follow through from their answer to part (d) (i).

(e)



(A1) for window
 (A1) for a smooth curve with the correct shape
 (A1) for axes intercepts in approximately the correct positions
 (A1) for M and N marked on diagram and in approximately correct position (A4)

Note: If window is not indicated award at most (A0)(A1)(A0)(A1)(ft).

(f) (i) $3x^2 - 6x - 24 = 21$ (M1)
 $3x^2 - 6x - 45 = 0$ (M1)
 $x = 5; x = -3$ (A1)(ft)(A1)(ft)(G3)

Note: Follow through from their derivative.

OR

Award (A1) for L_1 drawn tangent to the graph of f on their sketch in approximately the correct position ($x = -3$), (A1)(ft)
 (A1) for a second tangent parallel to their L_1 , (A1)(ft)
 (A1) for $x = -3$, (A1) for $x = 5$. (A1)(A1)

*Note: If only $x = -3$ is shown without working award (G2).
 If both answers are shown irrespective of working award (G3).*

(ii) $f(5) = -40$ (M1)(A1)(ft)(G2)

*Notes: Award (M1) for attempting to find the image of their $x = 5$. Award (A1) only for $(5, -40)$.
 Follow through from their x -coordinate of B only if it has been clearly identified in (f) (i).*

[21]

2. (a) $2^2 \times p + 2q - 4 = -10$ (M1)

Note: Award (M1) for correct substitution in the equation.

$4p + 2q = -6$ or $2p + q = -3$ (A1)

Note: Accept equivalent simplified forms.

(b) (i) $\frac{dy}{dx} = 2px + q$ (A1)(A1)

*Note: Award (A1) for each correct term.
 Award at most (A1)(A0) if any extra terms seen.*

(ii) $4p + q = 1$ (A1)(ft)

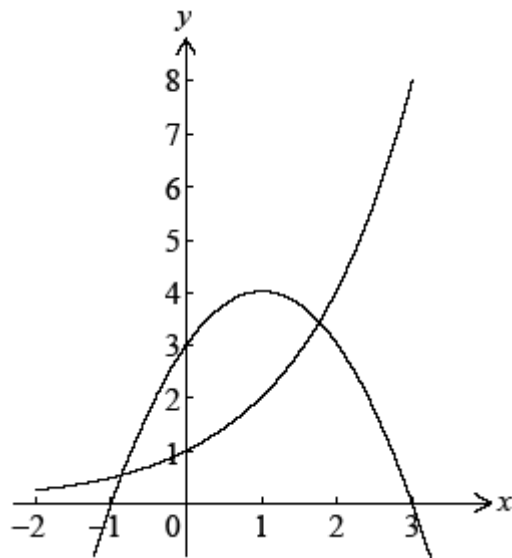
(c) $4p + 2q = -6$
 $4p + q = 1$ (M1)

Note: Award (M1) for sensible attempt to solve the equations.

$p = 2, q = -7$ (A1)(A1)(ft)(G3)

[8]

3. (a)



(A1)(A1)(A1)

Note: Award (A1) for correct domain, (A1) for smooth curve, (A1) for y-intercept clearly indicated.

(b) $y = 0$

(A1)(A1)

Note: Award (A1) for $y = \text{constant}$, (A1) for 0.

(c)

Note: Award (A1) for smooth parabola, (A1) for vertex (maximum) in correct quadrant. (A1) for all clearly indicated intercepts $x = -1$, $x = 3$ and $y = 3$. The final mark is to be applied very strictly. (A1)(A1)(A1)

(d) $x = -0.857$ $x = 1.77$

(G1)(G1)

Note: Award a maximum of (G1) if x and y coordinates are both given.

(e) 4

(G1)

Note: Award (G0) for (1, 4).

(f) $f'(x) = 2 - 2x$ (A1)(A1)

*Note: Award (A1) for each correct term.
Award at most (A1)(A0) if any extra terms seen.*

$2 - 2x = 0$ (M1)

Note: Award (M1) for equating their gradient function to zero.

$x = 1$ (A1)(ft)

$f(1) = 3 + 2(1) - (1)^2 = 4$ (A1)

Note: The final (A1) is for substitution of $x = 1$ into $f(x)$ and subsequent correct answer. Working must be seen for final (A1) to be awarded.

[16]

4. (a) $f'(x) = 4x + 1$ (A1)(A1)(A1) (C3)

*Note: Award (A1) for each term differentiated correctly.
Award at most (A1)(A1)(A0) if any extra terms seen.*

(b) $f'(-3) = -11$ (A1)(ft) (C1)

(c) $4x + 1 = 0$ (M1)

$x = -\frac{1}{4}$ (A1)(ft) (C2)

[6]

5. (a) (i) $y = 0$ (A1)

(ii) $(0, -2)$ (A1)(A1)

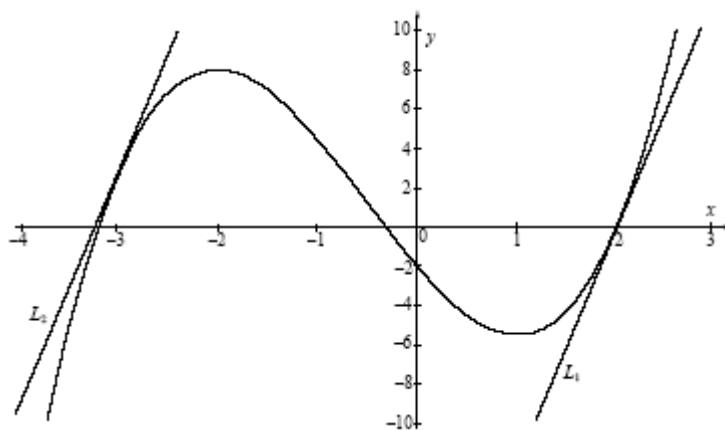
Note: Award (A1)(A0) if brackets missing.

OR

$x = 0, y = -2$ (A1)(A1)

*Note: If coordinates reversed award (A0)(A1)(ft).
Two coordinates must be given.*

(b)



(A4)

Note: (A1) for appropriate window. Some indication of scale on the x-axis must be present (for example ticks). Labels not required. (A1) for smooth curve and shape, (A1) for maximum and minimum in approximately correct position, (A1) for x and y intercepts found in (a) in approximately correct position.

(c) $\frac{dy}{dx} = 3x^2 + 3x - 6$ (A1)(A1)(A1)

Note: (A1) for each correct term. Award (A1)(A1)(A0) at most if any other term is present.

(d) (i) $3 \times 4 + 3 \times 2 - 6 = 12$ (M1)(A1)(AG)

Note: (M1) for using the derivative and substituting $x = 2$. (A1) for correct (and clear) substitution. The 12 must be seen.

(ii) Gradient of L_2 is 12 (can be implied) (A1)

$3x^2 + 3x - 6 = 12$ (M1)

$x = -3$ (A1)(G2)

Note: (M1) for equating the derivative to 12 or showing a sketch of the derivative together with a line at $y = 12$ or a table of values showing the 12 in the derivative column.

(iii) (A1) for L_1 correctly drawn at approx the correct point (A1)

(A1) for L_2 correctly drawn at approx the correct point (A1)

(A1) for 2 parallel lines (A1)

Note: If lines are not labelled award at most (A1)(A1)(A0).

Do not accept 2 horizontal or 2 vertical parallel lines

(e) (i) $b = 1$ (G2)

(ii) The curve is decreasing. (A1)

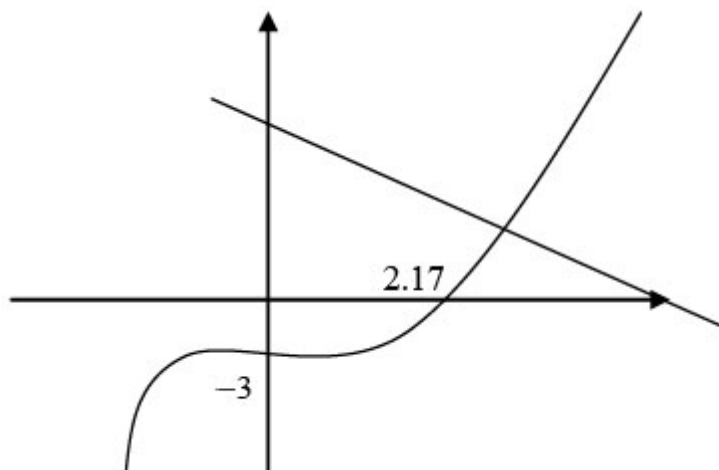
Note: Accept any valid description.

(iii) $y = 8$ (A1)(A1)(G2)

Note: (A1) for “y = a constant”, (A1) for 8.

[23]

6. (a)



(G3) 3

(b) line drawn with -ve gradient and +ve y-intercept
(2.45, 2.11)

(G1)

(G1)(G1) 3

(c) $f(1.7) = 3(1.7)^2 - 4(1.7) + 1$

(M1) 2

Note: Award (M1) for substituting in their $f(x)$

2.87

(A1)(G2)

[8]

7. (a) for attempt at substituted $\frac{y \text{ distance}}{x \text{ distance}}$

(M1)

gradient = 2

(A1)(G2) 2

(b) $2x - 3$

(A1)(A1) 2

(A1) for $2x$, (A1) for -3

(c) for their $2x - 3 =$ their gradient and attempt to solve

(M1)

$x = 2.5$

(A1)(ft)

$y = -5.25$ ((ft) from their x value)

(A1)(ft)(G2) 3

- (d) *for seeing* $\frac{-1}{\text{their}(a)}$ solving $2x - 3 = -\frac{1}{2}$ (*or their value*) (M1)(M1)
 $x = 1.25$ (A1)(ft)(G1)
 $y = -6.1875$ (A1)(ft)(G1) 4
- (e) (i) $2 \times 2 - 3 = 1$ ((ft) *from (b)*) (A1)(ft)(G1)
(ii) $y = mx + c$ *or equivalent method to find c* $\Rightarrow -6 = 2 + c$ (M1)
 $y = x - 8$ (A1)(ft)(G2) 3
- (f) $x = 1.5$ (A1) 1
- (g) *for substituting their answer to part (f) into the equation of the parabola* (M1)
 $(1.5, -6.25)$ *accept* $x = 1.5, y = -6.25$ (A1)(ft)(G2)
gradient is zero (*accept* $\frac{dy}{dx} = 0$) (A1) 3

[18]

8. (a) $f(x) = ax^2 + 4x^{-1} - 3$
 $f'(x) = 2ax - 4x^{-2}$ (A3) (C3)
Note: (A1) for $2ax$, (A1) for $-4x^{-2}$ and (A1) for derivative of -3 being zero.
- (b) $2ax - 4x^{-2} = 0$ (M1)
 $2a(-1) - 4(-1)^{-2} = 0$ (M1)
 $-2a - 4 = 0$
 $a = -2$ (A1)(ft) (C3)
*Notes: (M1) for setting derivative function equal to 0
(M1) for inserting $x = -1$ but do not award (M0)(M1)*

[6]

9. (a) $y = x + z + z$ (M1)

Note: Award (M1) for writing a sensible equation.

$$xz = 162$$

$$z = \frac{162}{x} \quad \text{(M1)}$$

$$y = x + \frac{2 \times 162}{x} \quad \text{(M1)}$$

$$y = x + \frac{324}{x} \quad \text{(AG)}$$

(b) $\frac{dy}{dx} = 1 - \frac{324}{x^2}$ (A1)(A1)(A1)

Note: Award (A1) for 1 and no other constant present, (A1) for -324 , (A1) for $\frac{1}{x^2}$ or x^{-2} .

(c) $\frac{dy}{dx} = 0$

$$1 - \frac{324}{x^2} = 0 \quad \text{(M1)}$$

Note: Award (M1) for putting candidate's derivative equal to zero.

$$x^2 = 324 \quad \text{(A1)(ft)}$$

$$x = 18 \quad \text{(A1)(ft)(G3)}$$

(d) $y = 18 + 9 + 9$ (M1)

Note: Award (M1) for adding three sides of rectangle.

$$= 36 \quad \text{(A1)(ft)(G2)}$$

OR

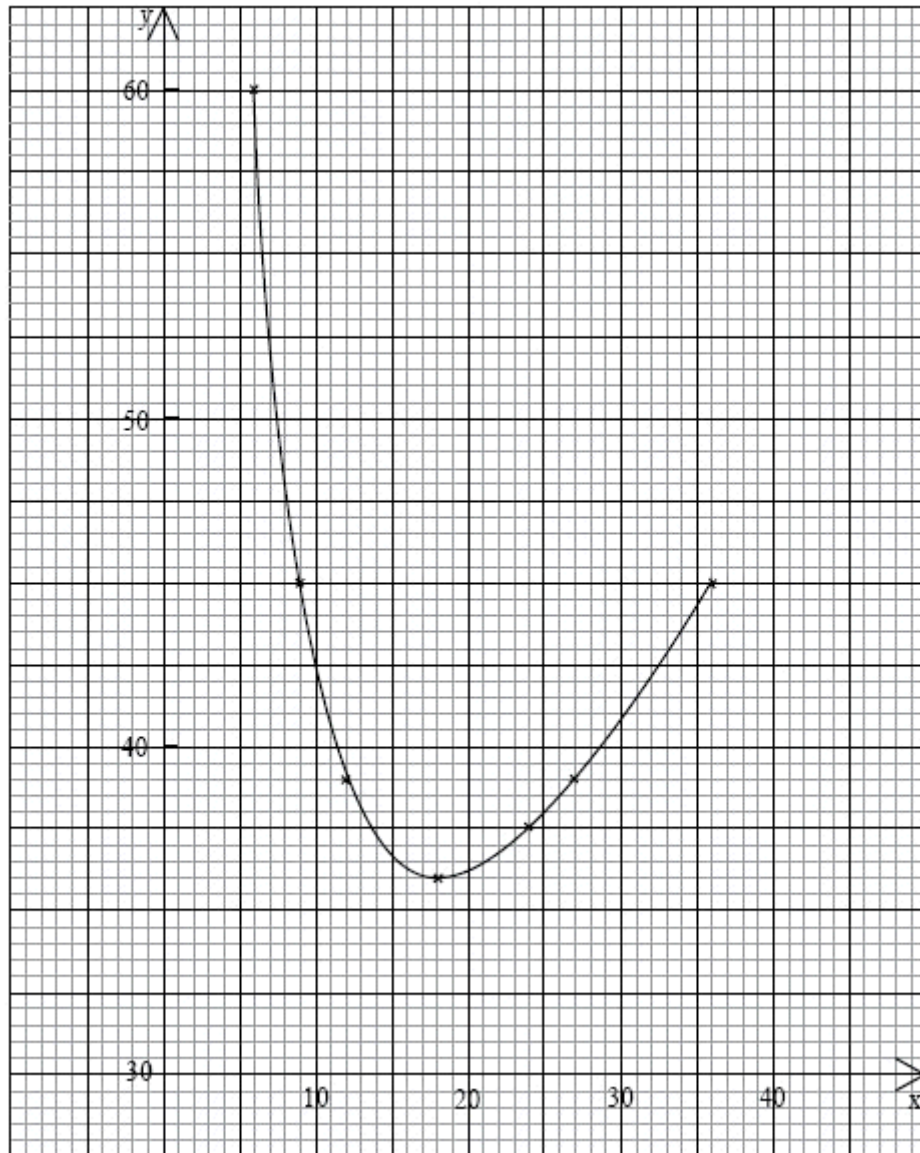
$$18 + \frac{324}{18} \quad \text{(M1)}$$

$$= 36 \quad \text{(A1)(ft)}$$

(e) $a = 36$ (A1)

$$b = 39 \quad \text{(A1)}$$

(f)



(A5)(ft)

Notes: Award (A1) for correct scales and labels, (A3) for correct points plotted, (A1) for smooth curve with (18, 36) as the minimum value.

Award (A2) for 5 or 6 points correctly plotted, (A1) for 3 or 4 points correctly plotted.

(g) $x \geq 18$

(A1)(A1)(ft)

Notes: Award (A1) for $x \geq$, (A1) for 18.

Accept $x > 18$

[20]

10. (a) $2x^3 - 2x$ (A1) (C1)

(b) $f'(x) = 6x^2 - 2$ (A1)(ft)(A1)(ft) (C2)

Note: Award (A1) for each term.

(c) gradient = $f'(-1)$
 $= 6(-1)^2 - 2$
 $= 4$ (M1)
(A1)(ft) (C2)

(d) $\tan \theta = 4$ (A1)(ft) (C1)

[6]

11. (a) $f(1) = \frac{3}{1^2} + 1 - 4$ (M1)
 $= 0$ (A1)

OR
 $f(1) = 0$ (G2) 2

(b) $f'(x) = -\frac{6}{x^3} + 1$ (A4) 4

*Note: Award (A2) for $\frac{3}{x^2}$ correctly differentiated
and (A1) for each other term correctly differentiated.*

(c) $f'(1) = -\frac{6}{1} + 1$ for substituting $f'(x)$ (M1)
 $= -5$ (A1)

OR
 $f'(1) = -5$ (G2) 2

(d) The gradient of the curve where $x = 1$. (A2) 2

Note: Award (A1) for gradient and (A1) for $x = 1$ or at point $(1, 0)$.

- (e) $y = 0, x = 1, m = -5$ for using $y = mx + c$ with their correct values of m, x and y . (M1)
 $0 = -5 \times 1 + c$ (A1)
 $c = 5$ (A1)
 $y = -5x + 5$ (A1)
- OR**
- $y = -5x + 5$ (G3) 3

- (f) $f'(x) = 0$
 $1 - \frac{6}{x^3} = 0$ (M1)(A1)
 $x^3 = 6$
 $x = \sqrt[3]{6} \text{ (1.82)}$ (A1)
- OR**
- 1.82 (G3) 3

[16]

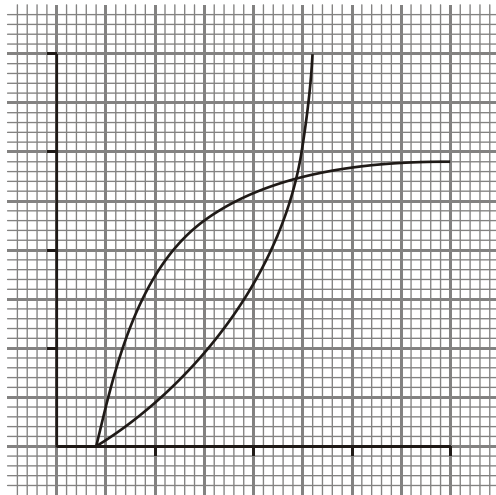
12. (a) $f'(x) = 2 + 25x^{-2}$ (A2) (C2)
- (b) $2 + 25x^{-2} = 6$ (M1)
 $25 = 4x^2$ (M1)
 $x^2 = \frac{25}{4}$
 $x = \pm 2.5$ (A1)(A1) (C4)

[6]

13. (a) $2x + 3$ (-1 for each extra term) (A2) (C2)
Note: If correct and an extra term included, award (A1) only.
- (b) Equating the gradient to 5 ($2x + 3 = 5$) (M1)
For solving attempt (M1)
For $x = 1$ (A1)
Co-ordinates (1, 2) (A1) (C4)

[6]

14. (a)



- For correct axes from 0 to 4. (A1)
 For correct curve $y = x^2$. (A1)
 For correct curve $y = 3 - \frac{1}{x}$. (A1)
 For two intersections. (A1) 4

- (b) (0.347, 0.121) or $x = 0.347, y = 0.121$ (by GDC) (G1)(G1)
 (1.53, 2.35) or $x = 1.53, y = 2.35$. (G1)(G1) 4

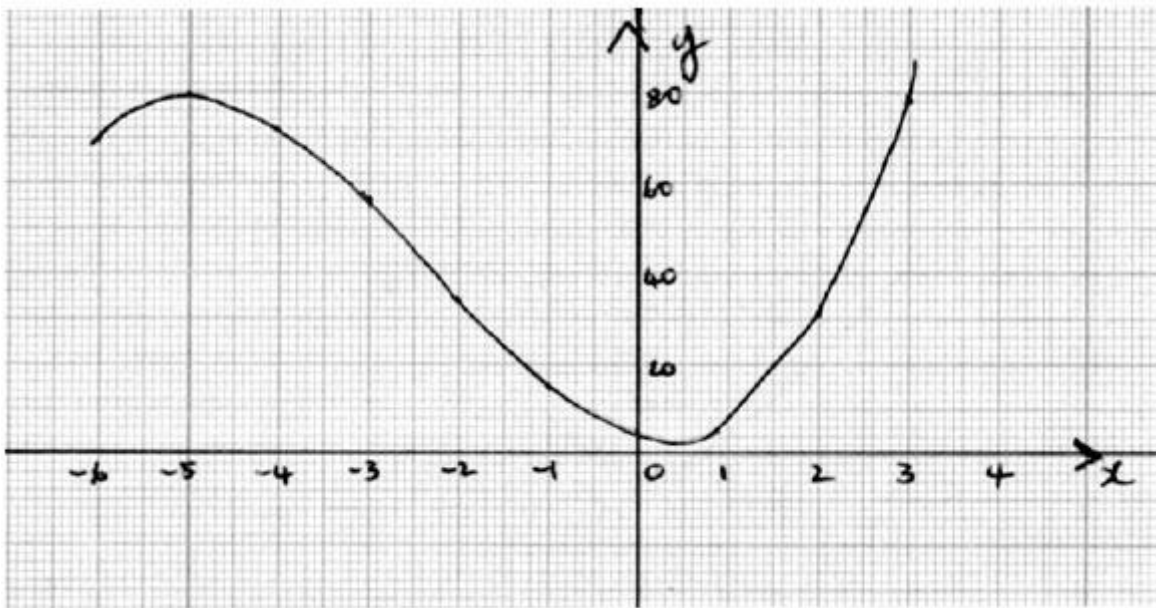
- (c) (i) $\frac{dy}{dx} = \frac{1}{x^2}$ for losing the constant. (A1)
 For attempting to write $\frac{1}{x}$ as a power (can be implied). (M1)
 For correct answer $\frac{1}{x^2}$ or x^{-2} . (A1)
- (ii) 1 (A1) 4

- (d) For using $y = mx + c$ or equivalent with their m , to find c . (M1)
 $c = 1$ (A1)
 $y = x + 1$ (A1) 3

[15]

15. (a) $f'(x) = 3x^2 + 14x - 5$ (A1)(A1)(A1) 3
- (b) $f'(1) = 3 + 14 - 5 = 12$ (M1)(A1) 2

- (c) $3x^2 + 14x - 5 = 0$ (M1)
 $(3x - 1)(x + 5) = 0$
 $x = \frac{1}{3}$ or -5 (A1)(A1) (or (G3)) 3
- (d) $\left(\frac{1}{3}, 3.15\right)$ $(-5, 79)$ (A1)(A1) (or (G2)) 2
- (e) (A4) 4



Note: Award (A1) for axes labelled, (A1) for maximum, (A1) for minimum, (A1) for y-intercept.

[14]

16. (a) $V = x^2h$ (A1) 1
- (b) $A = 2x^2 + 4xh$ (A1) 1
- (c) $1000 = x^2h$ (M1)
 $h = \frac{1000}{x^2}$ (A1) 2

$$(d) \quad A = 2x^2 + 4x\left(\frac{1000}{x^2}\right) \quad (M1)$$

$$A = 2x^2 + \frac{4000}{x} \quad (A1)$$

$$= 2x^2 + 4000x^{-1} \quad (AG) \quad 2$$

$$(e) \quad \frac{dA}{dx} = 4x - 4000x^{-2} \quad (A2) \quad 2$$

$$(f) \quad 4x - 4000x^{-2} = 0 \quad (M1)$$

$$4x^3 - 4000 = 0 \quad (M1)$$

$$4x^3 = 4000$$

$$x^3 = 1000 \quad (A1)$$

$$x = 10 \quad (A1)$$

OR

$$x = 10 \quad (G4) \quad 4$$

$$(g) \quad h = \frac{1000}{100} = 10 \quad (A1)$$

$$A = 2(100) + 4(10)(10) \quad (M1)$$

$$= 200 + 400 = 600 \quad (A1)$$

OR

$$A = 600 \quad (G3) \quad 3$$

[15]

17. (a) $3x^{-2}$ (A1) (C1)

Note: Award mark for -2 .

(b) $-2 \times 3x^{-3}$ (A1)(A1)

Note: Award (A1) for -2×3 , (A1) for -3 .

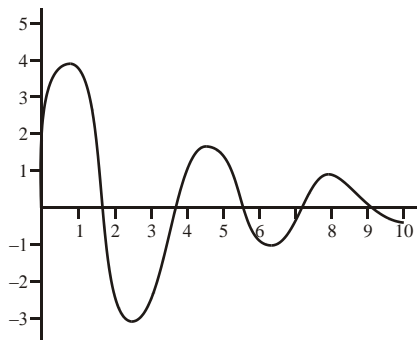
$$= -6x^{-3} \quad (A1)$$

$$= -\frac{6}{x^3} \quad (A1)(A1) \quad (C5)$$

Note: Award (A1) for positive power on denominator, (A1) for 3.

[6]

18. (a)



For labels and scales.

3 maxima drawn.

2 minima drawn.

General shape

(A1)

(A1)

(A1)

(A2)

5

(b) (0.827, 4.12)

(G2)

2

(c) 0, 1.8, 3.6, 5.4, 7.2, 9 (for any one of these answers).

(G1)

1

(d) $r = 1$

Perfect positive correlation.

(G2)

(R1)

3

(e) $y = 3x$ (accept $y = 3x + 0.000274$)

(G2)

2

(f) line on graph

(A1)

1

(g) (0, 0) or (1.16, 3.48)

(G1) (G1)

2

[16]

19. (a) $x - 15$

(A1)

1

(b) Profit = $(x - 15)(100\,000 - 4000x)$

(M1)

$$= 100000x - 4000x^2 - 1500\,000 + 60\,000x$$

(A2)

Note: Award (A1) for one error, (A0) for 2 or more errors.

$$= 160\,000x - 4000x^2 - 1500\,000$$

(AG)

3

- (c) (i) $\frac{dP}{dx} = 160000 - 8000x$ (A1)(A1)
- (ii) $0 = 160000 - 8000x$ (M1)
- $x = \frac{160000}{8000}$
- $x = 20$ (A1) 4
- (d) Books sold = $100\,000 - 4000 \times 20$ (M1)
- $= 20000$ (A1)

OR

Books = 20 000 (A2) 2

[10]

20. (a) $y = ax^2 + bx + 6$ (A1)(A1) 2
- $\frac{dy}{dx} = 2ax + b$
- (b) Gradient = 2 when $x = 6$.
- Therefore, $2 = 2a \times 6 + b$ (M1)
- $2 = 12a + b$ (A1) 2
- (c) $y = -15$ when $x = 3$.
- Therefore, $-15 = 9a + 3b + 6$
- or $-21 = 9a + 3b$ or $-7 = 3a + b$ (M1)(A1) 2

[6]

21. (a) $B \rightarrow D, G \rightarrow L$ (or $G \rightarrow K$ and $K \rightarrow L$) (both correct) (accept C, H, L) (A1) 1
- (b) $A \rightarrow B, D \rightarrow G$ (both correct) (accept A, E, F) (A1) 1
- (c) D (A1) 1
- (d) B or G (accept either) (A1) 1
- (e) Point of inflexion (A1) 1

[5]

22. (a) $g'(x) = 4x^3 + 9x^2 + 4x + 1$ (A3) 3

Note: Award (A3) for all five terms correctly differentiated, (A2) for four terms, (A1) for three terms, (A0) for two or less terms correctly differentiated.

(b) $g'(1) = 4(1)^3 + 9(1)^2 + 4(1) + 1$ (M1)
 $= 4 + 9 + 4 + 1$
 $= 18$ (A1)

OR

18 (G2) 2

[5]

23. (a) $g'(x) = 3x^2 + 12x + 12$ (A3) 3

(b) $3x^2 + 12x + 12 = 0$
 $x^2 + 4x + 4 = 0$ (M1)
 $(x + 2)^2 = 0$
 $x = -2$ (A1)
 or (G2) 2

(c) (i) $x = -3 \Rightarrow \frac{dy}{dx} = 3$ (A1)

(ii) $x = 0 \Rightarrow \frac{dy}{dx} = 12$ (A1)

(iii) (a) Increasing (A1)

(b) Increasing (A1) 4

[9]

24. (a) $x = 3$ (A1)
 $x = 5$ (A1)
 $x = 6.8 - 7.2$ (A1) 3

(b) $3 < x < 5$ (A1)(A1) 2

[5]

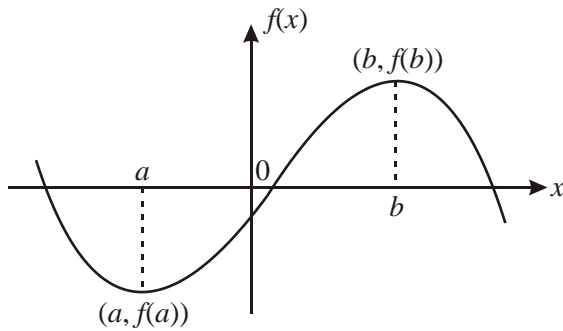
25. (a) (i) $f'(x) = 6x^2 - 6x - 12 (+0) = 6x^2 - 6x - 12$ (A2)
- Note: Award (A2) for all four items correctly differentiated,
(A1) for 3 correct derivatives.*
- (ii) $f'(3) = 6(3)^2 - 6(3) - 12 = 24$ (M1) (A1) 4
- (b) $6x^2 - 6x - 12 = -12$ (M1)
 $\Rightarrow 6x^2 - 6x = 0$
 $\Rightarrow 6x(x - 1) = 0$
 $\Rightarrow x = 0$ or $x = 1$ (A1) (A1) 3
- (c) (i) $f'(x) = 0 \Rightarrow 6x^2 - 6x - 12 = 0$ (M1)
 $\Rightarrow 6(x^2 - x - 2) = 0$
 $\Rightarrow 6(x - 2)(x + 1) = 0$ (M1)
 $\Rightarrow x = 2$ or $x = -1$ (A1) (A1)
- (ii) $x = 2, y = -15$ (A1)
Therefore, minimum is $(2, -15)$ (A1) 6
- (d) $x < -1$ and $x > 2$ (A1) (A1) 2

[15]

26. (a) $s(50) = 250(50) + 5(50)^2 - 0.06(50)^3$
 $= 17500$ m (A1) 1
- (b) (i) 1 second = $\frac{1}{60}$ minute (M1)
50 m and 1 sec = $50 \frac{1}{60}$ minute (AG)
- (ii) $s\left(50 \frac{1}{60}\right) = 250\left(50 \frac{1}{60}\right) + 5\left(50 \frac{1}{60}\right)^2 - 0.06\left(50 \frac{1}{60}\right)^3$ (M1)
 $= 17505$ (A1)
- (iii) $\frac{17505 - 17500}{\frac{1}{60}} = \frac{5}{\frac{1}{60}}$ (M1)
 $= 300$ m per minute (A1) 5

[6]

27. (a)



(A2) 2

Note: The curve need not be exactly like this one. The candidate's sketch must have $(a, f(a))$ as a minimum with $a < 0$, and $(b, f(b))$ as a maximum with $b > 0$. The turning points do not need to be on opposite sides of the x -axis.

- (b) (i) False (A1)
 (ii) True (A1)
 (iii) False (A1)
 (iv) True (A1)
 (v) False (A1) 5

[7]

28. (a) (i) $l = 24 - 2x$ (A1)
 (ii) $w = 9 - 2x$ (A1) 2

(b) $B = x(24 - 2x)(9 - 2x)$ (M1)
 $= 4x^3 - 66x^2 + 216x$ (AG) 1

(c) $\frac{dB}{dx} = 12x^2 - 132x + 216$ (A1) 1

(d) (i) $\frac{dB}{dx} = 0 \Rightarrow x^2 - 11x + 18 = 0$
 $(x - 2)(x - 9) = 0$ (M1)
 $\Rightarrow x = 2$ or $x = 9$ (not possible)
 Therefore, $x = 2$ cm. (A1)

(ii) $B = 4(2)^3 - 66(2)^2 + 216(2)$ (or $2 \times 20 \times 5$) (M1)
 $= 200 \text{ cm}^3$ (A1) 4

[8]

29. (a) $g'(x) = 2px + q$ (A1) 1

(b) $2px + q = 2x + 6$
 $\Rightarrow p = 1$ and $q = 6$ (A1)(A1) 2

(c) (i) $g'(x) = 0$
 $\Rightarrow 2x + 6 = 0$ (M1)
 $\Rightarrow x = -3$ (A1)

(ii) $-12 = (-3)^2 + 6(-3) + c$
 $-12 = 9 - 18 + c$ (M1)
 $\Rightarrow c = -3$ (A1) 4

[7]

30. (a) $\frac{dy}{dx} = 0$ at point C (A1) 1

(b) $\frac{dy}{dx}$ changes from +ve to -ve and is decreasing (A2) 2

*Notes: Award (A1) for “+ve to -ve” and, (A1) for “decreasing”.
 Accept equivalent answers, e.g. “decreasing, becomes zero, and then begins to increase negatively”.*

[3]

31. (a) $AE^2 + OE^2 = OA^2$ (3)
 $\Rightarrow AE^2 + k^2 = 8^2$ (M1)

Note: Award (M1) for using and substituting correctly in equation (3).

$AE^2 = \sqrt{64 - h^2}$ (A1) 2

(b) Volume (V) = $2h\pi r^2$ (M1)
 $= 2\pi h(AE^2)$ (M1)
 $= 2\pi h(64 - h^2) \text{ cm}^3 \dots\dots\dots (4)$ (AG) 2

(c) (i) From (b) $V = 128\pi h - 2\pi h^3$ (M1)
Note: Award (M1) for using equation (4) or any other correct approach.

$\frac{dV}{dh} = 128\pi - 6\pi h^2 = 0$ at maximum/minimum points (M2)

Note: Award (M2) for correctly differentiating V w.r.t. x .

$\Rightarrow h = \sqrt{\frac{64}{3}} = \pm 4.62 \text{ cm (3 s.f.)}$ (A1)

Test to show that V is maximum when $h = 4.62$ (R1) 5

Note: Award (R1) for testing to confirm V is indeed maximum.

(ii) $AE^2 = 64 - h^2$
 $= 64 - \frac{64}{3} = \frac{128}{3}$ (M1)

Notes: Follow through with candidate's AE from part (a) (M1) is for correctly obtaining candidate's AE^2 .

Therefore maximum volume = $\pi r^2(2h) = \pi \left(\frac{128}{3}\right) \left(2\left(\sqrt{\frac{64}{3}}\right)\right)$ (M1)

Note: Follow through with candidate's AE^2

$= 1238.7187\dots = 1239 \text{ cm}^3$ (nearest cm^3) (A1) 3

Notes: Correct answer only.

Accept 1238 cm^3 if and only if candidate uses $\pi = 3.14$

[12]

32. (a)

PR	0.1	0.01	0.001
QN	2.42	2.0402	2.004002
QR	0.42	0.0402	0.004002
Gradient $\frac{QR}{PR}$	4.2	4.02	4.002

(A4) 4

Note: Award $\frac{1}{2}$ mark for each correct entry. Round up to a maximum of 4 marks.

(b) (i) From (a) as $Q \rightarrow P$, $PR \rightarrow 0$ and $\frac{QR}{PR} \rightarrow 4$,
hence gradient = 4 at (1, 2) (A1) 1

Note: Accept gradient (slope) = 4.

(ii)

PR	0.1	0.01	0.001	
QN	5.42	5.0402	5.004002	
QR	0.42	0.0402	0.004002	
Gradient $\frac{QR}{PR}$	4.2	4.02	4.002	(M2)

Hence $\frac{d}{dx}(2x^2 + 3) = 4$ at $(1, 5)$ (A1) 3

Notes: Award (M2) for any correct method towards conclusion.

Accept $\frac{d}{dx}(2x^2 + 3) = 4x$ (M1)

$= 4(1)$ at $(1, 5)$ (M1)

$= 4$ (A1)

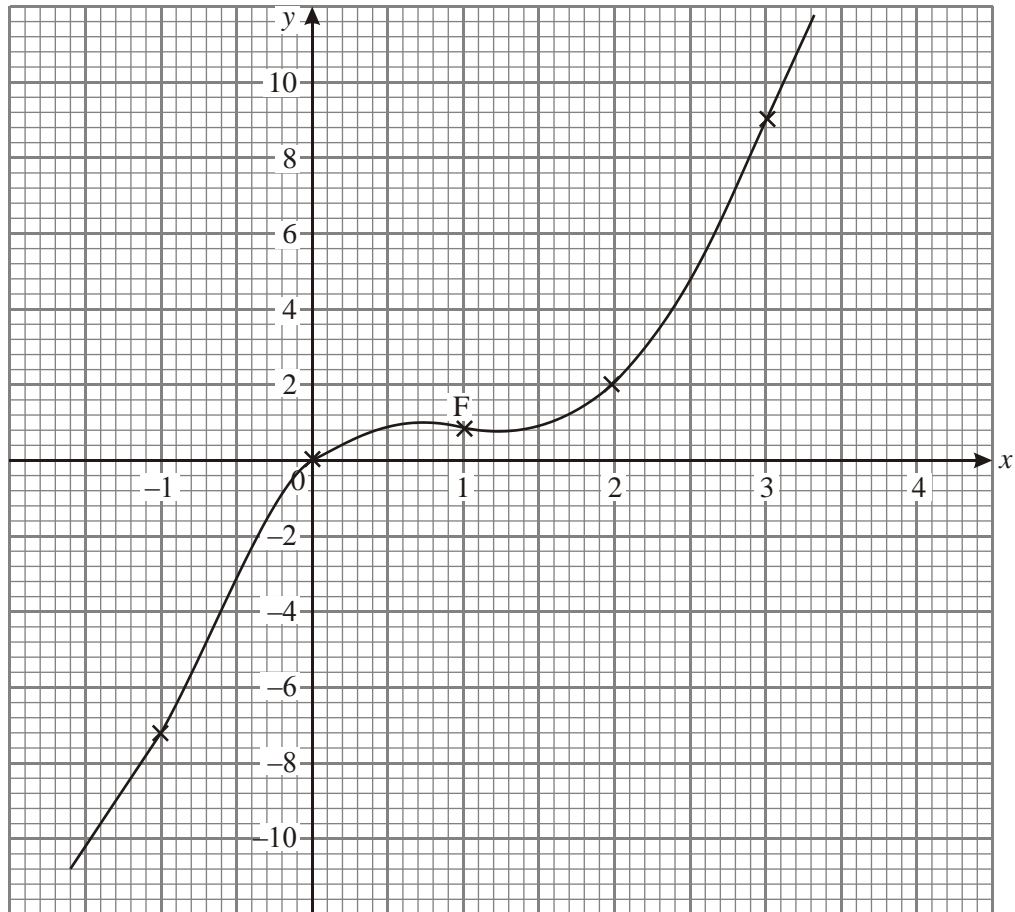
[8]

33. (a) $f'(x) = 3x^2 - 6x + 3$ (A2) 2

(b)

x	-1	0	1	2	3	
$f(x)$	-7	0	1	2	9	
$f'(x)$	12	3	0	3	12	(A3) 3

(c)



Note: The graph does not have to be on graph paper as long as it is reasonable. (A2) 2

(d) 12 (A1) 1

[8]

34. (a) $a = 2, b = 20, c = 9, d = 8, e = 32$ (A2) 2

Note: Award (A2) for all 5 correct, (A1) for 3 or 4 correct, (A0) for 2 or less correct.

(b) $A = 12x - x^2$ (C1) 1

(c) $\frac{dA}{dx} = 12 - 2x$ (A1)
 A is maximum when $12 - 2x = 0$ (M1)
 \Rightarrow length = 6m and width = 6m (A1)
OR
length = 6m and width = 6m (A2) 3

[6]