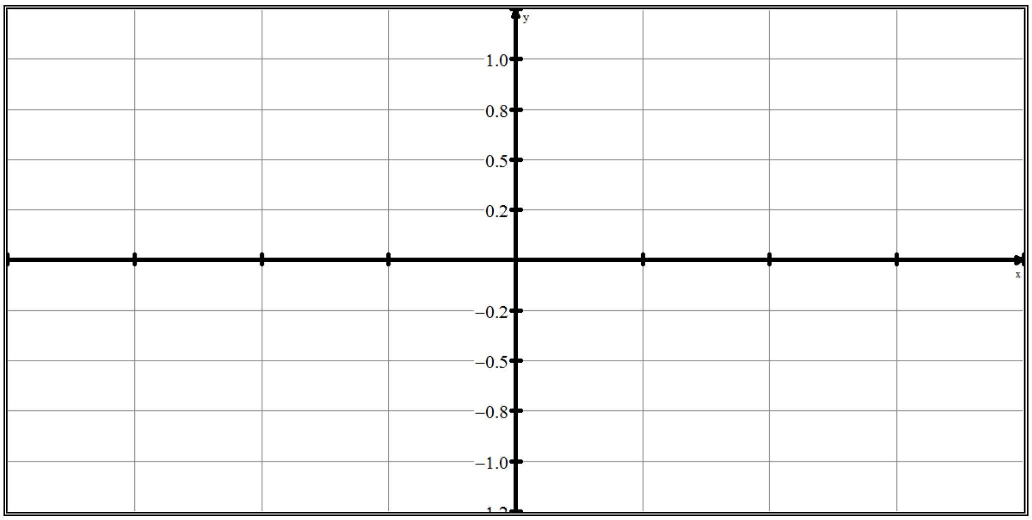
(A) Lesson Objectives

a. Work with the equation $f(x) = A\sin(B(x - C)) + D$ in the context of word problems using the TI-84 and graphic approaches to answering application questions.

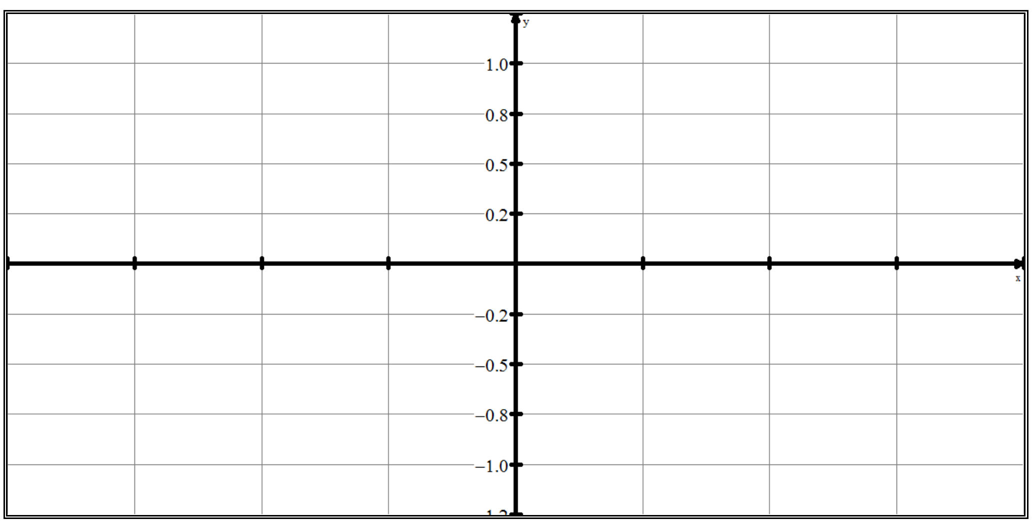
b. Use the graphing calculator to answer application questions involving the key terms related to periodic phenomenon (periodic, period, amplitude, axis of the curve (equilibrium axis)) and relate them back to the context of the problem/equation

(B) Review of Basics

a. Basic sinusoidal functions ➔ Graph and analyze 2 periods of $f(x) = \sin(x)$



b. Basic sinusoidal functions ➔ Graph and analyze 2 periods of $f(x) = \cos(x)$

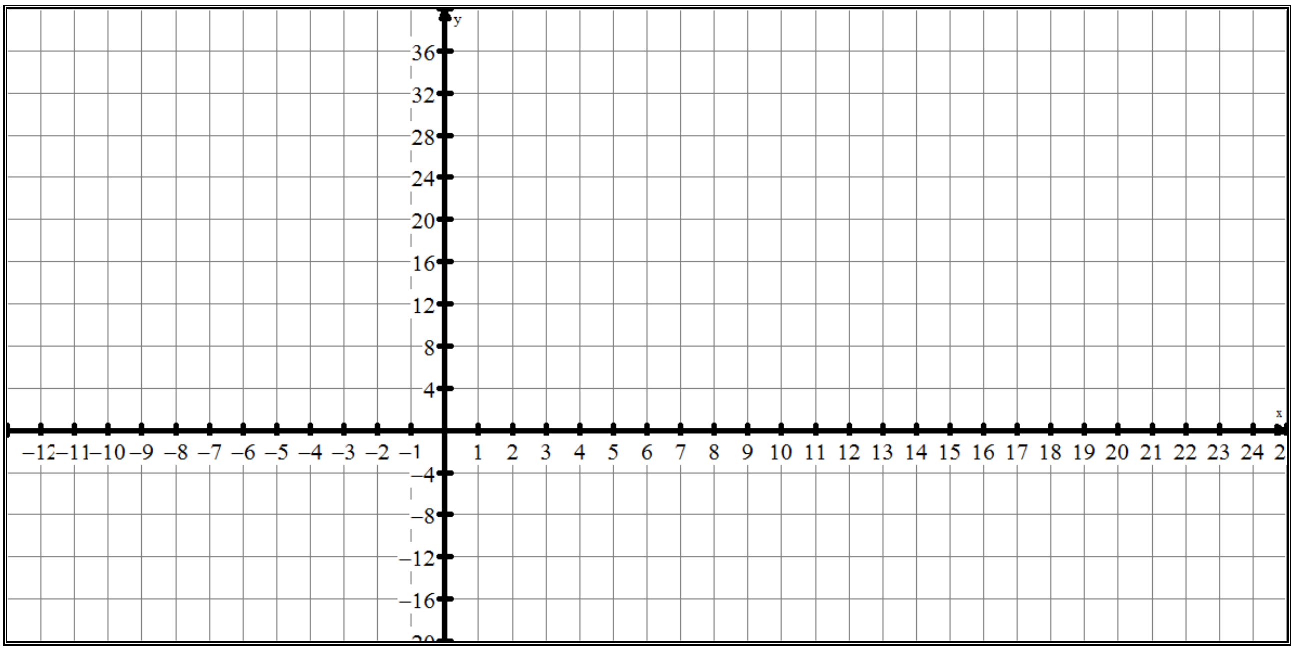


c. Basic Transformations of Sinusoidal Functions:

(C) In-Class Example to Work Through

The average monthly temperature, *T*, in degrees Celsius in the Kawartha Lakes was modelled by $T(t) = -22\cos(30t) + 10$, where *t* represents the number of months. For *t* = 0, the month is January; for *t* = 1, the month is February, and so on.

a. Sketch the graph from your GDC.



b. What is the period? Explain the period in the context of the problem.

c. What is the amplitude? Explain the amplitude in the context of the problem.

d. What is the maximum temperature? the minimum temperature?

e. What is the range of temperatures for this model?

f. What is the annual/yearly average temperature?

g. What is the predicted temperature on April 15th?

h. Evaluate $T(18.75)$ and explain the solution in the context of the problem.

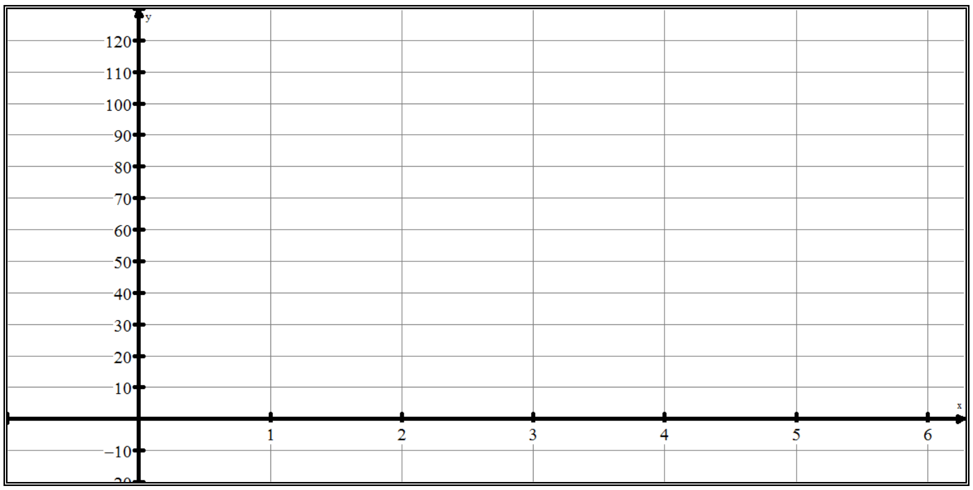
i. When will the temperature be predicted to be 12°?

j. Solve the equation $0 = -22\cos(30t) + 10$ and explain the solution in the context of the problem.

(D)In-Class Example

Each person's blood pressure is different. But there is a range of blood pressure values that is considered healthy. The function $P(t) = -20\cos(300t) + 100$ models the blood pressure, P, in millimetres of mercury, at time, t, in seconds of a person at rest.

a. Sketch the graph of $P(t) = -20\cos(300t) + 100$ for $0 \le t \le 6$.



b. What is the period of the function? What does the period represent for an individual?

c. What is the amplitude? Explain the amplitude in the context of the problem

d. How many times does this person's heart beat each minute?

e. What is the range of the function? Explain the meaning of the range in terms of a person's blood pressure.

f. What is the predicted blood pressure at 4 seconds of rest??

g. Evaluate $P(24)$ and explain the solution in the context of the problem.

h. When will the blood pressure be predicted to be 90 mm Hg?

i. Solve the equation $88 = -20\cos(300t) + 100$ and explain the solution in the context of the problem.

(E) In-Class Example

The function $D(t) = 4\sin\left[\frac{360}{365}(t-80)\right]^{\circ} + 12$ is a model of the number of hours of daylight, *D*, on a specific day, *t*, on the 50° of north latitude.

a. Explain why a trigonometric function is a reasonable model for predicting the number of hours of daylight.

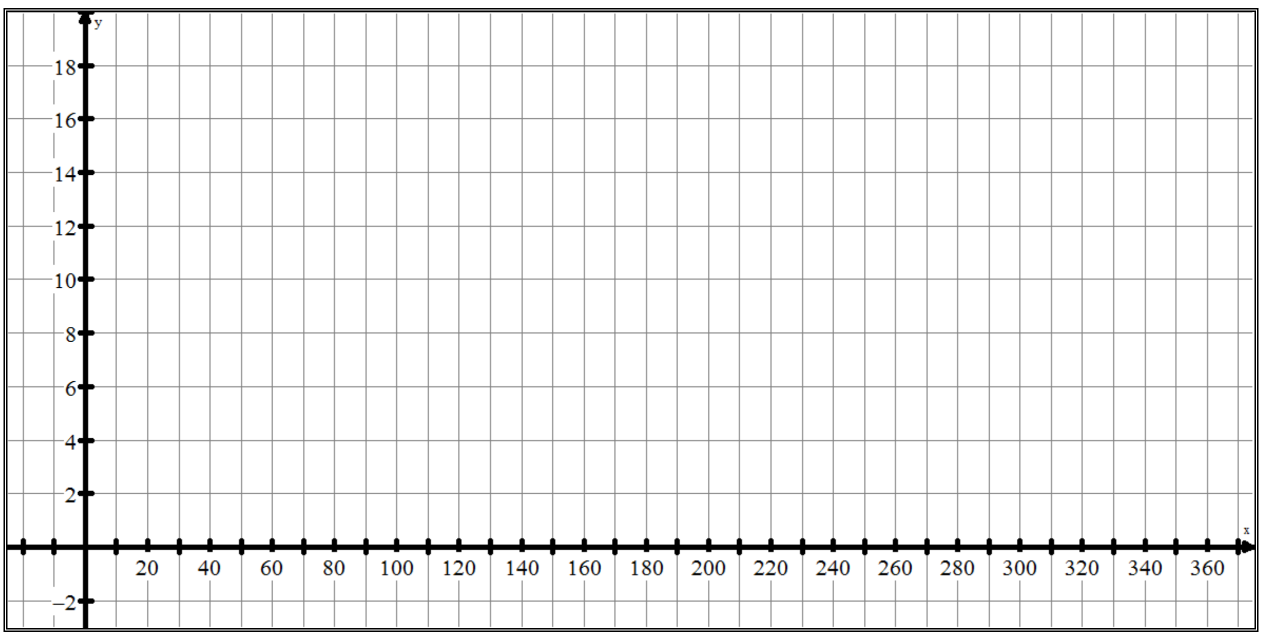
b. How many hours of daylight do March 21 and September 21 have? What is the significance of each of these days?

c. What is the significance of the number 80 in the model?

d. How many hours of daylight do June 21 and December 21 have? What is the significance of each of these days?

e. Explain what the number 12 represents in the model.

f. Graph the model.



g. What are the maximum hours of daylight? the minimum hours of daylight? On what days do these values occur?

h. Use the graph to determine *t* when $D(t) = 15$. What dates correspond to *t* ?

i. Evaluate $D(246)$ and explain the solution in the context of the problem.

Lesson 39 – Problem Solving with Periodic Phenomenon Date:

(F) Example

Thinking, Inquiry, Problem Solving: The population, *R*, of rabbits and the population, *F*, of foxes in a given region are modelled by the functions $R(t) = 10{,}000 + 5{,}000\cos(15t)$ and $F(t) = 1{,}000 + 500\sin(15t)$ where *t* is the time in months. Explain, referring to each graph, how the number of rabbits and the number of foxes are related by answering the following questions:

a. When does each population reach their maximum populations? Their minimum populations?
b. When does the rabbit population increase? How do you know? What is happening to the fox population in the same time interval?
c. Evaluate $R(5)$ as well as $F(5)$.
d. Unfortunately, a pathogenic bacteria gets introduced to the ecosystem. The rabbit population halves and now fluctuates by 2000 rabbits per period.
   i. Write a new equation to represent this situation.
   ii. What would happen to the fox population? Write a new equation to present a realistic model for the fox population. Explain your equation.

(G) Example

The average monthly temperature in a region of Australia is modelled by the function $T(t) = 9 + 23\cos(30t)$, where *T* is the temperature in degrees Celsius and *t* is the month of the year. For *t* = 0, the month is January.

a. Prepare a table for $0 \le t \le 13$.
b. Graph the data.
c. Explain how to use the axis of the curve and the amplitude to determine the maximum and minimum values of the function.
d. Determine the period of the function from the graph. Verify your answer algebraically.
e. Verify the graph in (b) by using a graphing calculator.
f. Explain how to sketch a similar graph using transformations of *y* = cos(t).

(H) Example

A skyscraper sways 55 cm back and forth from "the vertical" during high winds. At *t* = 5 s, the building is 55 cm to the right of vertical. The building sways back to the vertical and, at *t* = 35 s, the building sways 55 cm to the left of the vertical.Write an equation that models the motion of the building in terms of time.

(I) Example

The maximum height of a Ferris wheel is 35 m. The wheel takes 2 min to make one revolution. Passengers board the Ferris wheel 2 m above the ground at the bottom of its rotation.

a. Write an equation to represent the position of a passenger at any time, *t*, in seconds.
b. How high is the passenger after 45 s?
c. The ride lasts for 4 min. When will the passenger be at the maximum height during this ride?